465.211 Mechanics in Energy Resources Engineering, Spring 2010

2nd Exam

08:30 - 11:00 28 April 2010

* Be careful about the units that you use.

* You may use Korean or English in answering above questions.

1. Indicate T(true) or F(false) for the following statements. <u>Note that an incorrect answer receives -1 mark while a correct answer receives +1 mark. You may leave the question blank if you wish.</u> (5)

(1) The moments of inertia of a plane area have dimensions of length to the third or fourth power depending on the cross sectional shape.

(2) When a beam has a circular cross section, we can no longer assume that the shear stresses acting throughout the entire section are parallel to the shear force.

(3) In a beam of square cross section subjected to a shear force V, minimum shear stress in the beam is zero and maximum shear stress is larger than the average shear stress (shear force divided by the cross sectional area).

(4) If the cross section of a beam is doubly symmetric (such as rectangular or circular), maximum tensile and compressive bending stresses are equal numerically.

(5) If the shear force is zero in a region of the beam, then the bending moment is also zero in that same region.

2. Determine the centroidal moments of inertia I_x and I_y for the wide-flange beam section shown in the below. (5)
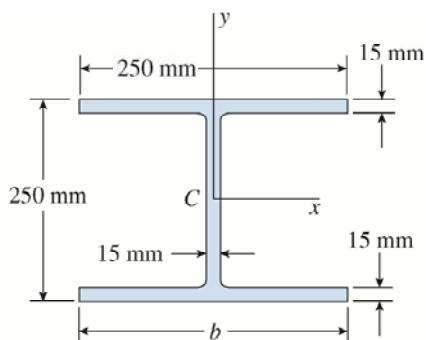
3. Determine the location of the centroid and the moments of Inertia with respect to x-axis and y-axis for the parabolic semisegment OAB shown in the below (10).
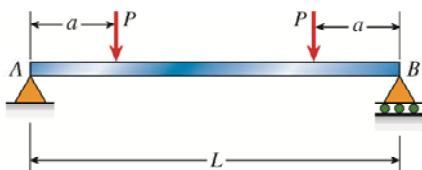

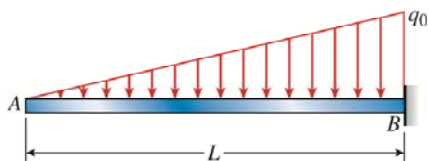$y = -8x^2 + 2$
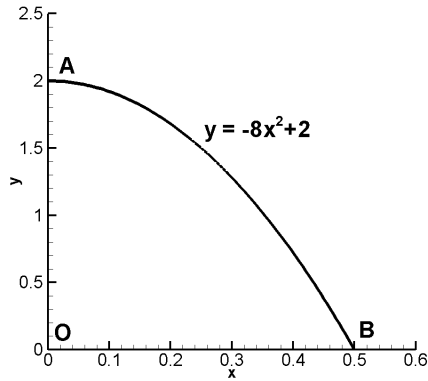
4. Derive the flexure formula or shear formula. (10)

$$\sigma_x = -\frac{My}{I}$$

or

$$\tau = \frac{VQ}{Ib}$$

* bonus (+5) if you derive both.

5.

(a) Draw the shear-force and bending-moment diagrams for a simple beam AB supporting two equal concentrated loads P. (5)


(b) Draw the shear-force and bending-moment diagrams for a cantilever beam AB supporting a linearly varying load of maximum intensity q0. (5)

