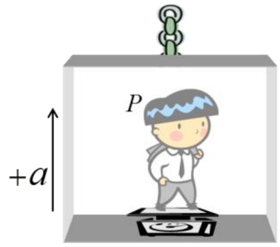
Midterm Exam						SNU ID #							
Date		2010. 10. 30				Name							
Problem Number		1	2	3	4	5	6	7	8	9	10	11	12	Total
Grader	Max													
	Score													

1. Suppose that a person "P" is standing in an elevator. The elevator has an upward acceleration "a"

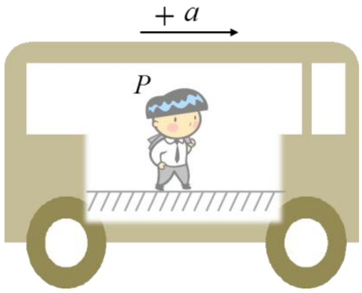


1.1. Derive the equations of motion for the person "P" in inertial reference frame.

1.2. Derive the equations of motion for the person "P" in non-inertial reference frame (that is fixed on the elevator) and explain the equation.

1.3. What is the force that the person "P" perceives to be exerted on him?

2. A bus is moving with acceleration of "a" in forward direction. And the person "P" is standing on the bus and moves with the same acceleration "a" with the bus.

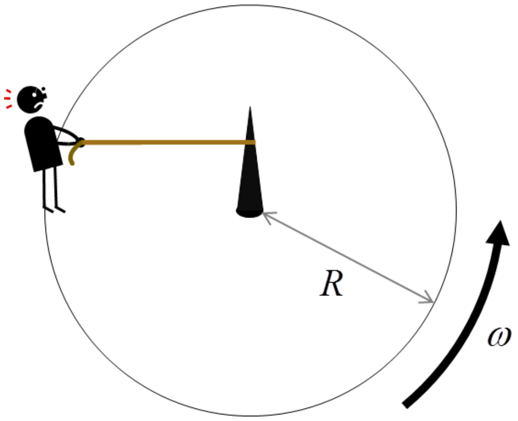


2.1. Derive the equations of motion for the person "P" in inertial reference frame.

2.2. Derive the equations of motion for the person "P" in non-inertial reference frame (that is fixed on the bus) and explain the equation.

2.3. What is the force that the person "P" perceives to be exerted on him?

3. A person is standing on the rim of a large disk holding the rope connected to the center of the disk to stand on the disk. And the disk is rotating with a constant angular velocity ω. Answer the following questions.

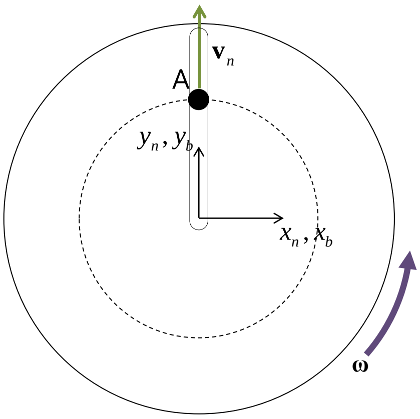


3.1. When the person is rotating with the disk, the velocity vectors have the same magnitude but they differ in direction. Because of this change of direction, the motion of the person is "accelerated motion". Derive the force exerted on the person in terms of the velocity of the person, radius of the disk R, angular velocity ω. And describe the motion of the person in inertial reference frame.

