465.211 Mechanics in Energy Resources Engineering, Spring 2010

Final Exam

08:30 - 11:00 14 June 2010

* Be careful about the units that you use.

* You may use Korean or English in answering the questions.

1. Indicate T(true) or F(false) for the following statements. Note that an incorrect answer receives -1 mark while a correct answer receives +1 mark. You may leave the question blank if you wish. (5)

(1) The strain energy of a structure supporting more than one load can be obtained by adding the strain energies for the individual loads acting separately. (F)

(2) In a circular bar in torsion, a stress element oriented at an angle of 45 degree is acted upon by equal tensile and compressive stresses in perpendicular directions without any shear stresses (T)

(3) The maximum shear stress is equal to one-half the difference of the principal stresses. (T)

(4) The shear stresses are always zero on the principal planes. (T)

(5) In linearly elastic isotropic materials, there are three independent material properties, namely, elastic modulus (E), shear modulus (G) and Poisson's ratio (ν). (F)

2.

(a) Explain what the Mohr's Circle is. Give an example if necessary. (10)

(b) Derive the following differential equations of the deflection curve (10).

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

(c) Derive the Transformation equations for plane stress as show in the below. (10)

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

3. The rails of a railroad track are welded together at their ends (to form continuous rails and thus eliminate the clacking sound of the wheels) when the temperature is 10°C. What compressive stress produced in the rails when they are heated by the sun to 52°C if the coefficient of thermal coefficient is 12 x 10^{-6} /°C and the modulus of elasticity is 200 GPa. Consider one-dimensional stress and strain only. (10)

The rail is prevented from expanding because of their great length and lack of expansion joints.

Therefore, the rail is in the same condition as a bar with fixed ends (see Example 2-7).

The compressive stress in the rails may be calculated from Eq. (2-18).

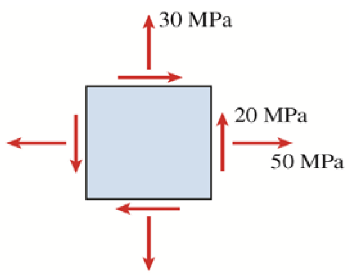
$$\Delta T = 52°C - 10°C = 42°C$$

$$\sigma = E\alpha(\Delta T)$$

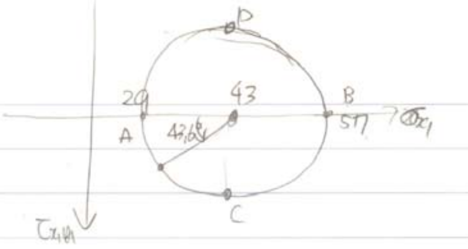
$$= (200 \text{ GPa})(12 \times 10^{-6}/°C)(42°C)$$

$$= 100.8 \text{ MPa} \quad \leftarrow \text{(compression)}$$

4. An element in *plane stress* is subjected to stresses σ_x = 50 MPa, σ_y = 30 MPa, and τ_{xy} = 20 MPa, as shown in the figure. Determine the principal stresses, principal angles and maximum shear stress. Show the principal stresses on a sketch of a properly oriented element. What is the stresses acting on an element oriented at an angle θ = 45° from the *x* axis, where the angle θ is positive when counterclockwise. (10)



5. $\sigma_{aver} = 43\,MPa$



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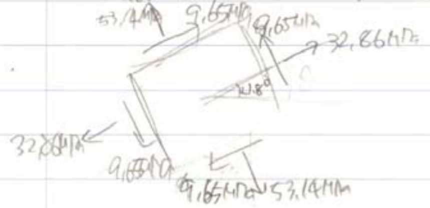
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 14$

(a) $\tan\theta = \frac{1}{2.5} \rightarrow \theta = 21.8°$

$\sigma_{x_1} = 43 - 14\cos(43.6) = 32.86\,MPa$ ✓

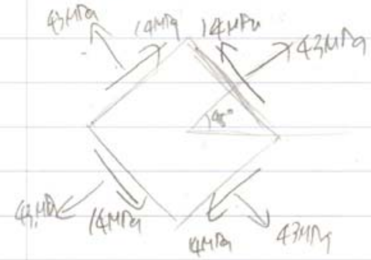
$\sigma_{y_1} = 53.14\,MPa$ ✓

$\tau_{x_1y_1} = 14\sin(43.6) = 9.65\,MPa$ ✓



(b) C점과 D점에서 Maximum shear stress가 된다.