6. A beam of T-section is supported and loaded as shown in the figure. The cross section has width b = 65 mm, height h = 75 mm, and thickness t = 13 mm. Determine the maximum tensile stress and maximum compressive stress in the beam. (10)
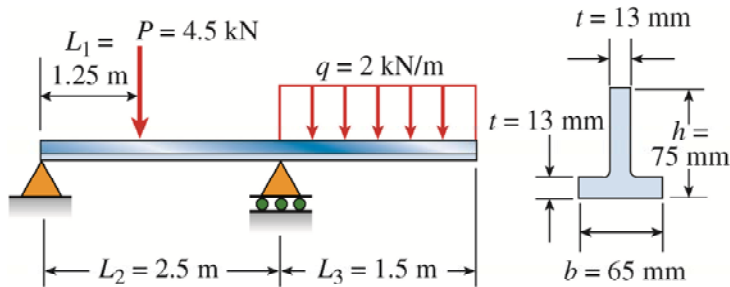
7. A temporary wood dam is constructed of horizontal planks A and supported by vertical wood posts B that are sunk into the ground so that they act as cantilever beams. The posts are of circular cross section (diameter d) with diameter d and spaced at distance s, center to center. Assume that the water level behind the dam is at its full height h = 2.0 m. The thickness of the plank is 75 mm and diameter of the pile is 300 mm. Assuming that the allowable stress in the wood is 11 MPa, calculate the maximum permissible spacing s of the piles. If the spacing is 5% larger than the permissible spacing of the piles, in which location will the fail occur? (15)
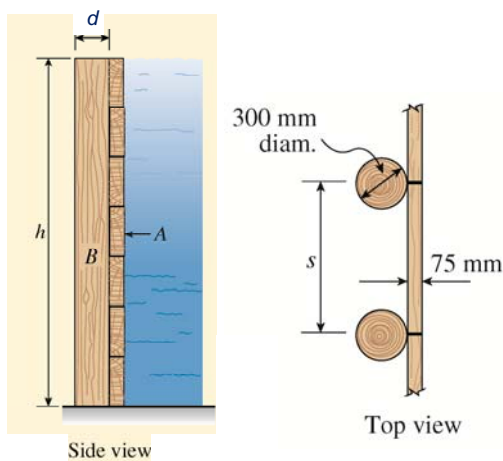
8. A metal beam with span L = 1 m is simply supported at points A and B. The uniform load on the beam (including its own weight) is q = 28 kN/m. The cross section of the beam is rectangular with width b = 25 mm and height h = 100 mm. The beam is adequately supported against sideways buckling. Determine the normal stress and shear stress at point C, which is located 25 mm below the top of the beam and 250 mm from the right-hand support. Show these stresses on a sketch of a stress element at point C. (10)
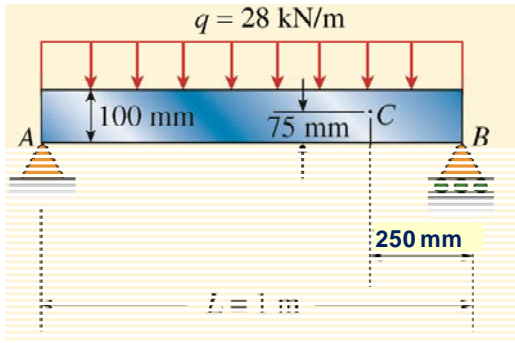