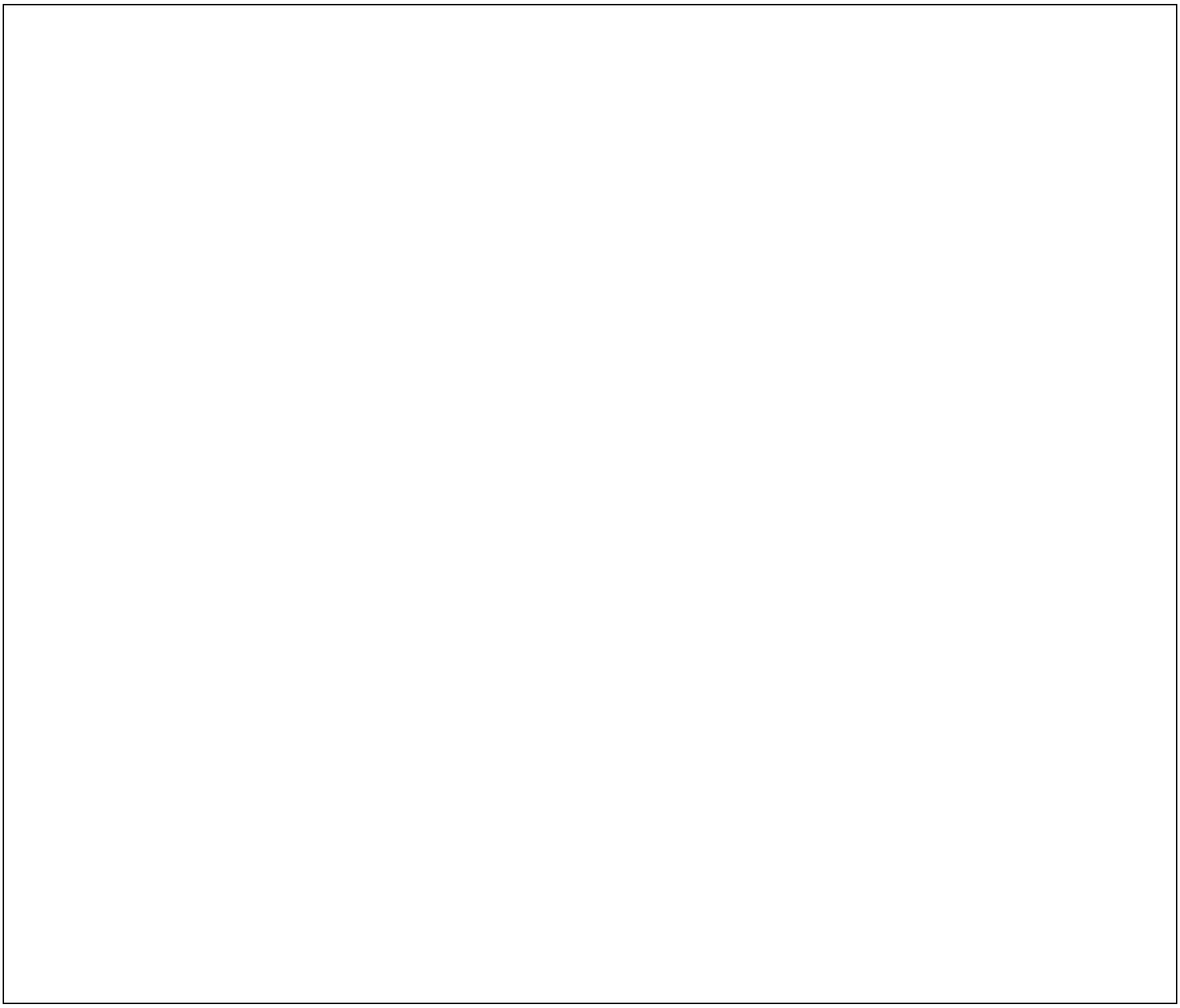
3.2. Describe the motion of the person in the reference frame attached to the disk and explain each term in the equations of motion.

3.3. In the reference frame attached to the disk, what forces does he perceive to be exerted on him?

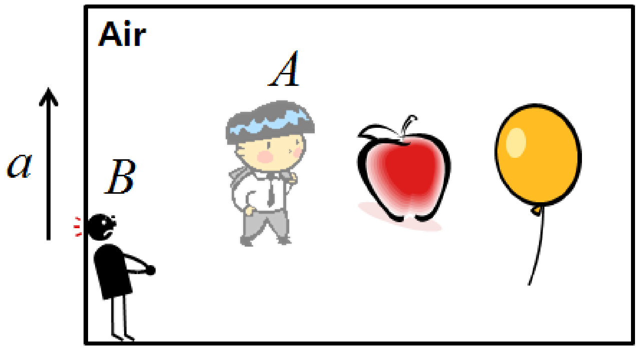
4. A point "A" is moving along a slot with a constant velocity $\mathbf{v}_n$, and the slot is on a disk rotating with a constant angular velocity $\boldsymbol{\omega}$. In this case, coriolis acceleration is $2(\boldsymbol{\omega}\times\mathbf{v}_n)$. Derive the term of coriolis acceleration.



n-frame: inertial frame
b-frame: body-fixed frame



5. An astronaut "A", an apple and a helium-filled balloon are in a spacecraft. There is no gravity, and none of them have weight. So they are all floating in space. The spacecraft is going to accelerate in the upper direction with acceleration "a". And an astronaut "B" who has the same acceleration "a" observes the motion of the astronaut "A", apple and balloon in the spacecraft. Answer the following questions.



