Plasma Electrodynamics 1 June 16, 2009

Final Exam

(15 points)

1. If each of the following statements is true, please mark O in the parenthesis. If false or incorrect, mark X and make corrections of the false statement or word in the parenthesis.

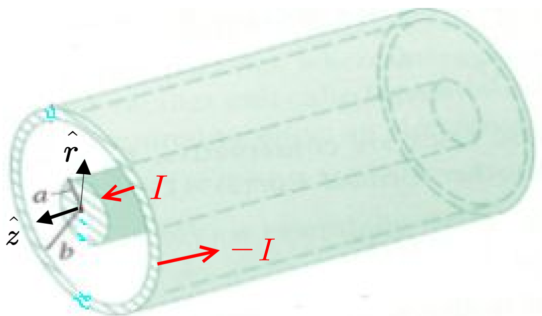
1) $\nabla \cdot \boldsymbol{B} = 0$ implies that magnetic flux lines always close upon themselves and there are no isolated magnetic charges. (   )
2) Biot-Savart law is a formula for determining $\boldsymbol{B}$ caused by current density $\boldsymbol{J}(\boldsymbol{r})$ flowing in a three-dimensional conducting fluid. (   )
3) The magnetic dipole moments of atoms determine a magnetization vector $\boldsymbol{M}$, which can be analytically proved by the equivalence to a magnetization surface current density $\boldsymbol{J}_{ms} = \nabla \times \boldsymbol{M}$. (   )
4) The normal component of $\boldsymbol{B}$ and the tangential component of $\boldsymbol{E}$ are continuous across the boundary of any two physical media in both static and time-varying cases.(   )
5) $\boldsymbol{B}$ is proportional to $\boldsymbol{H}$ in ferromagnetic materials having very large χ_m.(   )
6) The magnetic flux denstiy $\boldsymbol{B}$ in a current-carrying toroidal coil is inversely proportional to the major radius (= radius from the axis of the torus), and its self-inductance is proportional to the square of the number of coil turns.(   )
7) The total internal inductance of a long copper coaxial transmission line is $\mu_o/8\pi$ (H/m).(   )
8) Hysteresis loss in a ferromagnetic material is the energy loss in the form of heat produced by eddy currents.(   )
9) $\boldsymbol{E}$ in a time-varying magnetic field is not conservative, and the time-varying electric field linking with a circuit induces an emf in the circuit by Faraday's law of electromagnetic induction.(   )
10) Only in an ideal transformer, the primary-to-secondary current ratio depends on the turns ratio of its windings.(   )

(35 points)

2. Answer the following questions.

   1) Write Ampere's circuital law in a time-varying case, and state its physical meaning.
   2) From the definition of the magnetic flux Φ through an area S bounded by contour C, show that Φ relates to the magnetic vector potential $\mathbf{A}$ in a way: $\Phi = \oint_C \mathbf{A} \cdot d\mathbf{l}$.
   3) Describe the magnetic dipole moments $\mathbf{m}$ of electrons and ions in a magnetized plasma, and explain why the plasma is a diamagnetic medium.
   4) Define remanent flux density and coercive field intensity, and show where they are located on a hysteresis loop.
   5) What are the magnetic force $\mathbf{F}_m$ and the torque $\mathbf{T}$ on a current(I)-carrying loop of radius a in a uniform magnetic field B ?
   6) How is the time-varying electric field $\mathbf{E}$ determined in terms of potentials V and $\mathbf{A}$ from Maxwell's equations? Show that the time-varying potentials V and $\mathbf{A}$ satisfy the wave equation.

(15 points)

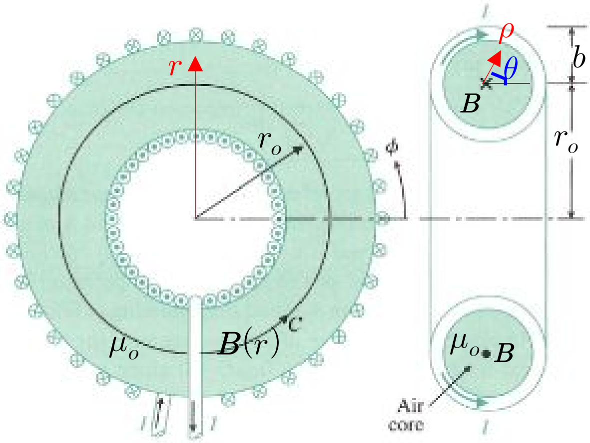


3. Consider an infinitely long coaxial line with a current I flowing in the inner conductor and returning via the outer conductor. The radii of the inner and outer conductors are a and b, respectively.
   Find the magnetic flux density $\mathbf{B}$ inside the two conductors ($a < r < b$) by applying the following two ways, repectively:

   1) Ampere's circuital law.
   2) Boundary-value problem (BVP) for vector potential $\mathbf{A}$.

(20 points)

4. Consider a toroidal coil of N turns of wire wound on an air frame with mean radius r_o and a circular cross section of radius b.



   1) Find $\boldsymbol{B}(r)$ inside the toroidal coil $[(r_o - b) < r < (r_o + b)]$ by using Ampere's circuital law.
   2) Show that the magnetic flux Φ crossing the circular cross section is derived as $\Phi = \mu_o NI(r_o - \sqrt{r_o^2 - b^2})$.

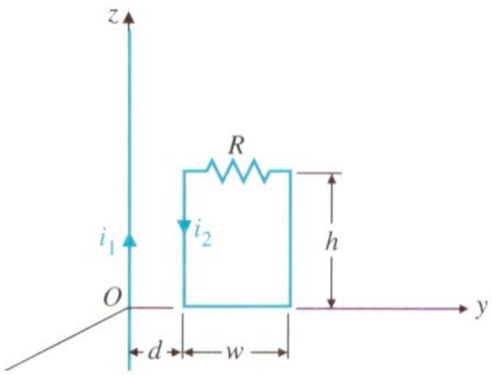
      (Note that $r = r_o + \rho\cos\theta$ where r is the major radial position from the major axis, ρ is the manor radius from the minor axis, and θ is the poloidal angle.

      Use the integral relation: $\int_0^{2\pi} \frac{dx}{a + b\cos x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$, if necessary)
   3) Determined the self-inductance of the toroidal coil.

(15 points)

5. In FIGURE, a constant current $i_1 = I_o$ is flowing along a long wire and the rectangular loop moves away with a constant velocity $\boldsymbol{u} = \hat{\boldsymbol{y}} u_o$.

FIGURE A rectangular loop near a long current-carrying wire



   1) Determine $\boldsymbol{B}_1$ in the loop due to the wire current $i_1 = I_o$.
   2) Find the motional emf in the moving loop.
   3) Determine the induced current i_2 flowing around the loop when the loop is at a position as shown in FIGURE.