9. The shear stress τ in a rectangular beam are given by

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

in which V is the shear force, I is the moment of inertia of the cross-sectional area, h is the height of the beam, and y1 is the distance from the neutral axis to the point where the shear stress is being determined. Show that the resultant of the shear stress is equal to the shear force V. What is the ratio of maximum shear stress to average shear stress? (5)
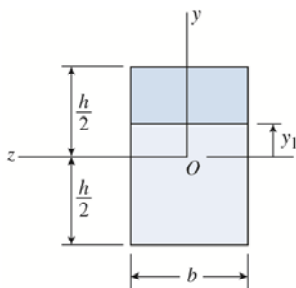
10. Determine the ratios of the weights of four beams that have the same length, are made of the same material, are subjected to the same maximum bending moment, and have the same maximum bending stress if their cross sections are (1) a rectangle with height equal to twice the width, (2) a rectangle with width equal to twice the height, (3) a square, and (4) a circle. (10)
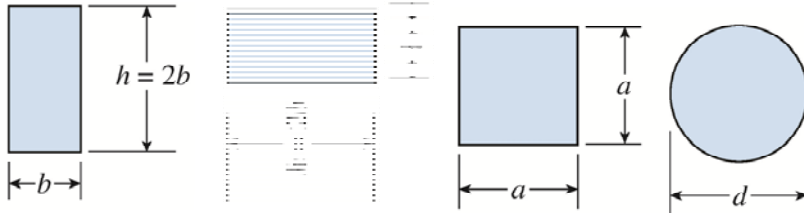
11. A simply supported wood beam of rectangular cross section and span length 1.2 m carries a concentrated load P at midspan in addition to its own weight. The cross section has width 140 mm and height 240 mm. The density of the wood is 5400 kg/m3. Calculate the maximum permissible value of the load P if (a) the allowable bending stress is 8.0 MPa, and (b) the allowable shear stress is 1.0 MPa. (10)
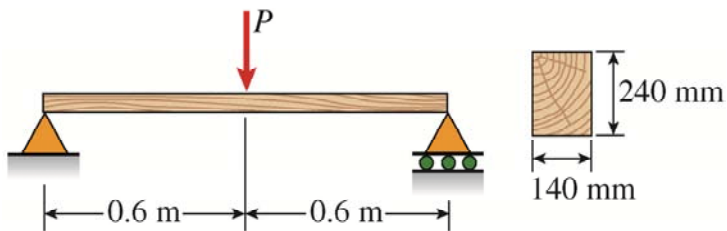
Sign convention.
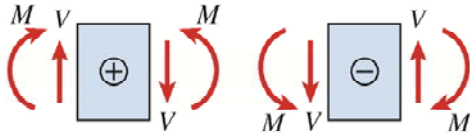
Formulas

$$\frac{dV}{dx} = -q \qquad \frac{dM}{dx} = V \qquad \kappa = \frac{1}{\rho} \qquad \kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \qquad \kappa = \frac{1}{\rho} = \frac{d\theta}{dx}$$

$$\varepsilon_x = -\frac{y}{\rho} = -\kappa y \qquad \sigma_x = E\varepsilon_x = -\frac{Ey}{\rho} = -E\kappa y \qquad I = \int_A y^2 dA \qquad \kappa = \frac{1}{\rho} = \frac{M}{EI} \qquad \sigma_x = -\frac{My}{I}$$

$$\sigma_x = -\frac{Mc_1}{I} = -\frac{M}{S_1} \qquad \sigma_x = \frac{Mc_2}{I} = \frac{M}{S_2} \qquad S_1 = \frac{I}{c_1} \qquad S_2 = \frac{I}{c_2}$$

rectangular cross section with width b and height h.
$$I = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$

circular cross section of diameter d
$$I = \frac{\pi d^4}{64} \qquad S = \frac{\pi d^3}{32}$$

wide flange beam
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3)$$

$$S = \frac{M_{max}}{\sigma_{allow}} \qquad \tau = \frac{VQ}{Ib} \qquad \text{shear stress in the web} \qquad \tau = \frac{VQ}{It} = \frac{V}{8It}\left[b\left(h^2 - h_1^2\right) + t\left(h_1^2 - 4y_1^2\right)\right]$$

$$Q_x = \int ydA \qquad Q_y = \int xdA \qquad \bar{x} = \frac{Q_y}{A} = \frac{\sum_{i=1}^{n} \bar{x}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{Q_x}{A} = \frac{\sum_{i=1}^{n} \bar{y}_i A_i}{\sum_{i=1}^{n} A_i} \qquad I_x = \int y^2 dA$$

$$I_y = \int x^2 dA \qquad I_x = I_{xc} + Ad_1^2 \qquad I_y = I_{yc} + Ad_2^2 \qquad I_p = \int \rho^2 dA \qquad I_p = I_x + I_y \qquad (I_p)_O = (I_p)_C + Ad^2$$