

Plasma Turbulence and Turbulent Transport

(2012 Semester 2)

1/4

Final Exam

I. Linear Onset condition for ion temperature gradient

(ITG) instability is approximately given by:

$$\eta_i = \frac{d \ln T_i}{d \ln n_0} = \frac{L_n}{L_{T_i}} \geq \frac{2}{1 + 2b_i \left(1 - \frac{I_1(b_i)}{I_0(b_i)}\right)} \quad (1)$$

with

$$b_i = k_y^2 \rho_i^2, \quad I_0, I_1 \text{ are modified Bessel functions,}$$

for normal density profiles with $\langle L_n \rangle < R_0$.

However Eq. (1) is no longer valid if L_n is too large.

For $L_n \rightarrow \infty$, at least two different theoretical formulas exist in the literature.

$$\frac{q}{s} \frac{R_0}{L_{T_i}} \geq 1.9 \left(\frac{T_i}{T_e} + 1 \right) \quad (2)$$

and

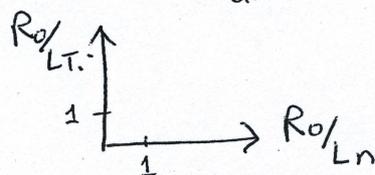
$$\frac{R_0}{L_{T_i}} \geq \frac{4}{3} \left(\frac{T_i}{T_e} + 1 \right) \quad (3)$$

a) List assumptions made in deriving the formulas

10 pts (1), (2) and (3) respectively.

b) For $q/s = 1.4$, $\frac{T_i}{T_e} = 2$, and $k_y^2 \rho_i^2 = 0.1$,

5 pts. draw the stability diagram for ITG in the following plane. (Indicate the area for instability and that for stability).



II, It has been observed that the energy confinement time τ_E of neutral beam heated plasmas scales like $\tau_E \propto P_{NBI}^{-0.5}$ roughly, in the early days of tokamak experiments. Here P_{NBI} is the heating power.

10 pts total

- a) Discuss if this is a good news or a bad news. State the reasons for your answer. 5 pts
- b) Discuss this result in the context of ITG driven turbulent transport. 5 pts.

III. Consider a high temperature tokamak plasma with strong ion temperature gradient and near flat density profile. Consider ion particle continuity equation:

50 pts total

$$\frac{\partial}{\partial t} n_i + \vec{\nabla} \cdot (n_i \vec{u}_i) = 0, \quad (1)$$

ion pressure convection equation:

$$\frac{\partial}{\partial t} P_i + \vec{u}_{E \times B} \cdot \vec{\nabla} P_i = 0, \quad (2)$$

here $\vec{u}_{E \times B} = \frac{c \hat{b} \times \vec{\nabla} \delta \phi}{B}$, $P_i = P_{i0} + \delta P_i$, $n_i = n_0 + \delta n_i$,

$\delta \phi$ is the electrostatic perturbation associated with ITG instability.

a) Linearize Eq.(1) and Eq.(2).

10pts

Assume \vec{u}_i consists of $\vec{E} \times \vec{B}$ drift $\vec{u}_{E \times B}$, parallel motion \vec{u}_{\parallel} , and $\vec{\nabla} B$ and curvature drifts

\vec{u}_{di} for thermal ions. Use the fact that

$|\vec{\nabla} n_0| \rightarrow 0$. Assume $k_{\perp}^2 \rho_i^2 \rightarrow 0$ and neglect polarization drift.

b)

20pts

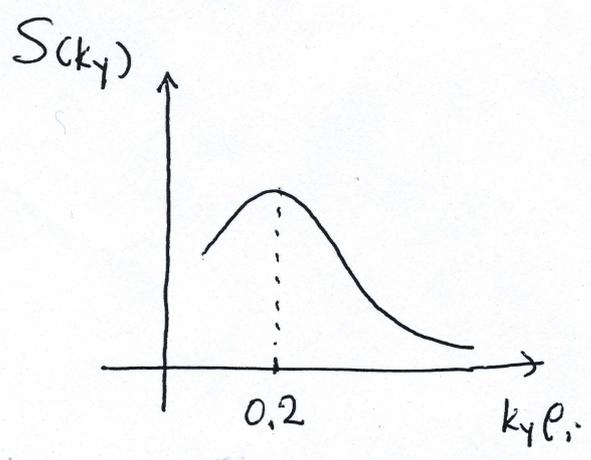
Sketch a derivation leading to ^{the following} simplified linear dispersion relation for toroidal ITG instability. List assumptions and approximations made at each step.

$$\omega^2 = -\frac{T_e}{T_i} |\bar{\omega}_{di} \omega_{*Ti}|, \quad (3)$$

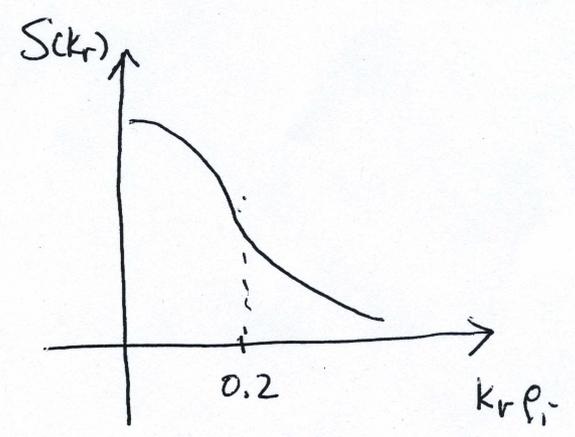
where $\omega_{*Ti} = -\frac{k_y \rho_i U_{Ti}}{L_{Ti}}$, $\bar{\omega}_{di} = -\frac{k_y \rho_i U_{Ti}}{R_0}$.

(Don't worry about a factor of 2 or 1/2).

Intensity of turbulent density fluctuations has been measured from ^{as} simulations functions of k_r and k_y . Two sets of numerical simulations have been performed.



①



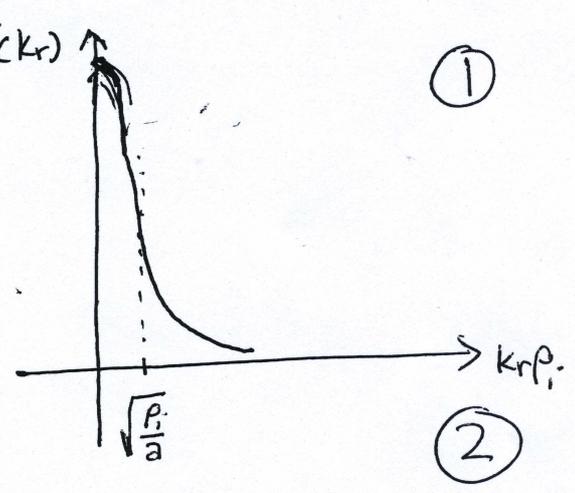
①

$S(k_y)$

$S(k_r)$

the same
//

②



②

c) Deduce the scaling of turbulent diffusion

10 pts coefficient from a random walk argument for case ① and for case ②. Use the information given in Eq.(3).

d) Discuss the possible origin of difference between the simulation ① and the simulation ②.

e) What experimental diagnostics (are) is capable of producing k_r and k_y spectra of density fluctuations as shown above?