C점에서 $\theta = 45°$ 일때 $\tau_{x_1y_1} = 14MPa$, $\sigma_{x_1} = \sigma_{y_1} = 43MPa$ 이된다 ✓ ✓

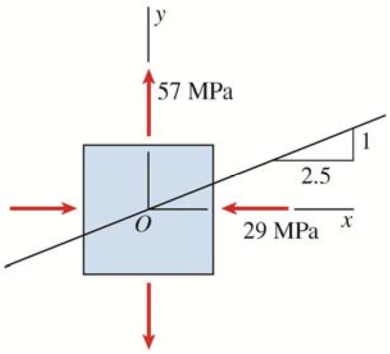


5. An element in *biaxial stress* is subjected to stresses σ_x = 29 MPa and σ_y = 57 MPa, as shown in the figure. Using Mohr's circle, determine: (10)

(a) The stresses acting on an element oriented at a slope of 1 on 2.5 (see figure).

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



$\sigma_x = -29$ MPa $\quad \sigma_y = 57$ MPa $\quad \tau_{xy} = 0$ MPa

(a) Element at a slope of 1 on 2.5

$\theta = \text{atan}\left(\frac{1}{2.5}\right) \quad \theta = 21.801^\circ \quad \leftarrow$

$2\theta = 43.603^\circ \quad R = \frac{|\sigma_x| + |\sigma_y|}{2} \quad R = 43.0$ MPa

Point C: $\sigma_c = \sigma_x + R \quad \sigma_c = 14.0$ MPa

Point D: $\sigma_{x1} = \sigma_c - R\cos(2\theta)$

$\sigma_{x1} = -17.1$ MPa $\leftarrow$

$\tau_{x1y1} = R\sin(2\theta) \quad \tau_{x1y1} = 29.7$ MPa $\leftarrow$

Point D′: $\sigma_{y1} = \sigma_c + R\cos(2\theta) \quad \sigma_{y1} = 45.1$ MPa

(b) Maximum shear stresses

Point S1: $\theta_{s1} = \frac{90^\circ}{2} \quad \theta_{s1} = 45.0^\circ \quad \leftarrow$

$\tau_{max} = R \quad \tau_{max} = 43.0$ MPa $\leftarrow$

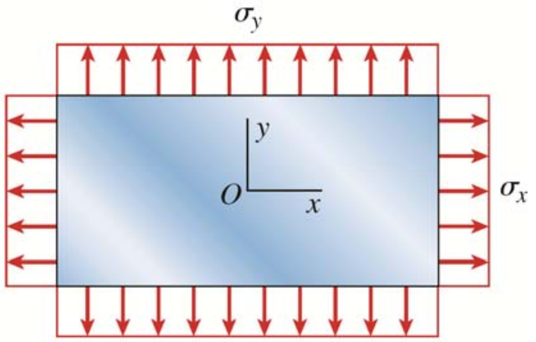
Point S2: $\theta_{s2} = \frac{-90^\circ}{2}$

$\theta_{s2} = -45.0^\circ \quad \leftarrow$

$\tau_{max} = -R \quad \tau_{max} = -43.0$ MPa $\leftarrow$

$\sigma_{aver} = \sigma_c \quad \sigma_{aver} = 14.0$ MPa $\leftarrow$

6. A magnesium plate in *biaxial stress* is subjected to tensile stresses σ_x =24 MPa and σ_y =12 MPa (see figure). The rectangular plate is of dimensions 200 mm (horizontal) x 100 mm (vertical) x 10 mm (thickness). The corresponding strains in the plate are ε_x = 440 x 10^{-6} and ε_y = 80 x 10^{-6}. Determine Poisson's ratio and the modulus of elasticity *E* for the material. What is the strain energy density and strain energy stored in the plate? (10)



6. $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$, $\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

