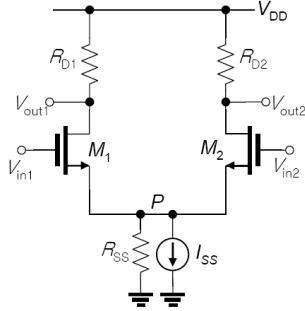


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1. For the circuit on the right, answer for the following questions. Assume  $\gamma=0$  (no back-bias effect),  $\lambda=0$ , and  $g_{m1}=g_{m2}=g_m$  for  $M_1$  and  $M_2$  transistors. Also assume  $R_{D1}=R_{D2}=R_D$ . (30)



(a) Briefly describe key advantages of fully symmetric differential amplifier. (4)

(b) Prove that P is a virtual ground for small, differential inputs. (5)

(c) Derive the expression for differential mode gain. ( $V_{in2}=-V_{in1}$ ) (4)

(d) Derive the expression for the common mode gain. (4)

$$(R_{D1}=0\Omega)$$

(e) Derive CMRR expression. How can you improve the CMRR. (3)

$$(R_{D1}=0\Omega)$$

(f) When  $R_{D1}$  is  $0\Omega$ , derive the expression for differential mode gain. (5)

(g) In the circuit,  $V_{CM}=1\text{ V}$ ,  $I_{SS}=1\text{ mA}$ , and  $R_D=1\text{ k}\Omega$ . What is the minimum allowable supply voltage if the transistors must remain in saturation? Assume  $V_{TH,n}=0.5\text{ V}$  and  $R_{SS}=\infty$ . (5)

**Answer)**

(a) i) If  $V_{DD}$  fluctuates, it can be canceled out. (high rejection of supply noise)

ii) the swing of  $V_{out1}-V_{out2}$  can reach  $2\{V_{DD} - (V_{GS}-V_{th})\}$  which is double the swing in a single CS amplifier. etc.

(b)  $V_{in1} = -V_{in2}$

$$V_{in1} = V_{CM} + \Delta V, \quad V_{in2} = V_{CM} - \Delta V$$

$$I_{D1} = \frac{I_{SS}}{2} + \Delta I, \quad I_{D2} = \frac{I_{SS}}{2} - \Delta I$$

$$\Delta I = g_m \Delta V$$

$$I_{D1} = g_m (\Delta V - \Delta V_p), \quad I_{D2} = g_m (\Delta V + \Delta V_p)$$

$$\Delta I_{D1} = -\Delta I_{D2}$$

$$g_m (\Delta V - \Delta V_p) = g_m (\Delta V + \Delta V_p)$$

$$\therefore \Delta V_p = 0 \quad (V_p \text{ is a virtual ground})$$

There can be other answers for this problem.

(c) Using half circuit analysis,

$$A_v = -g_m R_D \quad (r_o = \infty)$$

(d) If  $R_{D1} = 0$ ,  $R_{D2} = R_D$ ,

$$\begin{aligned} \Delta V_{CM} &= \Delta V_{GS} + 2\Delta I_D R_{SS} \\ &= \Delta I_D \left( \frac{1}{g_m} + 2R_{SS} \right) \end{aligned}$$

$$\Delta I_D = \frac{\Delta V_{CM}}{\frac{1}{g_m} + 2R_{SS}}$$

$$V_{out} = \Delta V_{out1} - \Delta V_{out2} = \Delta I_D R_D = \frac{\Delta V_{CM}}{\frac{1}{g_m} + 2R_{SS}} R_D$$

$$A_{CM-DM} = \frac{\Delta V_{out}}{\Delta V_{CM}} = \frac{R_D}{\frac{1}{g_m} + 2R_{SS}} \approx \frac{R_D}{2R_{SS}}$$

(f)  $V_{in2} = -V_{in1}$

$$V_{out1} = 0, \quad V_{out2} = -g_m R_D V_{in2}$$

$$A_v = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = \frac{g_m R_D v_{in2}}{-2v_{in2}} = -\frac{g_m R_D}{2}$$

$$(e) CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \frac{1 + 2g_m R_{SS}}{2}$$

From this result, you can improve the CMRR by increasing  $g_m$  and  $R_{SS}$ .

