Advanced rock mechanics

Semester 1, 2009

Mid-term Exam (20 April) 14:30 - 17:30

* Answer the questions in English.

1. Discuss the conditions in which the symmetry condition holds for stress and strain tensors (10).

2. Derive the equations of stress equilibrium using the divergence theorem (15).

3. Evaluate the following expressions involving the Kronecker delta δ_{ij} for a range of three on the indices (10).
   (a) $\delta_{ij}\delta_{ij}$
   (b) $\delta_{ij}\delta_{ik}\delta_{jk}$
   (c) $\delta_{ij}A_{ik}$

4. Derive compatibility equation in two dimensions and explain the physical meaning of the compatibility equation (15).

5. Derive the Navier's Equations, which is the equations of stress equilibrium expressed in terms of displacements (15).

6. Explain the Saint Venant's Principle (10).

7. The relationship between horizontal and vertical stress can be expressed as $\sigma_H = \frac{\nu}{1-\nu}\sigma_V$ if we assume uniaxial strain condition. Derive this equation from the constitutive equations of isotropic material. Using this equation and assuming that a rock element is subjected to $\sigma_V = \sigma_H$ at a depth of 1000 m and erosion causes a removal of 500 m of overburden over millions of years, determine the stress state at a depth of 500m and ratio of horizontal to vertical stress after erosion. The density and Poisson's ratio of rock is 2600 kg/m^3 and 0.25, respectively (15).

8. Number of independent elastic constants for anisotropic material reduces from 81 to 21. Explain how the reduction is done (10).

* Reference equations are listed below, however, feel free to use other equations that may be relevant in answering the questions.

$$T_i = \tau_{ji} n_j$$

$$\tau_{ij}' = \beta_{im} \beta_{jn} \tau_{mn}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$\tau_{ji,j} + F_i = 0$$

$$\varepsilon_{ij} = \frac{1+\nu}{E}\tau_{ij} - \frac{\nu}{E}\delta_{ij}\tau_{kk}$$

$$\tau_{ij} = \lambda \varepsilon_{\alpha\alpha}\delta_{ij} + 2\mu\varepsilon_{ij}$$

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + F_i = 0$$