10

$E \times (440 \times 10^{-6}) = (24 \times 10^6) - \nu(12 \times 10^6)$ — ①

$E \times (80 \times 10^{-6}) = (12 \times 10^6) - \nu(24 \times 10^6)$ — ②

① × 2 − ②

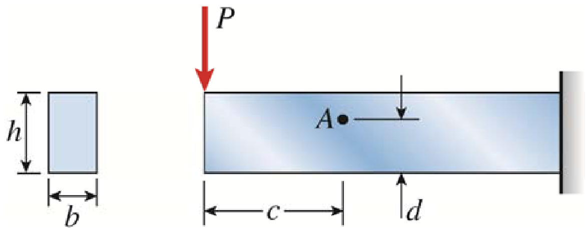
$E \times (800 \times 10^{-6}) = (36 \times 10^6)$ → $E = 45\,GPa$ ✓

$\nu = 0.35$ ✓

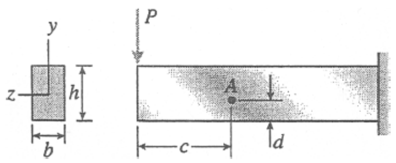
$u = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y) = 5760\ J/m^3$ ✓

$U = uV_0 = 1.152\,J$ ✓

7. A cantilever beam of rectangular cross section is subjected to a concentrated load P = 70 kN acting at the free end (see figure). The beam has width b = 100 mm and height h = 250 mm. Point *A* is located at distance c = 600 mm from the free end and distance d = 75 mm from the bottom of the beam. Calculate the principal stresses σ_1 and σ_2 and the maximum shear stress τ_{max} at point *A*. Show these stresses on sketches of properly oriented elements. (15)



CANTILEVER BEAM



$P = 70\text{ kN} \quad c = 600\text{ mm} \quad b = 100\text{ mm} \quad d = 75\text{ mm}$

$h = 250\text{ mm}$

STRESS AT POINT *A*

$$I = \frac{bh^3}{12} \qquad I = 130.2 \times 10^6\text{ mm}^4$$

$$M = -Pc \qquad M = -42\text{ kN}\cdot\text{m} \qquad V = P \qquad V = 70\text{ kN}$$

$$y_A = -\frac{h}{2} + d \qquad y_A = -50\text{ mm}$$

$$\sigma_x = -\frac{My_A}{I} \qquad \sigma_x = \frac{(-42 \times 10^6\text{ N}\cdot\text{mm})(-50\text{ mm})}{(130.2 \times 10^6\text{ mm}^4)}$$

$$= -16.13\text{ MPa}$$

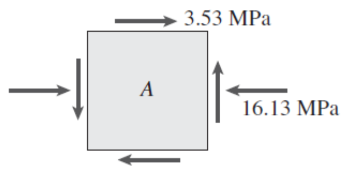
$$Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) \qquad Q = 656250\text{ mm}^3 \qquad \tau = \frac{VQ}{Ib}$$

$$\tau = \frac{(70\text{ kN})(656250\text{ mm}^3)}{(130.2 \times 10^6\text{ mm}^4)(100\text{ mm})} = 3.53\text{ MPa}$$

$$\tau_{xy} = \tau \qquad \sigma_x = -16.13\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 3.53\text{ MPa}$$

PRINCIPAL STRESSES

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.4377$$



$$\theta_p = \frac{1}{2}\operatorname{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad 2\theta_p = -23.64^\circ$$

and $\theta_p = -11.82^\circ$

$2\theta_p = 156.36^\circ$ and $\theta_p = 78.18^\circ$

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

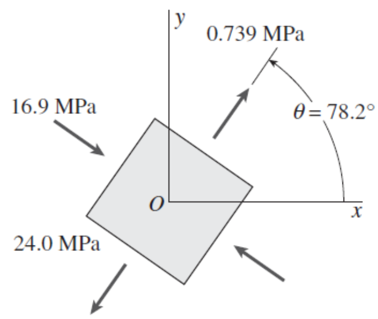
$\sigma_{x1} = 60.306\text{ MPa}$

For $2\theta_p = 156.36^\circ \quad \sigma_1 = 0.739\text{ MPa}$

For $2\theta_p = -23.64^\circ \quad \sigma_2 = -16.90\text{ MPa}$

Therefore

$\sigma_1 = 0.739\text{ MPa} \quad \leftarrow \qquad \theta_{p1} = 78.2^\circ \quad \leftarrow$