(g)  $V_{DD} - I_D R_D = V_{DS}$

$$V_{DS} \geq V_{GS} - V_{TH} \text{ (sat)}$$

$$V_{DD} - I_D R_D \geq 1 - 0.5$$

$$V_{DD} - 0.5 \geq 0.5$$

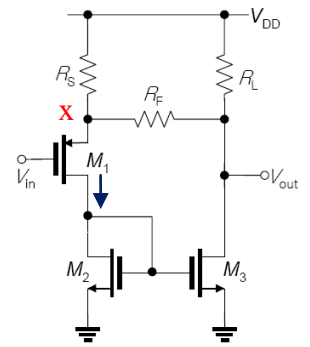
$$V_{DD} \geq 1$$

$\therefore$  minimum allowable supply voltage is 1[V].

2. For the circuit on the right, answer for the following questions. Assume  $(W/L)_2$  and  $(W/L)_3$  are different. Assume  $\lambda=0$  for all transistors, and  $\gamma=0$  for  $M_1$ . (12)

(a) Assume  $R_F$  is much larger than  $R_S$  and  $R_L$ . Also assume  $R_S=0\Omega$ . Calculate small signal voltage gain. (7)

(b) Determine the polarity of feedback when  $R_S$  is finite. (5)



**Answer)**

(a)  $\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3$ ,  $r_o = \infty$

$R_F \gg R_S, R_L$  means  $R_F$  path is open. (open loop)

$$I_{D1} = I_{D2}$$

$$I_{D3} = \frac{(W/L)_3}{(W/L)_2} I_{D2}$$

$$V_{out} = \frac{(W/L)_3}{(W/L)_2} I_{D2} R_D = \frac{(W/L)_3}{(W/L)_2} g_m v_{in} R_L$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{(W/L)_3}{(W/L)_2} g_m R_L$$

(b)  $V_{in} \uparrow \Rightarrow I_{D1} \downarrow \Rightarrow I_{D2} \downarrow \Rightarrow I_{D3} \downarrow \Rightarrow V_{out} \uparrow \Rightarrow V_X \uparrow \Rightarrow I_{D1} \uparrow$   
or

$$V_{in} \uparrow \Rightarrow V_Y \downarrow \Rightarrow I_{D3} \downarrow \Rightarrow V_{out} \uparrow \Rightarrow V_X \uparrow \Rightarrow V_Y \uparrow$$

Thereby opposing the effect produced by  $V_{in}$ , the feedback is therefore **negative**.

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3. Assume that the MOSFETs ( $M_1$  and  $M_2$ ) have the same output resistance  $r_o$  ( $\lambda > 0$ ). Load capacitance  $C_L$  is connected to output. Neglect other capacitances.  $M_1$  and  $M_2$  are n- and p-type MOSFETs, respectively. Answer for the following questions. (8)

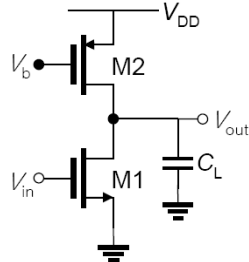


Fig. 1

- (a) Assume that there is a capacitance ( $C_F$ ) between the gate and the drain of  $M_1$ . By using Miller theorem, calculate the input and output capacitances. (3)
- (b) Estimate the -3 dB bandwidth in (a) as the  $r_o$  goes to infinite ( $\lambda=0$ ). Here assume that there is a finite input resistance connected with  $V_{in}$  in series. (2)
- (c) Assume that  $V_{in}$  and  $V_b$  are exchanged. That is to say,  $V_{in}$  is connected to the gate of  $M_2$  and  $V_b$  is connected to the gate of  $M_1$ . Compute voltage gain at low frequency. (3)

**Answer)**

(a) low frequency gain,  $A_v = -g_m(r_o \parallel r_o) = -\frac{g_m r_o}{2}$

Using Miller's Theorem,

$$C_{F,in} = \left(1 + \frac{g_m r_o}{2}\right) C_F$$

$$C_{F,out} = \left(1 + \frac{2}{g_m r_o}\right) C_F + C_L$$

- (b)  $r_o = \infty$ , a finite input resistance =  $R_s$

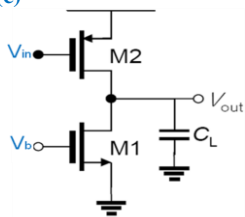
Since  $C_{F,in}$  is infinite, the input pole is zero ( $\omega_{pin} = 1/R_s C_{F,in} = 0$ ).

