Computer Aided Ship Design

- Midterm Exam -

Saturday, October 22nd, 2011

Time: 16:00-19:00 (3 hours)

Name	
SNU ID #	

Note: Budget your time wisely. Some parts of this exam could take you much longer time than others. Move on if you are stuck and return to the problem later.

Optimization Method

Problem Number		1			2		3	4	Total
		1.1.	1.2.	1.3.	2.1.	2.2.	3.	4.	
Grader	Max	10	10	10	10	10	20	30	100
	Score								

1. Unconstrained Optimization Problem

[1] Find the minimum design point of the following nonlinear unconstrained optimization problem with two unknown variables by using three kinds of different unconstrained optimization methods.

Given: Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1 x_2 + x_2^2$

Starting design point: $\mathbf{x}^{(0)} = (0, 0)$,

Convergence tolerance $\varepsilon = 0.001$

① *Conjugate Gradient method*

② *Newton's method*

③ *Broyden-Fletcher-Goldfarb-Shanno(BFGS) method*

[Reference] Broyden-Fletcher-Goldfarb-Shanno(BFGS) Method

■ **Update the matrix $\tilde{\mathbf{H}}^{(k)}$ - approximation for the Hessian matrix of the objective function - as**

$$\tilde{\mathbf{H}}^{(k+1)} = \tilde{\mathbf{H}}^{(k)} + \mathbf{D}^{(k)} + \mathbf{E}^{(k)} \quad : \quad n \times n \text{ matrix}$$

where the correction matrices $\mathbf{D}^{(k)}$ and $\mathbf{E}^{(k)}$ are given as follows:

$$\mathbf{D}^{(k)} = \frac{\mathbf{y}^{(k)} \mathbf{y}^{(k)^T}}{(\mathbf{y}^{(k)} \cdot \mathbf{s}^{(k)})}; \qquad \mathbf{E}^{(k)} = \frac{\mathbf{c}^{(k)} \mathbf{c}^{(k)^T}}{(\mathbf{c}^{(k)} \cdot \mathbf{d}^{(k)})};$$

$\mathbf{s}^{(k)} = \alpha_k \mathbf{d}^{(k)}$: change in design

$\mathbf{y}^{(k)} = \mathbf{c}^{(k+1)} - \mathbf{c}^{(k)}$: change in gradient

$\mathbf{c}^{(k+1)} = \nabla f(\mathbf{x}^{(k+1)})$

$\mathbf{d}^{(k)}$: search direction

$\alpha^{(k)}$: optimum step size

[2] The function of the shipbuilding cost with two unknown variables, L/B and C_B, is plotted as in the following figures, find the optimal values for L/B and C_B in order to minimize the shipbuilding cost. Minimize the objective functions by using two different kind of unconstrained optimization methods and plot the procedures in the graph.

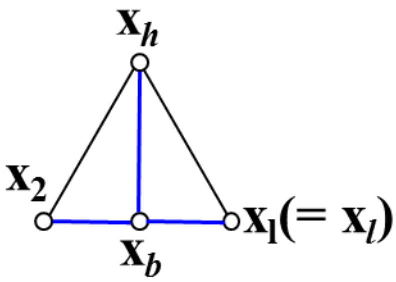
[2.1] Hooke & Jeeves direct search method

A. Starting design point: $L/B = 1.0$, $C_B = 0.9$
B. Step size at the starting design point: $\Delta(L/B) = 0.5$, $\Delta(C_B) = 0.1$
C. Stopping criterion: $\Delta(L/B) \le 0.125,\ \Delta(C_B) \le 0.025$,

