465.211 Mechanics in Energy Resources Engineering, Spring 2010

1st Exam

09:00 - 11:00 31 March 2010

* Be careful about the units that you use.

* You may use Korean or English in answering above questions.

1. Indicate T(true) or F(false) for the following statements. <u>Note that an incorrect answer receives -2 mark while a correct answer receives +2 mark. You may leave the question blank if you wish.</u> (10)

(1) When a material follows the Hooke's Law upon loading, this material can be said to behave 'elastically'. (T, F)

(2) When a material have the same properties at every point, this material is said to be 'homogeneous'. (T,F)

(3) Materials having the same properties in all directions are said to be 'isotropic'. (T,F)

(4) While normal strain is related to the change in length, shear strain is related to the change of the shape. (T, F)

(5) When additional strains are generated with time even without stress increase, this phenomenon is called 'Creep'. (T, F)

2.

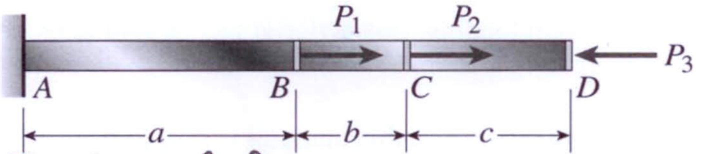
(1) Explain Saint Venant's Principle. Do you think the material properties such as modulus of elasticity (elastic modulus, E) affect the behavior explained by this principle? (5)

(2) Explain the 'statically indeterminate structures' and list an example of this type and another example of 'statically determinate structure' with brief explanations. (5)

(3) When a circular bar is subject to a torsion, shear stresses vary linearly with the distance from the center of the bar and maximum shear stress is generated at the outer surface of the bar. Explain why this is the case. (5)

(4) Explain the mechanism of how thermal stress is generated when temperature increases. List the important properties of materials that may affect the magnitude of the thermal stress with brief explanations. (5)

3. A homogeneous steel bar AD (see figure) has a cross-sectional area of 260 mm^2 and is loaded by forces P_1 = 12 kN, P_2 = 10 kN, and P_3 = 6 kN. The lengths of the segments of the bar are a = 1.5 m, b = 0.6 m, and c = 0.9 m. By what amount P should the load P_3 be increased so that the bar does not change in length when the three loads are applied? (10)



Internal axial forces

$N_{AB} = P_1+P_2-P_3 = 16$ kN

$N_{BC} = P_2-P_3 = 4$ kN

$N_{CD} = -P_3 = -6$ kN

change in length, $\delta_D = 0$.

$$\sum \frac{N_i L_i}{EA_i} = \frac{1}{EA}\cdot\{(22-P_3)\times 1.5 + (10-P_3)\times 0.6 - P_3\times 0.9\} = 0$$

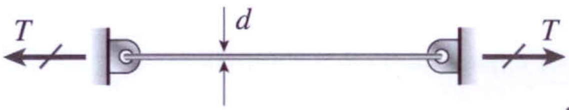
$P_3 = 13$ kN.

$\therefore$ increase of $P_3 = 13 - 6 = 7$ kN

4. At room temperature (25 °C), a brass wire of diameter d = 3.0 mm is stretched tightly between rigid supports so that the tensile force is T = 200 N (see figure). The coefficient of thermal expansion for the wire is 19.5 x 10^{-6} /°C and the modulus of elasticity is E = 110 GPa. (10)

(a) At what temperature does the maximum shear stress in the wire reach 30 MPa?

(b) At what temperature does the wire go slack?



can be obtained from 2.(4)

(a) thermal stress, $\sigma_T = -E\alpha\Delta T$ ← with tension positive.

Stress in the wire, $\sigma = \frac{T}{A} + \sigma_T = \frac{T}{A} - E\alpha\Delta T$

maximum shear stress occur at 45° from the axis of uniaxial stress & the magnitude is half of σ. → $\tau_{max} = \frac{\sigma}{2}$

$$\tau_{max} = \frac{T}{2A} - \frac{1}{2}E\alpha\Delta T = \frac{200}{2\times\frac{\pi}{4}(3\times10^{-3})^2} - \frac{1}{2}110\times10^9\times19.5\times10^{-6}\times\Delta T = 30\times10^6 \text{ (MPa)}$$

→ $\Delta T = -14.8$ °C. $T = T_i + \Delta T = 25 - 14.8 = 10.2$ °C

(b) τ_{max} or $\sigma = 0$. $\frac{T}{A} - E\alpha\Delta T = 0$. $\Delta T = 13.2$ °C → $T = 25 + 13.2 = 38.2$ °C