The output pole is also zero because  $r_o$  is infinite.

$$(\omega_{p,out} = 1/(r_o/2)(C_L + C_{F,out}) = 0)$$

$$\therefore BW \cong 0$$

- (c)



Including the impedance of  $C_L$  ( $\frac{1}{j\omega C_L}$ ),

$$R_{out} = r_o \parallel r_o \parallel \frac{1}{j\omega C_L}$$

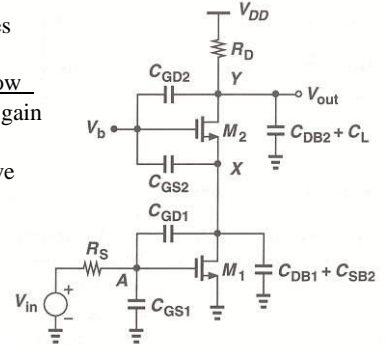
So the gain of the circuit is  $-g_m(r_o \parallel r_o \parallel \frac{1}{j\omega C_L})$ .

At a low frequency, we can ignore  $C_L$  (opened).

$$\therefore A_v = -g_m \frac{r_o}{2}$$

4. For the following cascode stage, answer for the following questions. Assume  $\gamma_n = \lambda_n = 0$ . (20)

- (a) Describe briefly key features of the amplifier. (5)
- (b). Derive an expression for low frequency small signal voltage gain between nodes A and X. (5)
- (c) Using Miller theorem, derive expressions of capacitances at nodes A and X. (5)
- (d) Suppose a resistor  $R_G$  appears in series with the gate of  $M_2$ . Including only  $C_{GS2}$ , neglecting other capacitances, determine the transfer function (5)



**Answer)**

- (a) Compared with the Miller approximation results obtained in CS stage, the input pole has risen considerably and hence the cascode bandwidth is larger.

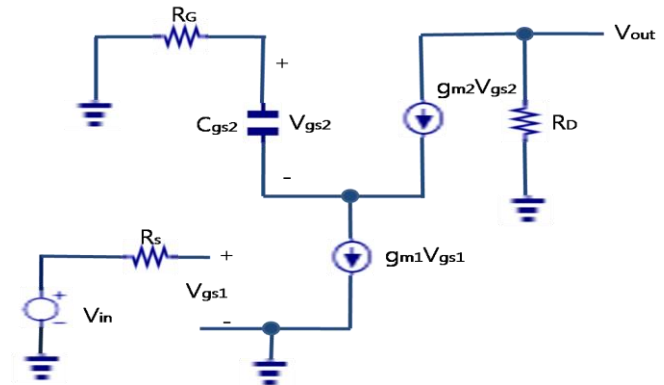
- (b) A low frequency gain from A to X is

$$A_v = -\frac{g_{m1}}{g_{m2}} \quad (r_{o1}, r_{o2} = \infty)$$

- (c)  $C_A = C_{GS1} + (1 + \frac{g_{m1}}{g_{m2}})C_{GD1}$

$$C_X = C_{DB1} + C_{GS2} + (1 + \frac{g_{m2}}{g_{m1}})C_{GD1} + C_{SB2}$$

- (d)



$$\frac{V_{out}}{V_{in}} = \frac{V_{gs2}}{V_{in}} \cdot \frac{V_{out}}{V_{gs2}}, \quad V_{in} = V_{gs1}$$

$$g_{m1} V_{in} = g_{m2} V_{gs2} + j\omega C_{gs2} V_{gs2} = V_{gs2} (g_{m2} + j\omega C_{gs2})$$

$$\frac{V_{gs2}}{V_{in}} = \frac{g_{m1}}{g_{m2} + j\omega C_{gs2}}, \quad \frac{V_{out}}{V_{gs2}} = -g_{m2} R_D$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{-g_{m1} g_{m2} R_D}{g_{m2} + j\omega C_{gs2}}$$

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5. Assume that MOS transistors  $M_1$  and  $M_2$  have a finite  $r_{o1}$  and infinite  $r_{o2}$ , respectively. It is also assumed that current sources  $I_1$  and  $I_2$  are ideal. Answer for the following questions. (30)

(a) Identify the sense and return mechanisms. (3)

(b) Determine the polarity of feedback. (4)

(c) Calculate open-loop  $Z_{in}$  and  $Z_{out}$ . (3)

(d) The feedforward system in the circuit above is a transimpedance amplifier. Calculate the gain of feedforward system. (4)

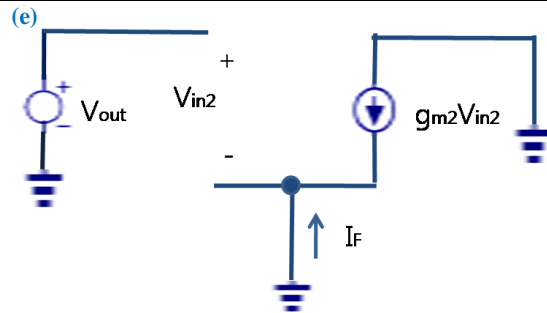
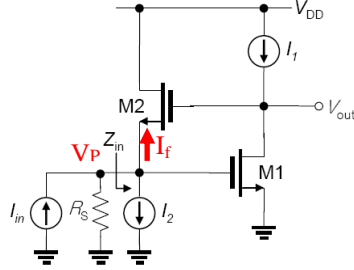
(e) Calculate the feedback factor  $K$ . (4)