$\sigma_2 = -16.9$ Mpa ← $\theta_{p2} = -11.82°$ ←

MAXIMUM SHEAR STRESS

$$\tau_{max} = \sqrt{\left(\frac{\tau_x - \tau_y}{2}\right)^2 + \tau_{xy}^2} \quad \tau_{max} = 8.80 \text{ MPa} \quad \leftarrow$$

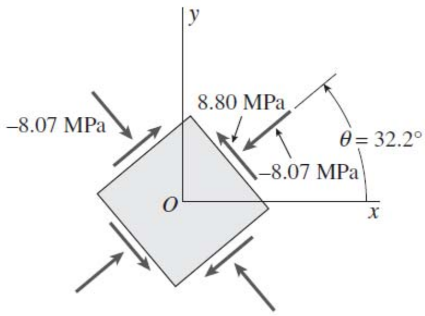
$\theta_{s1} = \theta_{p1} - 45°$

$\theta_{s1} = 32.2°$ and $\tau_{max} = 8.80$ MPa ←

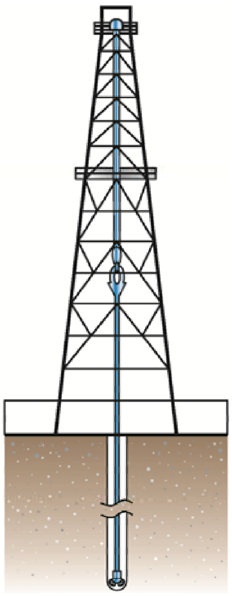
$\theta_{s2} = \theta_{s1} + 90°$

$\theta_{s2} = 123.2°$ and $\tau = -8.80$ MPa ←

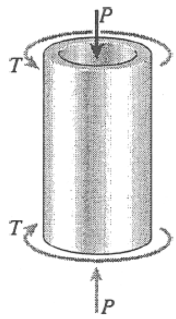
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad \sigma_{avg} = -8.07 \text{ MPa} \quad \leftarrow$$



8. The hollow drill pipe for an oil well (see figure) is 150 mm in outer diameter and 15 mm in thickness. Just above the bit, the compressive force in the pipe (due to the weight of the pipe) is 265 kN and the torque (due to drilling) is 19 kN m. Determine the maximum tensile, compressive, and shear stresses in the drill pipe. (15)



Drill pipe



P = compressive force $\quad T$ = Torque

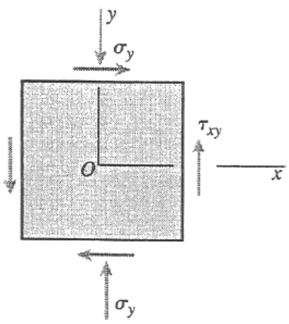
d_2 = outer diameter $\quad d_1$ = inner diameter

$P = 265$ kN $\quad T = 19$ kN·m $\quad d_2 = 150$ mm

$d_1 = d_2 - 2t \quad d_1 = 120$ mm

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) \qquad A = 6.362 \times 10^{-3}\text{m}^2$$

$$I_p = \frac{\pi}{32}\left(d_2^4 - d_1^4\right) \qquad I_p = 29.34 \times 10^{-6}\text{ m}^4$$



Stresses at the outer surface

$$\sigma_y = -\frac{P}{A} \qquad \sigma_y = -265\text{ kN}/\ 6.362 \times 10^{-3}\text{ m}^2$$

$$= -41.655\text{ MPa}$$

$$\sigma_x = 0$$

$$\tau_{xy} = \frac{T\,r}{I_p} = \frac{(19\text{ kN·m} \times 75\text{ mm})}{29.34 \times 10^{-6}\text{ m}^4} \qquad \tau_{xy} = 48.563\text{ MPa}$$

Principal stresses

$$\sigma_{1,2} = \frac{\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{41.65}{2} \pm \sqrt{\left(\frac{41.65}{2}\right)^2 + (48.57)^2}$$

$$= -20.83\text{ MPa} \pm 52.85\text{ MPa}$$

$\sigma_1 = 32.0$ MPa, $\sigma_2 = 73.7$ MPa

Maximum tensile stress $\qquad \sigma_t = \sigma_1$

$\sigma_t = 32.0$ MPa $\quad \leftarrow$

Maximum compressive stress $\qquad \sigma_c = \sigma_2$

$\sigma_c = -73.7$ MPa $\quad \leftarrow$

Maximum in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \qquad \tau_{max} = 52.8\text{ MPa} \quad \leftarrow$$

Note: Since the principal stresses have opposite signs, the maximum in-plane shear is larger than the maximum out-of-plane shear stress.