($\rho_{person} = 1030\ \mathrm{kg/m^3}$, $\rho_{apple} = 760\ \mathrm{kg/m^3}$, $\rho_{balloon} = 0.18\ \mathrm{kg/m^3}$, $\rho_{air} = 1.23\ \mathrm{kg/m^3}$)

5.1. Assume **that there is no air inside**.

(1) What will be the motion of the astronaut "A", apple and balloon observed by the astronaut "B"? Do they go downward or upward?

(2) What will be the **"relative motion"** of the astronaut "A", apple and balloon?

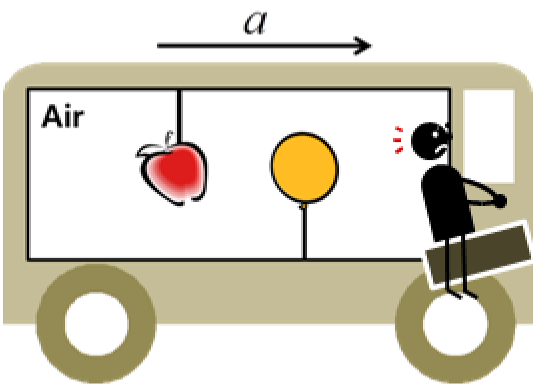
5.2. **Assume that air is filled in the spacecraft**. Thus you have to consider the "air" in the spacecraft.

(1) The height of the spacecraft is "h". Then what will be **the difference in the air pressure** between **the pressure** at the **bottom** and at the **ceiling**?

(2) What will be the **motion** of the astronaut "A", apple and balloon observed by **the astronaut "B"**? Do they go downward or upward?

(3) What will be the **"relative motion"** of the astronaut "A", apple and balloon?

6. An apple and a helium-filled balloon are in the car. The apple is connected to the top of the bus by a strap, whereas the balloon is connected to the bottom. The bus driver is sitting on the chair of the bus and has the same acceleration of the bus. The car is going to accelerate in the forward direction with an acceleration "*a*".



Answer the following questions.

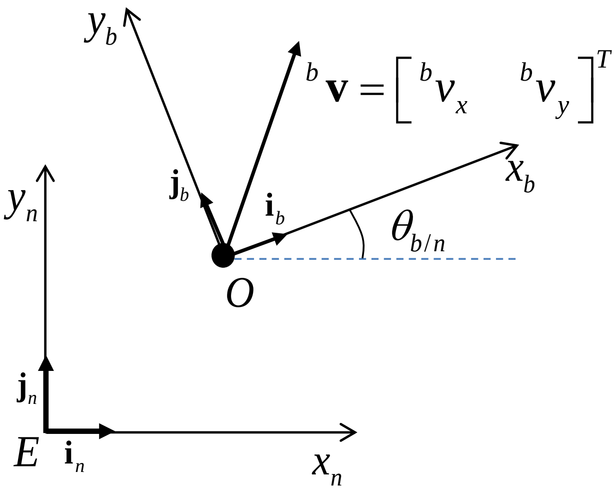
($\rho_{apple} = 760\ \mathrm{kg/m^3}$, $\rho_{balloon} = 0.18\ \mathrm{kg/m^3}$, $\rho_{air} = 1.23\mathrm{kg/m^3}$)

6.1. **Assume that there is no air inside**. Thus the force due to the air does not exist. What will be the motion of the apple and balloon observed by the bus driver?

6.2. **Assume** that **air is filled in the bus**. Thus you have to consider the "air" in the bus. What will be the motion of the apple and balloon observed by the bus driver?