(Hint: Note the direction or polarity of feedback quantity ( $I_F$  or  $V_F$ ))

(f) Calculate open-loop and closed-loop gains. (4)

(g) Calculate closed loop  $Z_{in}$  and  $Z_{out}$ . When  $r_{o1}$  goes to infinite, calculate again closed loop  $Z_{in}$  and  $Z_{out}$ . (4)

(h) When p-type MOSFET substitutes for n-type MOSFET  $M_2$ , determine the polarity of the feedback. Note the source of PMOS is connected to  $V_{DD}$ . (4)



$$I_F = -g_{m2}V_{in2}$$

$$V_{in2} = V_{out}$$

$$I_F = -g_{m2}V_{out}$$

$$\therefore K = \frac{I_F}{V_{out}} = -g_{m2}$$

(f) open loop gain =  $R_o = \frac{-g_{m1}r_{o1}}{g_{m2}}$

closed loop gain  $\frac{V_{out}}{I_{in}} = \frac{R_o}{1 + KR_o} = \frac{-g_{m1}r_{o1}/g_{m2}}{1 + g_{m1}r_{o1}}$

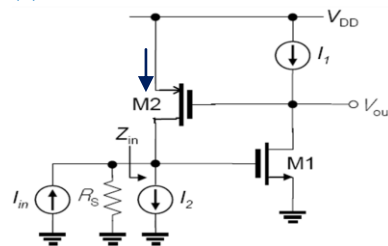
$$= -\frac{g_{m1}r_{o1}}{g_{m2}(1 + g_{m1}r_{o1})}$$

(g)  $Z_{in,closed} = \frac{R_{in}}{1 + KR_o} = \frac{1/g_{m2}}{1 + g_{m1}r_{o1}} \cong 0$

$$Z_{out,closed} = \frac{R_{out}}{1 + KR_o} = \frac{r_{o1}}{1 + g_{m1}r_{o1}} \cong \frac{1}{g_{m1}}$$

( $r_{o1} = \infty$ )

(h)



$$I_{in} \uparrow \rightarrow V_p \uparrow \rightarrow I_{D1} \uparrow \rightarrow V_{out} \downarrow \rightarrow I_{D2} \uparrow \rightarrow V_p \uparrow$$

Since the returned signal enhances the effect produced by  $I_{in}$ , The polarity of feedback is **positive**.

**Answer)**

(a) This topology employs a transimpedance amplifier as the forward system. The feedback network sense the output voltage and return a current to the subtractor.

(voltage-current feedback)

$M_2$  both senses the output voltage and returns a current to the input.  $M_2$  also serves as the feedback network.

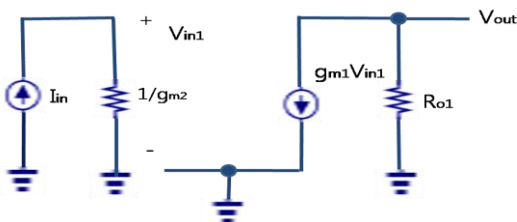
(b) If  $I_{in}$  increases,  $V_p$  increases and  $I_{D1}$  increases. As a result,  $V_{out}$  decreases (like inverter), thereby decreasing  $I_{D2}$ . Finally,  $V_p$  decreases. Since the current injected by  $M_2$  into the input node change in opposite directions, **the feedback is negative**.

$$I_{in} \uparrow \rightarrow V_p \uparrow \rightarrow I_{D1} \uparrow \rightarrow V_{out} \downarrow \rightarrow I_{D2} \downarrow \rightarrow V_p \downarrow$$

(c)  $Z_{in} = \frac{1}{g_{m2}}$  ( $r_{o2} = \infty$ )

$$Z_{out} = r_{o1}$$

(d) the gain of feedforward system  $\Rightarrow$  open loop gain



$$V_{out} = -g_m V_{in1} r_{o1}$$

$$V_{in1} = \frac{1}{g_{m2}} I_{in}$$

$$V_{out} = -\frac{g_{m1}}{g_{m2}} r_{o1} I_{in}$$

$$\therefore R_o = \frac{V_{out}}{I_{in}} = \frac{-g_{m1}r_{o1}}{g_{m2}}$$