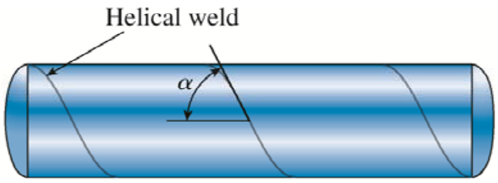
9. A pressurized steel tank is constructed with a helical weld that makes an angle α = 55° with the longitudinal axis (see figure). The tank has radius r = 0.5 m, wall thickness t = 10 mm, and internal pressure p = 2.0 MPa. Also, the steel has modulus of elasticity E = 100 GPa and Poisson's ratio v = 0.30. The equations for circumferential and longitudinal stresses are as follows. (15)

Circumferential stress $\sigma_1 = \dfrac{pr}{t}$

Longitudinal stress $\sigma_2 = \dfrac{pr}{2t}$

Determine the following quantities for the cylindrical part of the tank.
(a) The circumferential and longitudinal stresses.

(b) The maximum in-plane and out-of-plane shear stresses.
(c) The circumferential and longitudinal strains.
(d) The normal and shear stresses acting on planes parallel and perpendicular to the weld (show these stresses on a properly oriented stress element).



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9. (a) $\sigma_1 = \frac{pr}{t} = 100\text{MPa}$ ✓
$\sigma_2 = \frac{pr}{2t} = 50\text{MPa}$

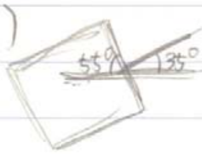
(b) maximum in-plane $\tau_{max}|_z = \frac{pr}{4t} = 25\text{MPa}$ ✓
maximum out-of-plane $\tau_{max} = \frac{pr}{2t} = 50\text{MPa}$

(c) circumferential strain ε_y

$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = 0.00085$ ✓

longitudinal strain ε_x

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}(\sigma_2 - \nu\sigma_1) = 0.0002$ ✓

(d)


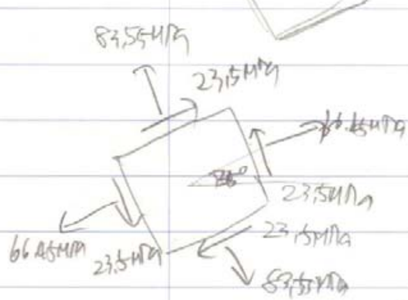
$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= 66.45\text{MPa}$ ✓

$\sigma_{y_1} = 83.55\text{MPa}$

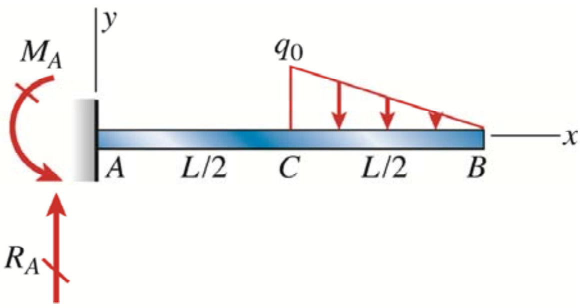
$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= 23.5\text{MPa}$ ✓



$\sigma_w = 66.45\text{MPa}$, $\tau_w = 23.5\text{MPa}$ ✓

10. Derive the equations of the deflection curve for a cantilever beam *AB* supporting a distributed load of peak intensity q_0 acting over one-half of the length (see figure). Also, obtain formulas for the deflections δ_B and δ_C at points *B* and *C*, respectively. (10)