7. There are two different reference frame, n-frame and b-frame, and the vector **v** can be represented by unit vectors of n-frame and b- frame.
   - $^{b}\mathbf{v}$: the vector **v**, represented by unit vectors of b-frame
   - $^{n}\mathbf{v}$: the vector **v**, represented by unit vectors of n-frame

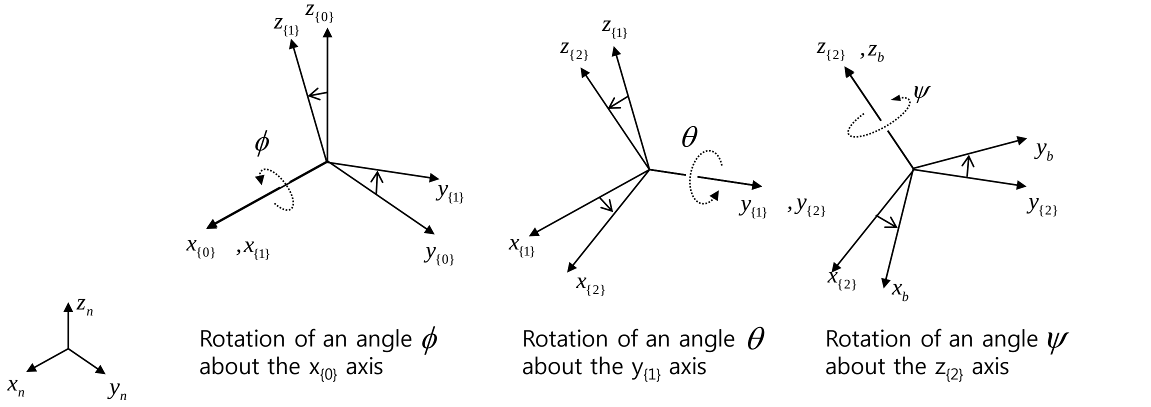
   Answer the following questions.



7.1. Derive the rotational transformation matrix, which can transfer the vector $^{b}\mathbf{v}$ to vector $^{n}\mathbf{v}$.

7.2. ${}^{b}\mathbf{v}$ is $[{}^{b}v_x \; {}^{b}v_y]^T$. Calculate ${}^{n}\mathbf{v}$ by using rotational transformation matrix derived in the problem 7.1.

8. Using xyz Euler angle $\boldsymbol{\gamma}$ = [Φ θ ψ]^T, the orientation of b-frame can be represented in 3D space as following figure.



- Axes of the {0}-frame are parallel to axes of the n-frame.

Some vector **v** can be represented by unit vector of n-frame and b- frame.

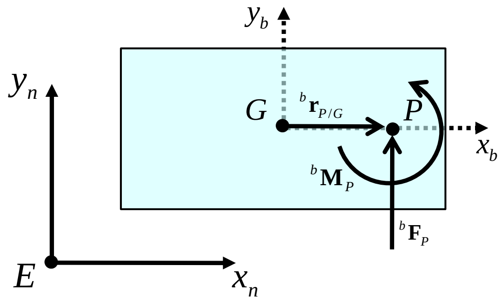
- ^{b}**v**: the vector **v**, represented by unit vector of b-frame
- ^{n}**v**: the vector **v**, represented by unit vector of n-frame

Answer the following questions.

8.1. Derive the rotational transformation matrix, which can transfer the vector ^{b}**v** to vector ^{n}**v.**

8.2. Suppose that the b-frame is rotating with angular velocity vector $\boldsymbol{\omega}_{b/n}$ with respect to n-frame. Then the angular velocity vector $\boldsymbol{\omega}_{b/n}$ can be calculated from the derivative of Euler angle using transformation matrix **G**. Derive the transformation matrix **G** and describe the meaning of the sequence of the deriving in detail.

9. Suppose that the force ${}^{b}\mathbf{F}_{P}$ and moment ${}^{b}\mathbf{M}_{P}$ is exerted on the the point P. Answer the following questions.



