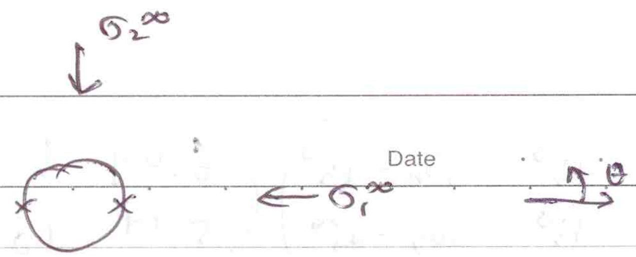
Q.2. (a)

① at the sidewalls



$\sigma_{\theta\theta} = (\sigma_1^\infty + \sigma_2^\infty) - 2(\sigma_1^\infty - \sigma_2^\infty)\cos 2\theta$

σ_2^∞ = gravitational stress $= 0.025 \times 500 = 12.5$ MPa

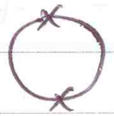
$\sigma_1^\infty = \begin{cases} k = 0.3 \rightarrow \sigma_1^\infty = 3.75 \text{ MPa} \\ k = 2.5 \rightarrow \sigma_1^\infty = 31.25 \text{ MPa} \end{cases}$

$\sigma_{\theta\theta} = \begin{cases} k=0.3 \rightarrow -\sigma_1^\infty + 3\sigma_2^\infty = 33.75 \text{ MPa} \\ k = 2.5 \rightarrow \quad '' \quad = 6.25 \text{ MPa} \end{cases}$

Stresses are between -3.0 MP and 60 MPa

$\rightarrow$ does not fail.

② at the crown or invert



$\sigma_{\theta\theta} = \begin{cases} k = 0.3 \rightarrow 3\sigma_1^\infty - \sigma_2^\infty = -1.25 \text{ MPa} > -3.0 \text{ MPa} \\ k = 2.5 \rightarrow 3\sigma_1^\infty - \sigma_2^\infty = 81.25 \text{ MPa} > 60 \text{ MPa} \end{cases}$

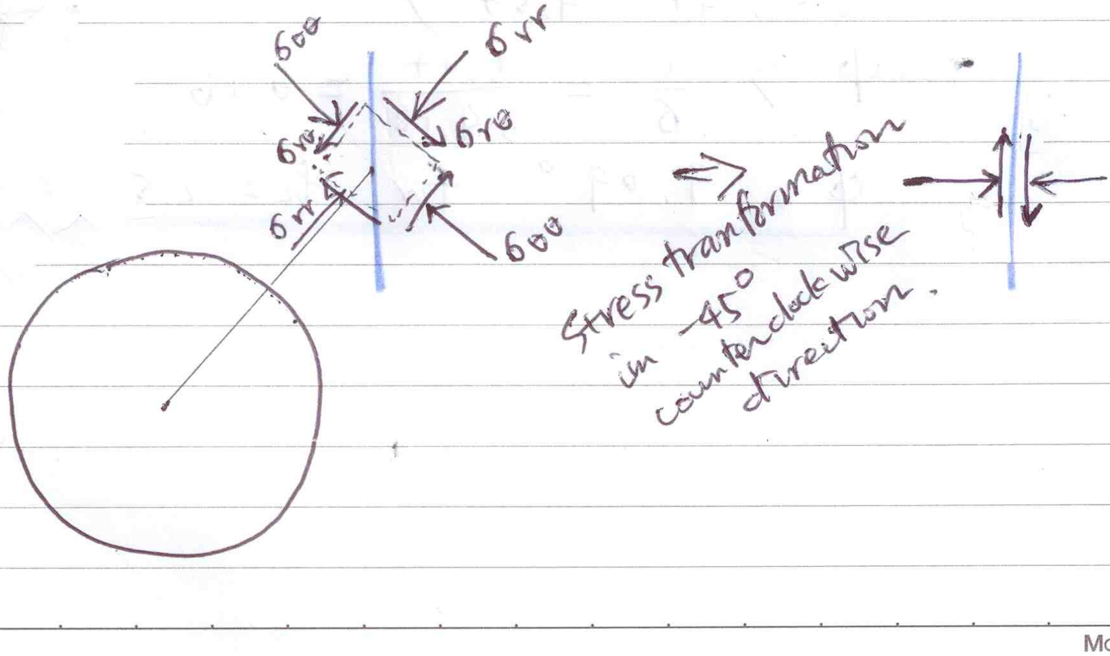
there can be compressive failure at the crown or invert when stress ratio is 2.5.