BENDING-MOMENT EQUATION

For $0 \le x \le \frac{L}{2}$

$$EIv'' = M(x) = \frac{q_0 L x}{4} - \frac{q_0 L^2}{6}$$

$$EIv' = \frac{q_0 L x^2}{8} - \frac{q_0 L^2 x}{6} + C_1$$

$$EIv = \frac{q_0 L x^3}{24} - \frac{q_0 L^2 x^2}{12} + C_1 x + C_2$$

B.C. $v'(0) = 0 \qquad C_1 = 0$

B.C. $v(0) = 0 \qquad C_2 = 0$

$$v'\left(\frac{L}{2}\right) = -\frac{5 q_0 L^3}{96 EI}$$

$$v(x) = \frac{q_0 L}{24 EI}(x^3 - 2Lx^2) \quad \leftarrow$$

$$\delta_C = -v\left(\frac{L}{2}\right) = \frac{q_0 L^4}{64 EI} \quad \leftarrow$$

For $\frac{L}{2} \le x \le L$

$$EIv'' = M(x) = \frac{q_0 L x}{4} - \frac{q_0 L^2}{6} - \frac{q_0}{L}(L - x)\left(x - \frac{L}{2}\right)^2 - \frac{1}{2}\left[q_0 - \frac{2 q_0}{L}(L - x)\right]\left(x - \frac{L}{2}\right)^2 \frac{2}{3}$$

$$EIv'' = M(x) = \frac{-q_0}{3L}(-3L^2 x + L^3 + 3Lx^2 - x^3)$$

$$EIv' = -\frac{q_0}{3L}\left(\frac{-3}{2}L^2 x^2 + L^3 x + Lx^3 - \frac{x^4}{4}\right) + C_3$$

$$EIv = -\frac{q_0}{3L}\left(\frac{-1}{2}L^2 x^3 + \frac{1}{2}L^3 x^2 + \frac{1}{4}Lx^4 - \frac{1}{20}x^5\right) + C_3 x + C_4$$

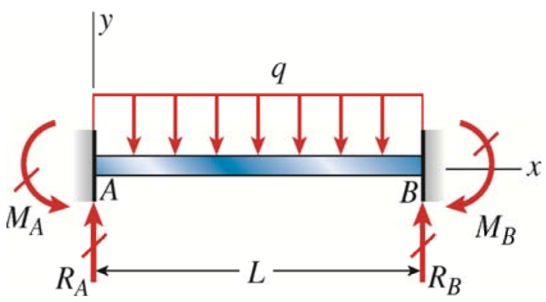
B.C. $v'\left(\frac{L}{2}\right) = -\frac{5 q_0 L^3}{96 EI} \qquad C_3 = \frac{5}{192} q_0 L^3$

B.C. $v\left(\frac{L}{2}\right) = -\frac{q_0 L^4}{64 EI} \qquad C_4 = \frac{-1}{320} q_0 L^4$

$$v(x) = \frac{-q_0}{960 LEI}(-160L^2 x^3 + 160L^3 x^2 + 80Lx^4 - 16x^5 - 25L^4 x + 3L^5) \quad \leftarrow$$

$$\delta_B = -v(L) = \frac{7 q_0 L^4}{160 EI} \quad \leftarrow$$

11. A fixed-end beam AB of length L supports a uniform load of intensity q (see figure). Beginning with the second-order differential equation of the deflection curve (the bending-moment equation), obtain the reactions, shear forces, bending moments, slopes, and deflections of the beam. Construct the shear-force and bending-moment diagrams, labeling all critical values. (10)



Select M_A as the redundant reaction.

REACTIONS (FROM SYMMETRY AND EQUILIBRIUM)

$$R_A = R_B = \frac{qL}{2} \qquad M_B = M_A$$

BENDING MOMENT (FROM EQUILIBRIUM)

$$M = R_A x - M_A - \frac{qx^2}{2} = -M_A + \frac{q}{2}(Lx - x^2) \qquad (1)$$

DIFFERENTIAL EQUATIONS

$$EIv'' = M = -M_A + \frac{q}{2}(Lx - x^2)$$

$$EIv' = -M_A x + \frac{q}{2}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) + C_1 \qquad (2)$$