${}^{b}\mathbf{F}_{P}$: Force acting on the point P decomposed in the b-frame

${}^{b}\mathbf{M}_{P}$: Moment about z-axis through the point P decomposed in the b-frame

$${}^{b}\mathbf{r}_{P/G} = \begin{bmatrix} {}^{b}r_{P/G,x} & {}^{b}r_{P/G,y} & {}^{b}r_{P/G,z} \end{bmatrix}^{T}$$

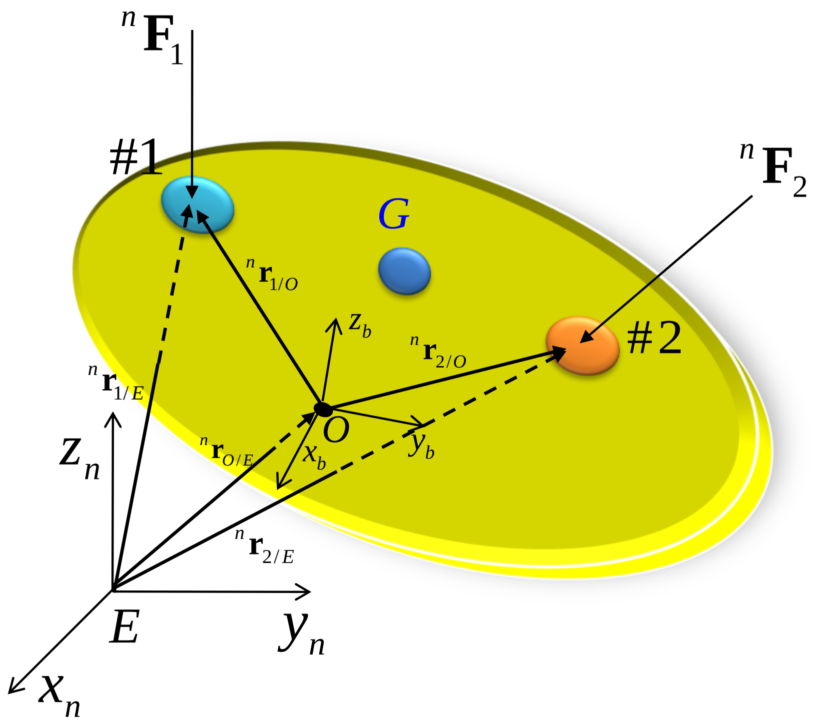
$${}^{b}\mathbf{F}_{P} = \begin{bmatrix} {}^{b}F_{P,x} & {}^{b}F_{P,y} & {}^{b}F_{P,z} \end{bmatrix}^{T}$$

$${}^{b}\mathbf{M}_{P} = \begin{bmatrix} {}^{b}M_{P,x} & {}^{b}M_{P,y} & {}^{b}M_{P,z} \end{bmatrix}^{T}$$

9.1. Calculate the force ${}^{b}\mathbf{F}_{G}$ and moment ${}^{b}\mathbf{M}_{G}$ exerted on the point G in force equilibrium system. (Calculate the components of the each vector)

10. Suppose the single rigid body is composed of two particles. The forces ${}^{n}\mathbf{F}_1$ and ${}^{n}\mathbf{F}_2$ are exerted on the each particle.

A single rigid body composed of two particles



$n-frame$: The inertial reference frame

m_1, m_2: The mass of the each particle

G : Center of mass

O : Body fixed point

The equations of motion for this single rigid body are as follows:

- Force equation

$$\sum \mathbf{F} = m_{system} \, {}^{n}\ddot{\mathbf{r}}_{G/E}$$

- Moment equation

$$\sum {}^{n}\mathbf{M}_O = m_{system} \, {}^{n}\mathbf{r}_{G/O} \times {}^{n}\ddot{\mathbf{r}}_{O/E} + {}^{n}\boldsymbol{\omega}_{b/n} \times {}^{n}\mathbf{I}_O \, {}^{n}\boldsymbol{\omega}_{b/n} + {}^{n}\mathbf{I}_O \, {}^{n}\dot{\boldsymbol{\omega}}_{b/n}$$

,where $m_{system} = m_1 + m_2$

10.1. Derive the force equation, and describe the meaning of the terms of force equation considering the following aspects

