The 2nd Midterm Exam for (2012-Fall)

459.666A. Special Topics in Fusion Plasmas

(Plasma Turbulence and Turbulent Transport)

(1) Write the current affiliation and the most significant scientific contribution of people listed below.

10 pts.

a. Liu Chen

b. Paul H. Rutherford

c. Katsumi Ida

d. William M. Tang

e. Mitsuru Kikuchi

(2) Name the institution and the country in which the following experiment is located.

10pts

(eg., Q: TFTR, A: Princeton Plasma Physics Lab, U.S.A.)

a) VEST b) ITER c) KSTAR

d) ASDEX-U e) HL-2A f) Alcator C-Mod

g) Joint European Torus h) Large Helical Device

i) DIII-D j) NSTX

③ What conditions (ordering assumptions) should be satisfied for the following quantities for the validity of the linear electrostatic ion gyrokinetic equation?

20pts.

Answer either by "$\sim$" or by "$\gg$".

(eg., $A \sim B$, $C \gg D$, ...).

a) $\Omega_{ci} = \frac{|e|B}{M_i c}$, ω, ω_{*e}, $\frac{v_{Ti}}{qR_0}$, $1/\tau_E$

4pts

(τ_E: energy confinement time).

b) R_0, L_n, ρ_i, $1/k_\perp$, $1/k_\parallel$, ρ_{bi} (banana orbit width)

4pts

($k_\perp$ and $k_\parallel$ are perpendicular and parallel component of $\vec{k}$ of fluctuation)

c) $\delta n_i/n_0$, $|e|\delta\phi/T_i$, $\delta f_i/f_0$, $\delta T_i/T_{i0}$, $\delta u_{\parallel i}/v_{Ti}$

4pts

d) Now, write essential assumptions for the nonlinear electrostatic ion gyrokinetic equation in terms of dimensionless equantities which can be obtained from combinations of quantities listed above. Use the following quantities and more.

8pts.

ω/Ω_{ci}, $k_\perp\rho_i$, $1/k_\parallel L_n$, $\delta f_i/f_0$.

④ Consider a drift wave eigenmode equation

30 pts. in a sheared magnetic field.

$$\left\{ \rho_s^2 \frac{\partial^2}{\partial x^2} + \frac{\omega_{*e}}{\omega} - 1 - k_y^2 \rho_s^2 + \frac{k_y^2 C_s^2}{\omega^2 L_s^2} x^2 \right\} \delta\phi_{k_y}(x) = 0 .$$

Notations are standard. The solution of this Weber equation is given by $\delta\phi_{k_y}(x) = \hat{\delta\phi}_{k_y} e^{-\frac{\sigma}{2} x^2} H_l(\sqrt{\sigma}\, x)$,

where $\sigma = \pm i \frac{k_y \Omega_{ci}}{\omega L_S}$, $l = 0, 1, 2, \cdots$

H_l is the Hermite polynomial. $\omega_{*e} = \frac{k_y \rho_s C_s}{L_n} > 0$, and $k_y > 0$.

a) Calculate the phase velocity of drift wave in y and x directions respectively for $l=0$,

5 pts.

b) Calculate the group velocity of drift wave in y and x directions respectively for $l=0$.

10 pts

c) Choose a physically acceptable eigenmode for $l=0$ with a justification (explanation).

5 pts

d) Write down the eigenvalues of this equation. Explain the physical meaning of each term.

10 pts

(5) 30pts Consider a drift wave problem in a toroidal plasma in which $\vec{B}(r,\theta) = B_\phi \hat{\zeta} + B_\theta \hat{\theta}$.

$$B \equiv |\vec{B}| = \frac{B_0 R_0}{R_0 + r\cos\theta}.$$

Notations are standard.

a) 10 pts Starting from a linearized ion density continuity equation, describe a derivation of the following ion density response in a uniform magnetic field.

$$\delta n_i / n_0 = \left(\frac{\omega_{*e}}{\omega} - \rho_s^2 k_\perp^2 + \frac{c_s^2 k_\parallel^2}{\omega^2} \right) |e| \delta\phi / T_e \quad (1)$$

* Now, consider a toroidal plasma with a magnetic field given above.

b) 5 pts. Write down the expressions for ∇B drift and curvature drift of thermal ions.

c) 5pts Show that the most important correction to Eq.(1) due to nonuniform $\vec{B}$ is the following additional term.

$$\text{"} \delta n_i / n_0 = (\text{terms in Eq.(1)}) - \frac{2\omega_{de}}{\omega} \frac{|e| \delta\phi}{T_e} \text{"}, \quad (2)$$

where $\omega_{de}/\omega = \frac{k_\theta \rho_s c_s}{R_0} (\cos\theta\, k_\theta + \sin\theta\, k_r)$

d) 10 pts Write down the full expression Eq.(2) in the extended poloidal (ballooning) coordinate "η". Sketch your derivation.