(b)-① for $k = 0.3$

from Kirsch's solution 8.113 ~ 8.115 in the textbook,

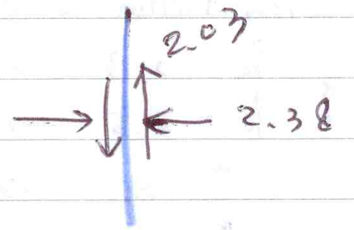
at $r = (5+5)$ m, $\theta = 45°$, $\sigma_1^\infty = 3.75$ MPa, $\sigma_2^\infty = 12.5$ MPa

$\sigma_r = 6.09$ (MPa), $\sigma_\theta = 10.16$ (MPa), $\sigma_{r\theta} = 5.74$ (MPa)



$$\begin{pmatrix} \cos -45^\circ & \sin -45^\circ \\ -\sin -45^\circ & \cos -45^\circ \end{pmatrix} \begin{pmatrix} 6.09 & 5.74 \\ 5.74 & 10.16 \end{pmatrix} \begin{pmatrix} \cos -45^\circ & -\sin -45^\circ \\ \sin -45^\circ & \cos -45^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 2.38 & -2.03 \\ -2.03 & 13.86 \end{pmatrix}$$



$$\tau = \sigma \tan\phi$$

$$\tan\phi > \frac{\tau}{\sigma} = \frac{2.03}{2.38} = 0.85$$

$$\underline{\phi = 40.47^\circ.} \quad \text{for } k = 0.3$$

② for $k = 2.5$

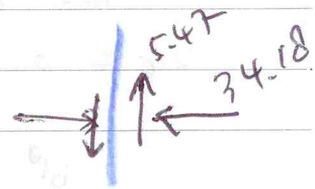
$r = 10\,m$, $\theta = 45$, $\sigma_1^\infty = 31.25$, $\sigma_2^\infty = 12.15$

$\sigma_r = 16.41$ $\sigma_{\theta\theta} = 27.34$, $\sigma_{r\theta} = -12.30$.

From stress rotation,

$$\begin{pmatrix} \cos -45^\circ & \sin -45^\circ \\ -\sin -45^\circ & \cos -45^\circ \end{pmatrix} \begin{pmatrix} 16.41 & -12.30 \\ -12.30 & 27.34 \end{pmatrix} \begin{pmatrix} \cos -45^\circ & -\sin -45^\circ \\ \sin -45^\circ & \cos 45^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 34.18 & -5.47 \\ -5.47 & 9.57 \end{pmatrix}$$