$\tau_{AC} = \frac{16 T_A}{\pi d_A^3} \rightarrow T_A = \frac{1}{16}\pi d_A^3 \tau_{allow} \rightarrow T_0 = \frac{1}{16}\pi d_A^3 \tau_{allow}\cdot\left(\frac{L_B I_{PA} + L_A I_{PB}}{L_B I_{PA}}\right)$

similarly $\tau_{CB} = \frac{16 T_B}{\pi d_B^3} \rightarrow T_B = \frac{1}{16}\pi d_B^3 \tau_{allow} \rightarrow T_0 = \frac{1}{16}\pi d_B^3 \tau_{allow}\cdot\left(\frac{L_B I_{PA} + L_A I_{PB}}{L_A I_{PB}}\right)$

T_0 from $\tau_{AC} = 30\text{MPa} \rightarrow 105$ N·m

" $\tau_{CB} = 30$ MPa $\rightarrow 167$ N·m.

smaller one governs $\Rightarrow$ 105 N·m

$I_p = \frac{\pi}{2} r^4$

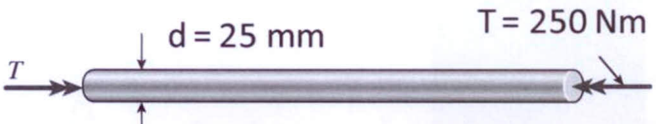
5. A solid aluminum bar of diameter d = 25 mm is subjected to torques $T = 250\cancel{0}$ Nm acting in the directions shown in the figure. Determine the maximum shear, tensile, and compressive stresses in the bar and show these stresses on sketches of properly oriented stress elements. (10)

$\tau_{max} = \frac{T \cdot r}{I_p} = \frac{2T}{\pi r^3} = \frac{2\times 250}{\pi \times (\frac{25}{2}\times 10^{-3})^3} = 81.5\text{ MPa}$

$\sigma_\theta = \tau \sin 2\theta$.

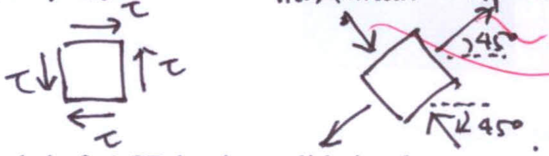
maximum tensile at 45° $\rightarrow \tau_{max} = 81.5$ MPa

maximum compressive at 135° $\rightarrow \tau_{max} = 81.5$ MPa



maximum shear

maximum compressive & tensile



6. A stepped shaft ACB having solid circular cross sections with two different diameters is held against rotation at the ends (see figure). If the allowable shear stress in the shaft is 30 MPa, what is the maximum torque $(T0)_{max}$ that may be applied at section C? (10)

$T_A + T_B = T_0 \leftarrow$ Equil. Eq.

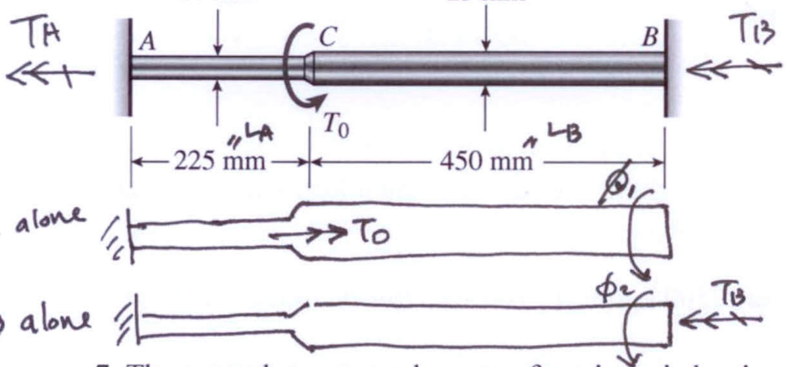
$\phi_1 + \phi_2 = 0 \leftarrow$ compatibility Eq.

$\phi_1 = \frac{T_0 L_A}{G I_{PA}},\quad \phi_2 = -\frac{T_B L_A}{G I_{PA}} - \frac{T_B L_B}{G I_{PB}}$

$\leftarrow$ Torque-displ eq.

combining $\rightarrow T_A = T_0\left(\frac{L_B I_{PA}}{L_B I_{PA} + L_A I_{PB}}\right)$

$T_B = T_0\left(\frac{L_A I_{PB}}{L_B I_{PA} + L_A I_{PB}}\right)$

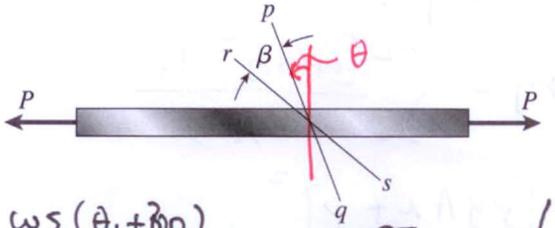


7. The normal stress on plane *pq* of a prismatic bar in tension (see figure) is found to be 57 MPa. On plane *rs*, which makes an angle $\beta = 30°$ with plane *pq*, the stress is found to be 23 MPa. Determine the maximum normal stress σ_{max} and maximum shear stress τ_{max} in the bar. (10)