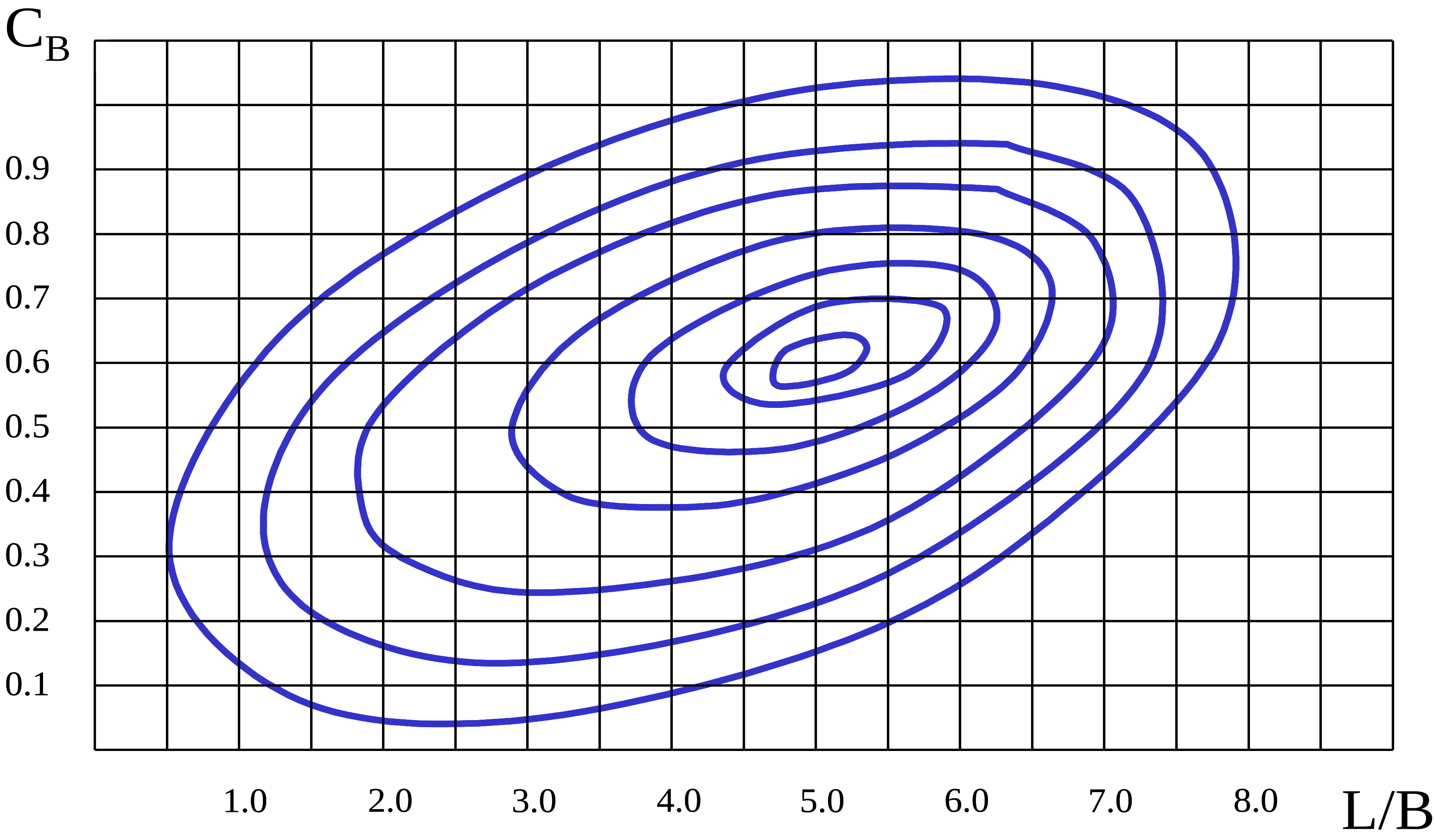
[2.2] Nelder & Mead Simplex method

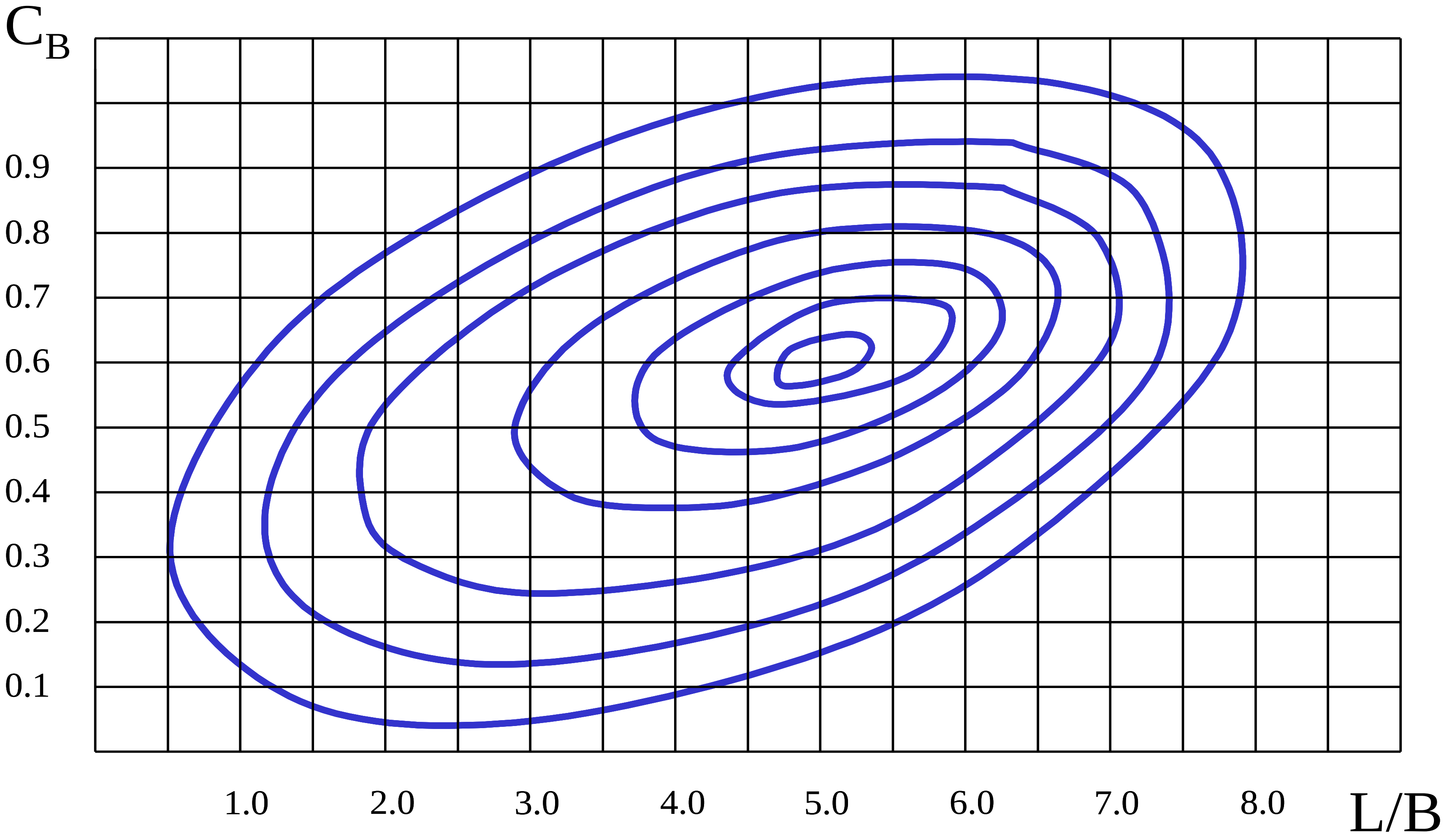
A. Starting corners of the simplex: $(L/B, C_B)$= (1.0, 0.9), (1.5, 0.9), (1.0, 0.8)
B. Stopping criterion: Average of the distance between each corners and $\mathbf{x}_b \le 0.01$

