Engineering Mathematics 1 (010.140)

1st Midterm Examination

2 April, 2009

1. (20 points) Find $y = f(p)$ by solving $p^2 + py = x,\ p = y'$

Hint) **d'Alembert-Lagrange equation**

$y = xf(p) + g(p),\ p = y'$

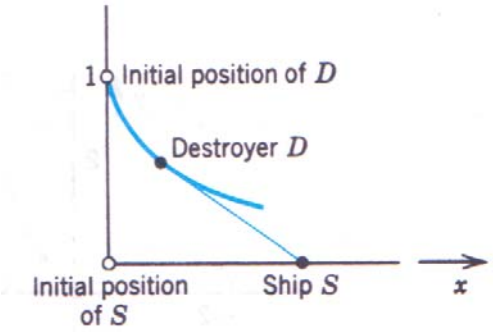
$p = y' = xf'(p)\frac{dp}{dx} + f(p) + g'(p)\frac{dp}{dx}$

$\frac{dx}{dp} - \frac{f'(p)}{p - f(p)}x = \frac{g'(p)}{p - f(p)} \Rightarrow \frac{dx}{dp} + M(p)x = N(p)$

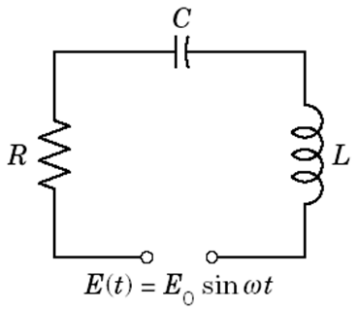
if, $p - f(p) \neq 0$

2. (20 points) The destroyer D (the pursuer) pursues the ship S (the target), that is, moves in the direction of S at all times. Assume that S moves along the x-axis and the distance a from D to S is constant. Show that $y' = -y/\sqrt{a^2 - y^2}\,dy$. Sketch a direction field (for a = 1 [nautical mile]) and the solution satisfying y(0) = 1. (This curve is called a tractrix, from Latin trahere, meaning "to pull.")
Separating variables, show that

$x = -\int y^{-1}\sqrt{a^2 - y^2}\,dy = -\sqrt{a^2 - y^2} + a\ln\left|y^{-1}(a + \sqrt{a^2 - y^2})\right| + c$



3. (1) (5 points) Find the governing equation of the electric circuit with respect to the charge, Q(t).
(2) (10 points) Find the general solution of the governing equation of (1) by considering the conditions for the circuit to be overdamped, critically damped and underdamped.
(3) (5 points) What is the critical resistance R_{crit}?



4. (1) (10 points) Solve the initial value problem and graph the solution
$y'' + 3y' + 2.25y = -10e^{-1.5x},\ y(0) = 1,\ y'(0) = 0$
(2) (10 points) Solve $y'' + 2y' - 15y = 17\sin 5x$ by undetermined-coefficient method and by variation of parameters.
(3) (10 points) Solve the initial value problem
$(x^3D^3 - x^2D^2 - 7xD + 16I)y = 9x\ln x,\ y(1) = 6,\ Dy(1) = 18, D^2y(1) = 65$

"Wisdom is supreme; therefore get wisdom. Though it cost all you have, get understanding." (Proverbs 4:7)