$\sigma_\theta = \sigma_x \cos^2\theta$

$\sigma_x \cos^2\theta_1 = 57$ MPa

$\sigma_x \cos^2(\theta_1 + 30°) = 23$ MPa



$\frac{\cos^2(\theta_1+30°)}{\cos^2\theta_1} = \frac{23}{57}$

$\cos\theta_1 > 0 \rightarrow \frac{\cos(\theta_1+30)}{\cos\theta_1} = \frac{1}{1.57}$.

$\frac{\cos\theta_1 \cos 30° - \sin\theta_1 \sin 30°}{\cos\theta_1} = \frac{\sqrt{3}}{2} - \frac{1}{2}\tan\theta_1 = \frac{1}{1.57} \Rightarrow \theta_1 = 24.6°$

$\sigma_x = \frac{57}{\cos^2\theta_1} \doteqdot 68.9\text{ MPa}$

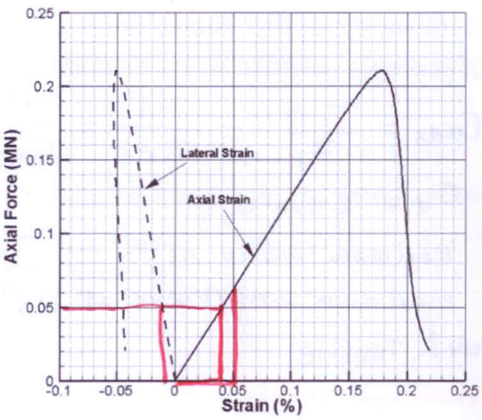
$\tau_{max} = \frac{1}{2}\sigma_x = 34.5\text{ MPa}$.

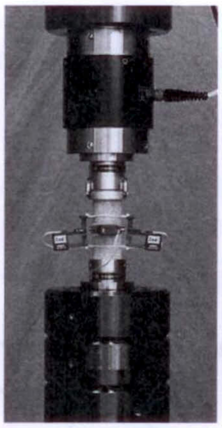
compressive strength $= \frac{P}{A} = \frac{0.21 \text{ MN}}{\frac{\pi}{4}\cdot(5.4\text{cm})^2} = \underline{91.7 \text{ MPa}}$

Elastic modulus, $E = \frac{\sigma}{\varepsilon}$ in linear portion $\rightarrow \frac{0.05 \text{ MN}/A}{0.04\times10^{-2}}$

$= \underline{54.6 \text{ GPa}}$

Poisson's ratio, $\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{axial}} = -\frac{-0.01}{0.04} = \underline{0.25}$





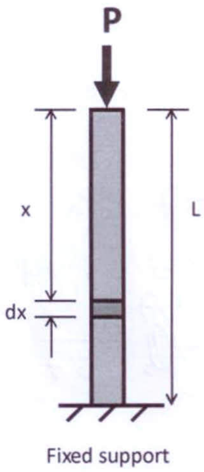
8.

The above graph was obtained from a uniaxial compressive test on a rock (granite) from a site for the underground storage of carbon dioxide. The graph shows the response of axial strain and lateral strains against the axial force applied on a cylindrical rock sample whose diameter is 5.4 cm and length is 10.0 cm (as shown in the picture). What are the uniaxial compressive strength (also called ultimate stress), elastic modulus and Poisson's ratio of the rock measured from the above experiment? Use the relevant units when necessary. (10).

9. Determine the strain energy of a cylindrical bar on a ground as shown in the below. Assume linearly elastic behavior and consider both the weight of the bar itself and a load P at the upper end. Dimensions and material properties are as follows. (10)

L: 10 m, Diameter: 20 cm, density of the bar (ρ): 2500 kg/m^3, P: 100 kN, Elastic modulus: 50 GPa,

g = acceleration = 9.8 m/sec^2.



internal axial force, $N(x)$ = force due to the weight + load P

$= \rho g A x + P.$

Strain energy $= \int_0^L \frac{[N(x)]^2\,dx}{2EA(x)}$

$= \int_0^L \frac{[\rho g A x + P]^2}{2EA}\,dx$

$= \frac{\rho^2 g^2 A L^3}{6E} + \frac{\rho g P L^2}{2E} + \frac{P^2 L}{2EA}$

$= \underline{34.4 \text{ J}}$