Example)



[2.1] Hooke & Jeeves direct search method




C_B
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
L/B

2. Constrained Optimization Problem

[3] The operation of the ballast tanks of a ship is as follows.

Tank	**Capacity of the tank(m^3)**	**Change of the trim(m) when 1,000m^3 ballast water is filled in the tank**	**Change of the GM(m) when 1,000m^3 ballast water is filled in the tank**
1	500	0	0.1
2	1,000	-2	0.1
3	1,500	0.5	0.5
4	2,000	-1	0.2
5	2,500	-0.2	0.4

The current value of the trim of the ship is 2.0 m and that of the GM is 0.1 m. However, the ship is required to maintain its trim at 0.0m and its GM larger than 0.6m. By filling the ballast tanks partially or fully, find the optimal amount of the ballast water to be filled in the ballast tanks for satisfying the requirements of the ship and minimizing the total amount of the required ballast water. Formulate this problem and solve that by using the Simplex method.

Hint:

Design variables: the amount of the ballast water to be filled in the each ballast tank

Constraints:

1) Limitation of the capacity of each ballast tank
2) Required value of GM of the ship
3) Required value of trim of the ship

[4] Solve the following nonlinear unconstrained optimization problem by using the sequential quadratic programming method.

Minimize: $f(\mathbf{x}) = x_1^2 + x_2^2 - 6x_1 - 8x_2 + 10$

The starting design point: $\mathbf{x}^{(0)} = (1,1),$

Subject to:

$$4x_1^2 + x_2^2 \le 16$$
$$3x_1 + 5x_2 \le 15$$
$$x_i \ge 0,\ i = 1,2$$

Penalty function (Pshenichny's descent function, $\Phi(x^{(k)})$

$$\Phi(\mathbf{x}^{(k)}) = f(\mathbf{x}^{(k)}) + R_k \cdot V(\mathbf{x}^{(k)})$$

where,

$$V(\mathbf{x}^{(k)}) = \max\{0; |h_1|, |h_2|, \cdots, |h_p|; g_1, g_2, \cdots, g_m\}$$

and,

h_p**: value of the equality constraint function at the design point** $\mathbf{x}^{(k)}$

g_p**: value of the inequality constraint function at the design point** $\mathbf{x}^{(k)}$

R_k **is a positive number called the penalty parameter**

$$R_k = \max\{R_0,\ r_k\}$$

Summation of all the Lagrange multipliers $r_k = \sum_{i=1}^{p} \left|v_i^{(k)}\right| + \sum_{i=1}^{m} u_i^{(k)}$

$v_i^{(k)}$ **: Lagrange multipliers for the equality constraints (free in sign)**

$u_i^{(k)}$ **: Lagrange multiplier for the inequality constraints (nonnegative)**