Computer Aided Ship Design

- 2nd Exam -

Saturday, December 3rd, 2011

Time: 10:00-13:00 (3 hours)

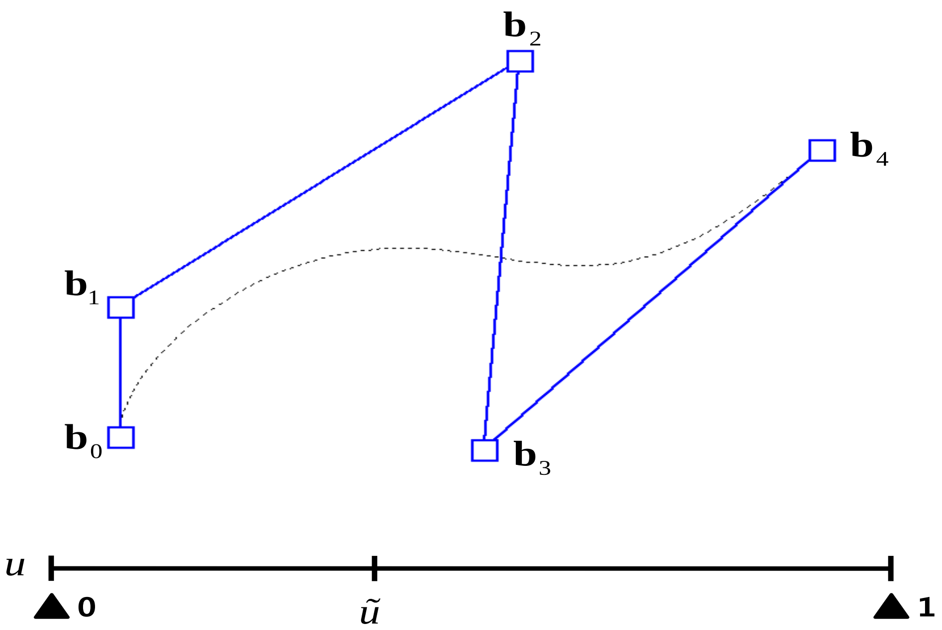
Name	
SNU ID #	

Note: Budget your time wisely. Some parts of this exam could take you much longer time than others. Move on if you are stuck and return to the problem later.

Curve & Surface

Problem Number		1		2			3			Total
		1.1	1.2	2.1	2.2	2.3	3.1	3.2	3.3	
Grader	Max	10	10	10	20	20	10	10	10	100
	Score									