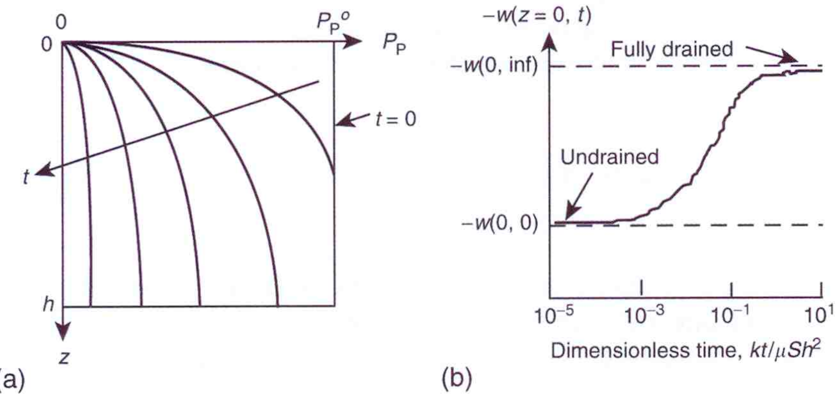


$$\tan\phi > \frac{\tau}{\sigma} = \frac{5.47}{34.18} = 0.16$$

$$\underline{\phi = 9.09^\circ \quad \text{for } k = 2.5}$$

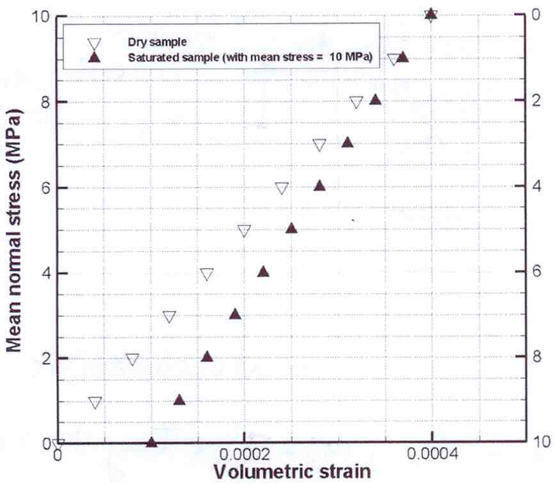
3. Explain the physical meaning of Biot's coefficient, α and Skempton's coefficient, B (10).

4. Explain the following two graph from Fig.7.7 of the textbook (10).



5. Discuss the similarity and differences between poroelasticity and thermoelasticity (10).

6. Following data are available from a laboratory test on a rock sample. The first test was conducted on a dry sample. The second test started when mean normal stress reached 10 MPa. In the second test, pore pressure was increased from zero to 10 MPa while maintaining the mean normal stress (confining stress) as 10 MPa. Calculate the Biot's coefficient α. Can you estimate the bulk modulus of mineral (matrix) from these data? If so please calculate it. Also, not using any equation, explain the graph below (10).



From $\varepsilon_b = \frac{1}{K}\sigma_m - \frac{\alpha}{K}P$ ... ①

From the first test,

$0.0004 = \frac{1}{K} \cdot 10$ (MPa).

$K = 25$ GPa.

From the second test, by taking the final point.

$\varepsilon_b = 0.0001$, $\sigma_m = 10$MPa, $p = 10$MPa

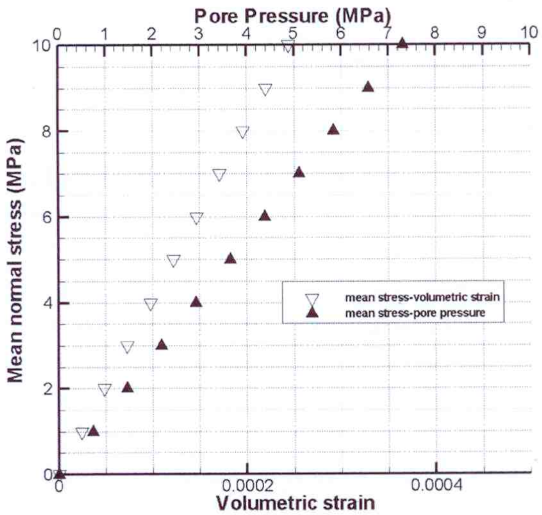
$0.0001 = \frac{1}{25000}(10 - 10\alpha) = \frac{10}{25000}(1-\alpha)$

$\alpha = 0.75$

From $\alpha = 1 - \frac{K}{K_m} \rightarrow K_m = \frac{K}{1-\alpha}$

bulk modulus of mineral ← $K_m = \frac{25}{1-0.75} = 100$ GPa.

7. Following data are available from undrained laboratory test on a rock sample. The graph shows the response of pore pressure and volumetric strain when mean normal stress (confining stress) was increased from zero to 10 MPa. Calculate the Skempton coefficient and drained bulk modulus of the rock when Biot's coefficient is 0.5. Also, not using any equation, explain the graph below (10).



From definition,

Skempton coefficient $B = \left(\frac{\partial P_p}{\partial P_c}\right)_{undrained}$

$B = \frac{7.3}{10} = 0.73$

undrained bulk modulus, $K = \left(\frac{\partial P_c}{\partial \varepsilon_b}\right)_{undrained}$

$K_u = \frac{10}{0.00025} \doteqdot 40$ GPa.

From $K_u = \frac{K}{1-\alpha B}$, $K = K_u(1-\alpha B)$

$K = 40\times(1-0.5\times0.73) = 25.4$ GPa

8. A rock sample in the size of 10 m × 10 m × 10 m is confined in both x- and y-directions with free surface (in z-direction). Calculate the stress and strain tensor when temperature was increased from 20°C to 80°C assuming that the initial strain and stress were zero. How much does the rock expand in the z-direction? (15)
The parameters for the rock, which was determined from the laboratory are as follows; Elastic modulus: 50 GPa, Poisson's ratio: 0.25, Linear expansion coefficient: 0.7×10^{-5} /°C, Heat conductivity: 2.7 W/m°C.
You first need to derive the equations for strain and thermal stress for this boundary condition using Eq.(7.125) and Eq.(7.128) and calculate the six strain and stress component using the input parameters.

