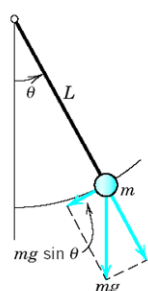# Engineering Mathematics 1 (010.140)
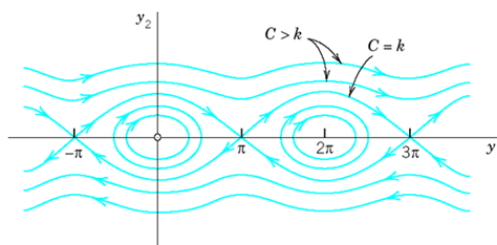## 2nd Midterm Examination
### 22 May, 2009

1. A pendulum consisting of a body of mass $m$ (the bob) and a rod of length $L$. Assume that the mass of the rod and air resistance are negligible.



    (1) (5 points) Set up the mathematical model.

    (2) (10 points) Determine the locations and types of the critical points.

    (2) (5 points) Describe the physical meaning the phase plane below.



2. (20 points) The associated Legendre functions $P_n^k(x)$ play a role in quantum physics. They are defined by $P_n^k(x) = (1-x^2)^{k/2}\dfrac{d^k P_n}{dx^k}$ and are solutions of ODE

$(1-x^2)y'' - 2xy' + [n(n+1) - \dfrac{k^2}{1-x^2}]y = 0$. Find $P_1^1(x)$ and $P_2^2(x)$.

3. (20 points) Find a solution of $x^2 y'' + (x^2 + \dfrac{5}{36})y = 0$.

4. (20 points) Derive the solution of the first kind of order $\nu$ of
$x^2 y'' + xy' + x^2 y = \nu^2 y$.

5. (20 points) Find $y$ at a certain time $t$ when the driving force is acting as $A\sin(\sqrt{k/m})t$ in an undamped mass-spring system, where the spring constant is $k$ and the mass of the body attached to the spring is $m$.

"Some trust in chariots and some in horses,
but we trust in the name of the LORD our God." (Psalms 20:7)