B.C. 1 $v'(0) = 0$ $\quad \therefore C_1 = 0$

$$EIv = -\frac{M_A x^2}{2} + \frac{q}{2}\left(\frac{Lx^3}{6} - \frac{x^4}{12}\right) + C_2 \qquad (3)$$

B.C. 2 $v(0) = 0$ $\quad \therefore C_2 = 0$

B.C. 3 $v(L) = 0$ $\quad \therefore M_A = \frac{qL^2}{12}$

REACTIONS

$$R_A = R_B = \frac{qL}{2} \qquad M_A = M_B = \frac{qL^2}{12} \qquad \leftarrow$$

SHEAR FORCE (FROM EQUILIBRIUM)

$$V = R_A - qx = \frac{q}{2}(L - 2x) \qquad \leftarrow$$

BENDING MOMENT (FROM EQ. 1)

$$M = -\frac{q}{12}(L^2 - 6Lx + 6x^2) \quad \leftarrow$$

SLOPE (FROM EQ. 2)

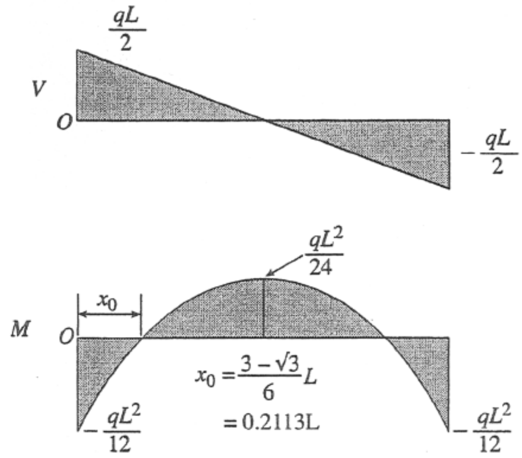
$$v' = -\frac{qx}{12EI}(L^2 - 3Lx + 2x^2) \quad \leftarrow$$

DEFLECTION (FROM EQ. 3)

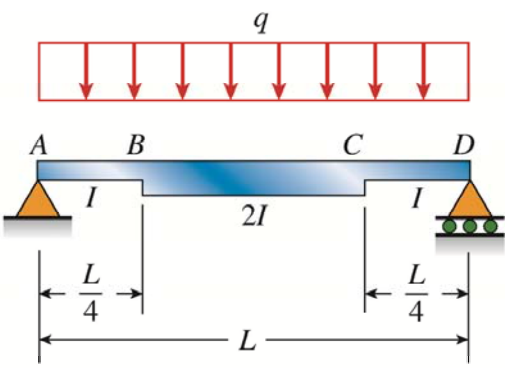
$$v = -\frac{qx^2}{24EI}(L - x)^2 \quad \leftarrow$$

$$\delta_{max} = -v\left(\frac{L}{2}\right) = \frac{qL^4}{384EI}$$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS

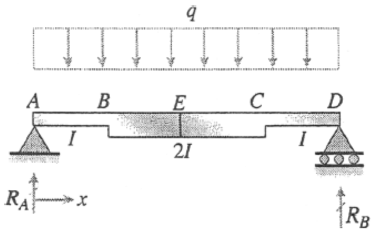


12. A simple beam *ABCD* has moment of inertia *I* near the supports and moment of inertia 2*I* in the middle region, as shown in the figure. A uniform load of intensity *q* acts over the entire length of the beam. Determine the equations of the deflection curve for the left-hand half of the beam. Also, find the angle of rotation θ_A at the left-hand support and the deflection δ_{max} at the midpoint. (10)

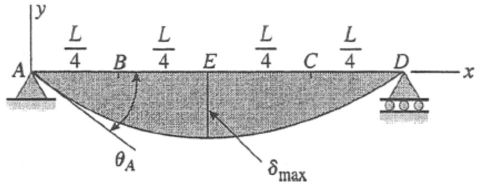


Use the bending-moment equation (Eq. 9-12a).