Thank you for your hard work during the spring semester!!!
Have a fruitful summer break.

Ki-Bok Min

8.

Equations of stress in terms of strain and thermal expansion.

$$\tau_{xx} = 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}) + 3\beta K\vartheta$$
$$\tau_{yy} = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}) + 3\beta K\vartheta$$
$$\tau_{zz} = 2G\varepsilon_{zz} + \lambda(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}) + 3\beta K\vartheta$$
$$\tau_{xy} = 2G\varepsilon_{xy},\quad \tau_{xz} = 2G\varepsilon_{xz},\quad \tau_{yz} = 2G\varepsilon_{yz}.$$

where $\vartheta = T - T_0$.

From B.C, $\underline{\varepsilon_{xx} = \varepsilon_{yy} = 0,\ \tau_{zz} = 0}$.

$$\tau_{xx} = \lambda\varepsilon_{zz} + 3\beta K\vartheta \quad \cdots ①$$
$$\tau_{yy} = \lambda\varepsilon_{yy} + 3\beta K\vartheta \quad \cdots ②$$
$$\tau_{zz} = (2G+\lambda)\varepsilon_{zz} + 3\beta K\vartheta = 0 \rightarrow \varepsilon_{zz} = -\frac{3\beta K\vartheta}{2G+\lambda} \quad ③ \cdots$$

put ③ into ① & ②.

$$\tau_{xx} = -\frac{\lambda}{2G+\lambda}\cdot 3\beta K\vartheta + 3\beta K\vartheta = \frac{2G}{2G+\lambda}\cdot 3\beta K\vartheta.$$

$$\frac{2G}{2G+\lambda} = \frac{2G}{2G+\frac{2G\nu}{1-2\nu}} = \frac{2G}{\frac{2G-2G\nu}{1-2\nu}} = \frac{1-2\nu}{1-\nu}.$$

$$\therefore\ \underline{\tau_{xx} = \tau_{yy} = \frac{1-2\nu}{1-\nu}3\beta K\vartheta}$$

From (7.125)

$$\varepsilon_{zz} = \frac{1}{2G}\tau_{zz} - \frac{\nu}{2G(1+\nu)}(\tau_{xx}+\tau_{yy}+\tau_{zz}) - \beta\vartheta$$

$$= -\frac{\nu}{\cancel{2}G(1+\nu)}\cdot\cancel{2}\times\frac{1-2\nu}{1-\nu}3\beta K\vartheta - \beta\vartheta$$

$$= \left\{-\frac{2\nu}{\cancel{E}}\cdot\frac{\cancel{1-2\nu}}{1-\nu}\cdot\cancel{3}\cdot\frac{\cancel{E}}{\cancel{3}(\cancel{1-2\nu})} - 1\right\}\beta\vartheta$$

$$= \left\{-\frac{2\nu}{1-\nu} - 1\right\}\beta\vartheta = \underline{-\frac{1+\nu}{1-\nu}\beta\vartheta}$$

$$K = \frac{E}{3(1-2\nu)} = \frac{50}{3\times(1-2\times0.25)} = \frac{50}{1.5} = 33.3\ \text{GPa}$$

$$\beta = 0.7\times10^{-5}$$

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$$\tau_{xx} = \tau_{yy} = \frac{1-2\nu}{1-\nu} 3\beta K v = \frac{1-2\times0.25}{1-0.25}\times 3\times 0.7\times10^{-5}\times 33.3\times 60$$ (GPa)

$$= 28 \text{ (MPa)}.$$

$$\varepsilon_{zz} = -\frac{1+\nu}{1-\nu}\beta v = -\frac{1+0.25}{1-0.25}\times 0.7\times10^{-5}\times 60$$

$$= -0.0007$$

$$\Delta l = -0.0007 \times 10 \text{ m} = -7 \text{ (mm)}$$

because of contraction (+), there is expansion of 7 mm in the positive z-direction.

thermal expansion does not generate shear strain, and hence, shear stress.

$$\tau_{ij} = \begin{pmatrix} 28 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \varepsilon_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.0007 \end{pmatrix}$$

contraction positive.