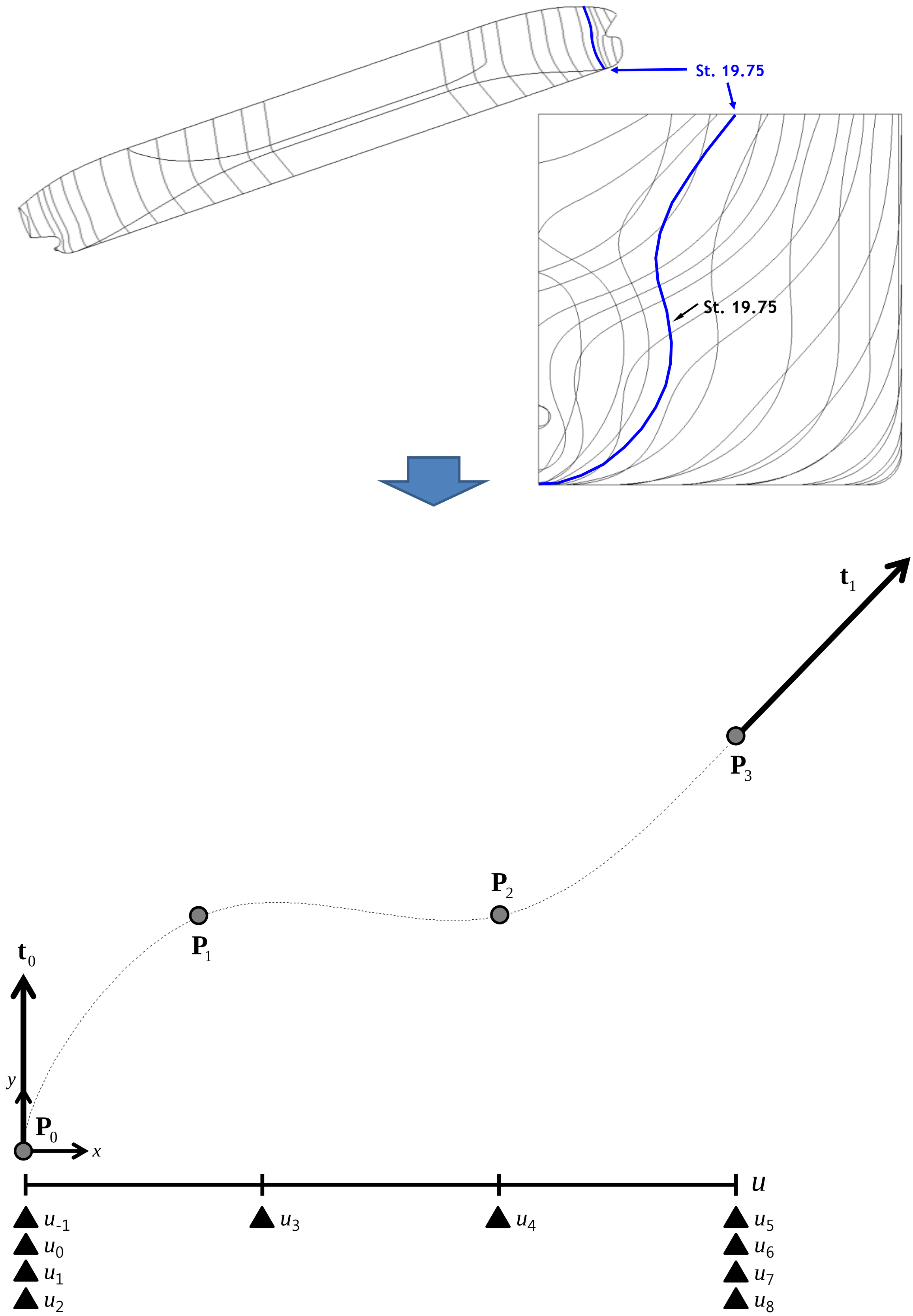
1. **(De Casteljau algorithm)** Let a 2D curve Bezier curve be given by the control points $\mathbf{b_0}$, $\mathbf{b_1}$, $\mathbf{b_2}$, $\mathbf{b_3}$ and $\mathbf{b}_4$. By using repeated linear interpolation at any given parameter $\hat{u}$, you can construct a Bezier curve of degree 4.



1.1 Sketch how to construct (e.g., using the de Casteljau algorithm) the point on the curve corresponding to $u = \hat{u}$. Show that this point is on the Bezier curve of degree 4, which is given by the control points $\mathbf{b_0}$, $\mathbf{b_1}$, $\mathbf{b_2}$, $\mathbf{b_3}$, $\mathbf{b}_4$.

1.2 Referring to the problem 1.1, we can define two segments of the curve corresponding to $[0,\ \hat{u}]$ and $[\hat{u}, 1]$. Find and sketch the control points for these two curves, and explain that these two curves are connected and satisfying the continuity conditions C^0, C^1 and C^2.

2. **(Cubic B-Spline Curve Interpolation)** Following figure shows a section line of a ship. Suppose you are given a set of data points $\mathbf{P_0}$, $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$. Determine the curve passing through them.



2.1 Sketch the control points of a cubic B-spline curve. By using the knot spacing Δ_i ($\Delta_i = u_{i+1} - u_i$), explain that the curve is satisfying the continuity conditions C¹ and C².

2.2 Find the control points of a cubic B-spline curve, which is passing through the points $\mathbf{P_0}$, $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$ with the two end conditions $\mathbf{t_0}$, $\mathbf{t_1}$.

