Plasma Electrodynamics 1 April 9, 2009

Mid-Term Exam 1

(15 points)

1. If each of the following statements is true, please mark O in the parenthesis. If false or incorrect, mark X and make corrections of the false statement or word in the parenthesis.

1) Two vectors are perpendicular to each other if their cross product is zero, and vice versa. (   )

2) The basic rules of vector dot and cross products are independent of coordinate system. (   )

3) In the deductive approach, electromagnetic theorems and laws are developed from the fundamental postulates for an idealized model, and then verified with experimental observations. (   )

4) Maxwell's equations describe the electromagnetic field quantities generated by distributed space sources in a circuit model. (   )

5) Coulomb C, which is the unit for charge, is a derived unit expressible in terms of MKSA units as A · s, and its dimension is *TI* . (   )

6) The gradient of a scalar field is a scalar point function. (   )

7) If $i = j$, then $\delta_{ij} = 1$ and $\epsilon_{ijk} = 0$. (   )

8) The divergence theorem transforms the surface integral of the divergence of a vector field to a closed line integral of the vector field, and vice versa. ( )

9) Helmholtz's theorem states that a vector field is determined if either its divergence or its curl is specified everywhere. (   )

10) If a vector field is curl-free, then it can be expressed as the gradient of a scalar field. (   )

(20 points)

2. Answer the following questions briefly.

1) What are the source quantities and four fundamental field quantities in the electromagnetic model? What are their SI units?

2) Does $\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \boldsymbol{C}$ imply $\boldsymbol{B} = \boldsymbol{C}$? Explain and show an example.
Does $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{A} \times \boldsymbol{C}$ imply $\boldsymbol{B} = \boldsymbol{C}$? Explain and show an example.

3) Which of the following four scalar triple products has a different magnitude from the others? Explain.
$(\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{C}$, $\boldsymbol{A} \cdot (\boldsymbol{C} \times \boldsymbol{B})$, $(\boldsymbol{A} \times \boldsymbol{C}) \cdot \boldsymbol{B}$, $\boldsymbol{B} \cdot (\boldsymbol{a}_A \times \boldsymbol{A})$

By using the representations of summation convention and Levi-Civita symbol, prove that the scalar triple product $(\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{C}$ can be calculated by the following determinant in Cartesian coordinate system.

$$(\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

4) What are the metric coefficients in cylindrical coordinates?
Write $d\boldsymbol{l}$, $d\boldsymbol{s}_z$, dv, and ∇ in cylindrical coordinates.

5) What is the physical definition of the curl of a vector field $\boldsymbol{A}$? Show that $\boldsymbol{A}$ is an irrotational field if $\boldsymbol{A} = \boldsymbol{a}_R f(R) \equiv \hat{\boldsymbol{R}} f(R)$ in spherical coordinates, where $f(R)$ is any function of the radial distance R.

(12 points)

3. Assuming a vector field expressed in cylindrical coordinates to be $\boldsymbol{A} = \hat{r}\,(3\cos\phi) - \hat{\phi}\,2r + \hat{z}\,z$.

1) What is the field at the point $P(4,\, 60^o,\, 1)$?
2) Expressed the field $\boldsymbol{A}_P$ at P in Cartesian coordinates.
3) The cylindrical coordinates of another point Q are: $Q(3,\, 180^o,\, -1)$.
Determine the distance between the two points P and Q.

(12 points)

4. 1) Express the r-component, A_r , of a vector $\boldsymbol{A}$ at the point $P_1(r_1, \phi_1, z_1)$ in terms of A_R and A_θ in spherical coordinates.

   2) Denote the position vector to a point $P_2(x, y, z)$ by $\boldsymbol{R}$. Determine $\nabla(1/R)$ in spherical coordinates.

   3) Find the divergence of the radial field, $f(\boldsymbol{R}) = \hat{\boldsymbol{R}}\, k/R^2$ where k is a constant.

(5 points)

5. Show that $\dfrac{\partial \hat{\boldsymbol{r}}}{\partial \phi} = \hat{\boldsymbol{\phi}}$ and $\dfrac{\partial \hat{\boldsymbol{\phi}}}{\partial \phi} = -\hat{\boldsymbol{r}}$ in cylindrical coordinates.

   (These relations state that the differentiation of a base vector may lead to a new vector in a different direction in a curvilinear coordinate system)

(16 points)

6. Assume a vector field $\boldsymbol{A} = \hat{\boldsymbol{x}}(2x+y) - \hat{\boldsymbol{y}}\, y$.

   1) Find $\oint_C \boldsymbol{A} \cdot d\boldsymbol{l}$ around the triangular contour shown in the figure below.

   2) Find $\int_S (\nabla \times \boldsymbol{A}) \cdot d\boldsymbol{s}$ over the triangular area.

   3) Verify the Stokes's theorem by using the results of 1) and 2).

   4) Can $\boldsymbol{A}$ be expressed as the gradient of a scalar? Explan.