Engineering Mathematics 1 (010.140)

Final Examination

11 June, 2009

1. (10 points) The associated Legendre functions $P_n^k(x)$ play a role in quantum physics. They are defined by $P_n^k(x) = (1-x^2)^{k/2}\dfrac{d^k P_n}{dx^k}$ and are solutions of ODE $(1-x^2)y'' - 2xy' + [n(n+1) - \dfrac{k^2}{1-x^2}]y = 0$. Find $P_1^1(x)$ and $P_2^2(x)$.

2. (10 points) Solve the initial value problem and graph the solution.
$y'' + 3y' + 2.25y = -10e^{-1.5x},\ y(0) = 1,\ y'(0) = 0$

p. 55 예제 2

3. (15 points) Find x (a is constant) to satisfy

$$|A| = \begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = 0.$$

→ $x=a$, $x=-3a$

p. 67 예제 14

4. (10 points) Solve the linear system using the Cramer's Rule.

$x_1 - 2x_2 + 3x_3 = 2$
$2x_1 - 3x_3 = 3$
$x_1 + x_2 + x_3 = 6$

→ $x_1 = 3,\ x_2 = 2,\ x_3 = 1$

8.3 Ex. 1

5. (15 points) Find u·v where u and v are transformation of a and b, respectively through a transformation y = Ax.

$$A = \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & -2/3 \end{bmatrix},\ a = \begin{bmatrix} 7/9 \\ 2/13 \\ -1/2 \end{bmatrix},\ b = \begin{bmatrix} 3/7 \\ 4/13 \\ 5 \end{bmatrix}$$

$$\rightarrow u \cdot v = \begin{bmatrix} 1/3 \\ 8/169 \\ -5/2 \end{bmatrix}$$

8.4 Ex. 4

6. (1) (10 points) Find Eigenvalues of A^T

→ 3, -4, 0

(2) (15 points) Evaluate A^6

$$A = \begin{bmatrix} 7.3 & 0.2 & -3.7 \\ -11.5 & 1.0 & 5.5 \\ 17.7 & 1.8 & -9.3 \end{bmatrix}$$

→

$$D = X^{-1}AX = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ D^6 = X^{-1}A^6X = \begin{bmatrix} 729 & 0 & 0 \\ 0 & 4096 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A^6 = X\begin{bmatrix} 729 & 0 & 0 \\ 0 & 4096 & 0 \\ 0 & 0 & 0 \end{bmatrix}X^{-1}$$

8.5. 16

7. (15 points) Find relationships between Pauli spin matrices as following;

$$S_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ S_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},\ S_z = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\rightarrow S_xS_y = iS_z,\ S_yS_x = -iS_z,\ S_x^2 = S_y^2 = S_z^2 = I$$

"Above all else, guard your heart, for it is the wellspring of life.
Put away perversity from your mouth; keep corrupt talk far from your lips.
Let your eyes look straight ahead, fix your gaze directly before you.
Make level paths for your feet and take only ways that are firm.
Do not swerve to the right or the left; keep your foot from evil." (Proverbs 4:23-27)