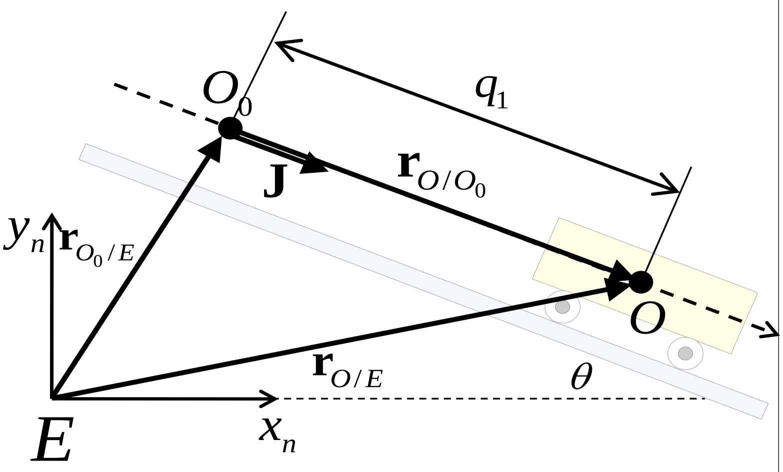
- What is the accelerated point, described in the force equation?
- The relationship between the forces exerted on the arbitrary points on the body and the accelerated point described in the force equation.

10.2. Represent the resultant moment, which is the left hand side of the moment equation, using the given notation in the figure of this problem.

10.3. Suppose that the rigid body is in xy-planar motion. Represent the mass moment of inertia ${}^{n}\mathbf{I}_{O}$ in the moment equation using the given notation in the figure of this problem and describe whether the mass moment of inertia is time variant or not with the reason.

10.4. Suppose that the kinematical reference point O coincides with center of mass G. Derive simplified version of the moment equation.

11. There is a vehicle constrained to move along the straight track. The external force exerted on the vehicle is gravitational force. Answer the following questions.



$n-frame$: Inertial reference frame. $\mathbf{J}$: unit vector along the vector $\mathbf{r}_{O/O_0}$

O: Center of mass of the vehicle.

$\mathbf{r}_{O_0/E}$: Initial position vector of the center of mass O_0
$\mathbf{r}_{O_0/E}$ is constant.

$\mathbf{r}_{O/E}$: Position vector of the center of mass O at time t.

$\mathbf{r}_{O/O_0}$: Directed vector from the point O_0 to the point O.

θ: Angle of inclination of the track. It is constant.

11.1. Derive the equations of motion represented by $x_{O/E}$, $y_{O/E}$ using augmented formulation.

$$\begin{bmatrix} \mathbf{M} & \nabla C(x_{O/E}, y_{O/E})^T \\ \nabla C(x_{O/E}, y_{O/E}) & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}}_{O/E} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{F}_O^e \\ 0 \end{bmatrix}$$

→ Absolute Coordinate Formulation (Augmented Formulation)

$C(x_{O/E}, y_{O/E}) = 0$: Constraint equation

λ : Lagrange Multiplier

$\mathbf{F}_O^e$: External force

$\mathbf{M}$: Mass matrix

11.2. In augmented formulation (refer problem 11.1), show which term is the constraint force to move the vehicle along the track.