REACTIONS, BENDING MOMENT, AND DEFLECTION CURVE



$$R_A = R_B = \frac{qL}{2} \qquad M = Rx - \frac{qx^2}{2} = \frac{qLx}{2} - \frac{qx^2}{2}$$



BENDING-MOMENT EQUATIONS FOR THE LEFT-HAND HALF OF THE BEAM

$$EIv'' = M = \frac{qLx}{2} - \frac{qx^2}{2} \qquad \left(0 \le x \le \frac{L}{4}\right) \qquad (1)$$

$$E(2I)v'' = M = \frac{qLx}{2} - \frac{qx^2}{2} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right) \qquad (2)$$

INTEGRATE EACH EQUATION

$$EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} + C_1 \qquad \left(0 \le x \le \frac{L}{4}\right) \qquad (3)$$

$$2EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} + C_2 \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right) \qquad (4)$$

B.C. 1 Symmetry: $v'\left(\frac{L}{2}\right) = 0$

From Eq. (4): $C_2 = -\frac{qL^3}{24}$

$$2EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} - \frac{qL^3}{24} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right) \qquad (5)$$

SLOPE AT POINT B (FROM THE RIGHT)

Substitute $x = \frac{L}{4}$ into Eq. (5):

$$EIv'_B = -\frac{11qL^3}{768} \qquad (6)$$

B.C. 2 CONTINUITY OF SLOPES AT POINT B

$(v'_B)_{\text{Left}} = (v'_B)_{\text{Right}}$

From Eqs. (3) and (6):

$$\frac{qL}{4}\left(\frac{L}{4}\right)^2 - \frac{q}{6}\left(\frac{L}{4}\right)^3 + C_1 = -\frac{11qL^3}{768} \qquad \therefore C_1 = -\frac{7qL^3}{256}$$

SLOPE OF THE BEAM (FROM EQS. 3 AND 5)

$$EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} - \frac{7qL^3}{256} \qquad \left(0 \le x \le \frac{L}{4}\right) \qquad (7)$$

$$EIv' = \frac{qLx^2}{8} - \frac{qx^3}{12} - \frac{qL^3}{48} \qquad \left(\frac{L}{4} \le x \le \frac{L}{3}\right) \qquad (8)$$

ANGLE OF ROTATION θ_A (FROM EQ. 7)

$$\theta_A = -v'(0) = \frac{7qL^3}{256EI} \text{ (positive clockwise)} \qquad \leftarrow$$

INTEGRATE EQS. (7) AND (8)

$$EIv = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{7qL^3x}{256} + C_3 \quad \left(0 \le x \le \frac{L}{4}\right) \qquad (9)$$

$$EIv = \frac{qLx^3}{24} - \frac{qx^4}{48} - \frac{qL^3x}{48} + C_4 \quad \left(\frac{L}{4} \le x \le \frac{L}{2}\right) \qquad (10)$$

B.C. 3 Deflection at support A

$v(0) = 0$ From Eq. (9): $C_3 = 0$

DEFECTION AT POINT B (FROM THE LEFT)

Substitute $x = \frac{L}{4}$ into Eq. (9) with $C_3 = 0$

$$EIv_B = -\frac{35\,qL^4}{6144} \qquad (11)$$

B.C. 4 Continuity of deflections at point B

$(v_B)_{\text{Right}} = (v_B)_{\text{Left}}$

From Eqs. (10) and (11):

$$\frac{qL}{24}\left(\frac{L}{4}\right)^3 - \frac{q}{48}\left(\frac{L}{4}\right)^4 - \frac{qL^3}{48}\left(\frac{L}{4}\right) + C_4 = -\frac{35qL^4}{6144}$$

$$\therefore\ C_4 = -\frac{13qL^4}{12{,}288}$$

DEFECTION OF THE BEAM (FROM EQS. 9 AND 10)

$$v = -\frac{qx}{768EI}(21L^3 - 64Lx^2 + 32x^3)$$

$$\left(0 \le x \le \frac{L}{4}\right) \quad \leftarrow$$

$$v = -\frac{q}{12{,}288EI}(13L^4 + 256L^3x - 512Lx^3 + 256x^4)$$

$$\left(\frac{L}{4} \le x \le \frac{L}{2}\right) \quad \leftarrow$$

MAXIMUM DEFLECTION (AT THE MIDPOINT E)

(From the preceding equation for v.)

$$\delta_{\max} = -v\left(\frac{L}{2}\right) = \frac{31qL^4}{4096EI} \quad \text{(positive downward)} \quad \leftarrow$$