- Points of the curve: $\mathbf{P_0} = (0, 0)$, $\mathbf{P_1} = (3, 4)$, $\mathbf{P_2} = (8, 4)$, $\mathbf{P_3} = (12, 7)$
- Tangent vector: $\mathrm{t}_0 = (0, 3)$, $t_1 = (3, 3)$

$$\alpha_i = \frac{(\Delta_{i+2})^2}{(\Delta_i+\Delta_{i+1}+\Delta_{i+2})(\Delta_{i+1}+\Delta_{i+2})}$$

$$\beta_i = \left\{\frac{\Delta_{i+2}(\Delta_i+\Delta_{i+1})}{\Delta_i+\Delta_{i+1}+\Delta_{i+2}} + \frac{\Delta_{i+1}(\Delta_{i+2}+\Delta_{i+3})}{\Delta_{i+1}+\Delta_{i+2}+\Delta_{i+3}}\right\} / (\Delta_{i+1} + \Delta_{i+2})$$

$$\gamma_i = \frac{(\Delta_{i+1})^2}{(\Delta_{i+1}+\Delta_{i+2}+\Delta_{i+3})(\Delta_{i+1}+\Delta_{i+2})}$$

2.3 Using the control points from the problem 2.2, find a point on this curve at $u = 1.5$.

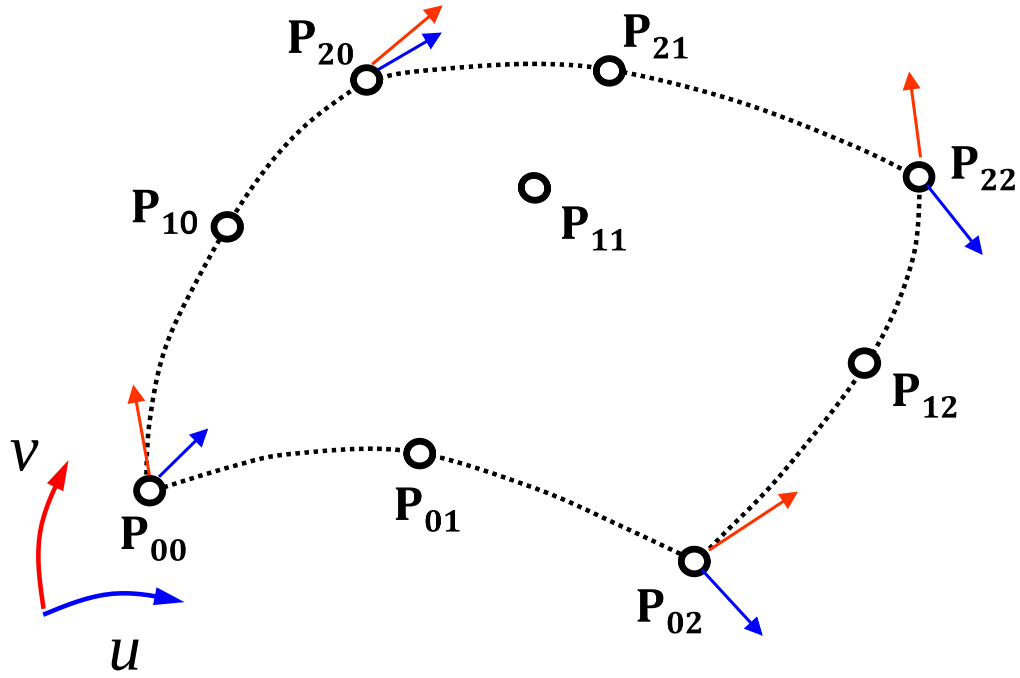
B-spline curve

$$\mathbf{r}(u) = \begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \sum_{i=0}^{D-1} \mathbf{d}_i N_i^n(u)$$ (D: the number of control points)

Cox-de Boor recurrence formula

$$N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u), \qquad N_i^0(u) = \begin{cases} 1 & \text{if } u_{i-1} \le u < u_i \\ 0 & \text{else} \end{cases}$$

3. **(Cubic B-Spline Surface Interpolation)** A set of data points $\mathbf{P_{00}}, \mathbf{P_{01}}, \dots, \mathbf{P_{22}}$ is given as in the following figure. Find a set of cubic B-Spline surface control points, such that the data points are on the resulting surface.



3.1 Determine the knot values in $\boldsymbol{u}$-direction.

3.2 Derive the cubic B-Spline curve in $\boldsymbol{u}$-direction.

3.3 Determine the knot values in $\boldsymbol{v}$-direction, and find the control points of the cubic B-Spline surface.

3.4 Using the control points from the problem 3.3, find a point on this surface at $(u, v) = (\hat{u}, \hat{v})$.