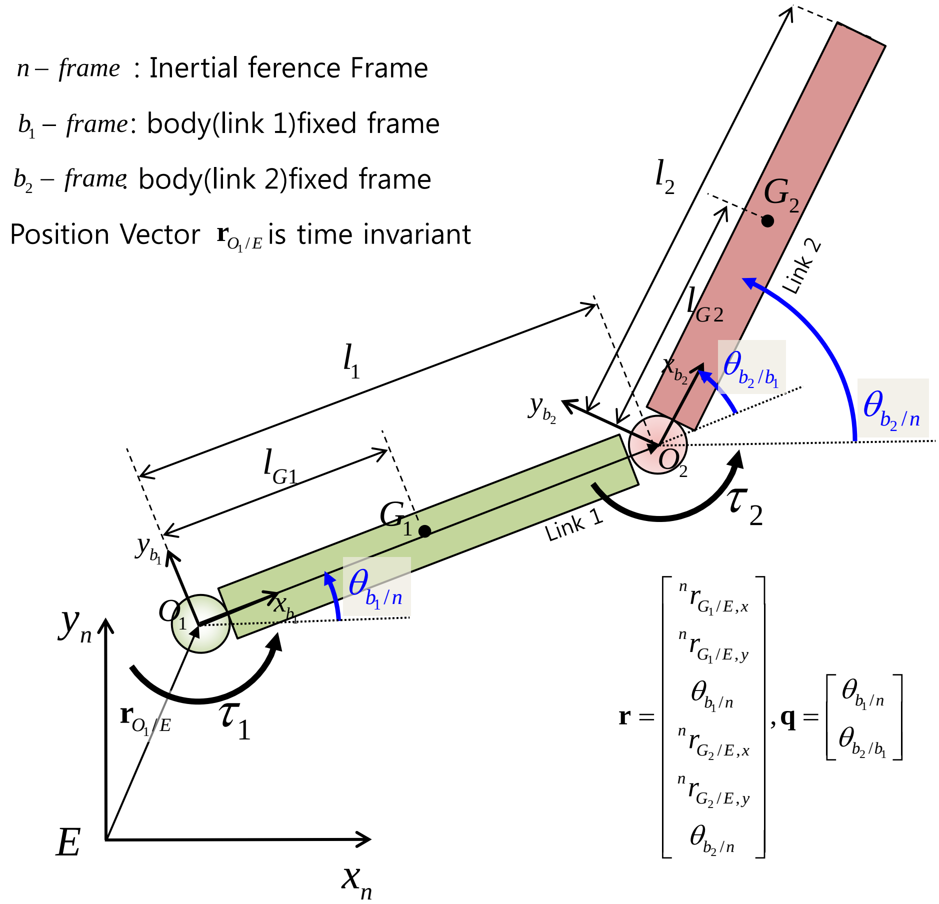
11.3. Suppose that the track is rotating about space fixed point O_0. Derive the equations of motion represented by q_1 and q_2 using imbedding technique. (In this question, q_2 represents the inclined angle of the track θ).

$$\boxed{\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{k}} = {}^{E}\tilde{\mathbf{F}}^{e}}$$

→ Relative coordinate formulation (Embedding technique)

$$,\left(\tilde{\mathbf{M}} = \mathbf{J}^{T}\mathbf{M}\mathbf{J},\ \tilde{\mathbf{k}} = \mathbf{J}^{T}\mathbf{M}\dot{\mathbf{J}}\dot{\mathbf{q}},\ {}^{E}\tilde{\mathbf{F}}^{e} = \mathbf{J}^{T}\,{}^{E}\mathbf{F}^{e}\right)$$

12. There is a 2-link arm moving in the xy-plane. The coordinates of this system are defined in the following figure. Only gravitational force is exerted on this system.



Mass and mass moment of inertia about point G_1 of link 1: m_1, I_1
Mass and mass moment of inertia about point G_2 of link 2: m_2, I_2

Answer the following questions.

12.1. Suppose that the only force exerted on this system is gravitational force. Derive the equations of motion represented by **q** using imbedding technique.

$$\boxed{\tilde{\mathbf{M}}\ddot{\mathbf{q}}+\tilde{\mathbf{k}} = {}^{E}\tilde{\mathbf{F}}^{e}}$$

→ Relative coordinate formulation (Embedding technique)

$$,\left(\tilde{\mathbf{M}} = \mathbf{J}^{T}\mathbf{M}\mathbf{J},\ \tilde{\mathbf{k}} = \mathbf{J}^{T}\mathbf{M}\dot{\mathbf{J}}\dot{\mathbf{q}},\ {}^{E}\tilde{\mathbf{F}}^{e} = \mathbf{J}^{T}\,{}^{E}\mathbf{F}^{e}\right)$$

12.2. Calculate the torque τ_1 and τ_2 to move the 2-link arm with given angles, angular velocities, and angular accelerations using recursive Newton-Euler formulation.

- Given angles, angular velocities, and angular accelerations:

$$\theta_{b_1/n}, \dot{\theta}_{b_1/n}, \ddot{\theta}_{b_1/n}$$

$$\theta_{b_2/b_1}, \dot{\theta}_{b_2/b_1}, \ddot{\theta}_{b_2/b_1}$$