Plasma Electrodynamics 2 **Mid-Term Exam 1** Oct. 20, 2009

(25점)

1. Choose a correct answer to each question by the corresponding number.

a) Which is the *time-average electromagnetic power flow per unit area?*

① $E \times H$ ② $\frac{1}{2} Re[\{E(z) \times H(z)\} e^{j\omega t}]$ ③ $\frac{1}{2} Re[E \times H^*]$ ④ $\oint_S E \times H \cdot ds$

b) Which is *incorrect* for the *reflection coefficient* $\Gamma_\perp$ in an oblique incident of plane electric wave with *perpendicular polarization*?

① $E_{ro} = \Gamma_\perp E_{io}$ ② $|\Gamma_\perp| = 1$ for total internal reflection.

③ $1 + \Gamma_\perp = \tau_\perp$ ④ $|\Gamma_\perp| = 0$ at any incident angle for nonmagnetic media ($\mu_1 = \mu_2$).

c) Which is *incorrect* for *electromagnetic wave* propagation in a medium?

① E and H of a TEM wave in a given medium must satisfy Maxwell's equations, and their amplitudes and phases can be specified independently.

② Electromagnetic waves of all frequencies propagate in a lossless medium with the same velocity, and E and H are in phase.

③ The intrinsic impedance of a low-loss dielectric is a complex quantity, and E and H thus have different time phases.

④ Wavenumber depends on medium characteristics as well as the wave frequency, but is always equal to $2\pi/\lambda$.

d) Which is *not* for the time-harmonic responses of an *inductor* to $i(t) = I\cos\omega t = Re[Ie^{j\omega t}]$?

① $v(t) = L\frac{di}{dt}$ ② $v(t) = \frac{1}{\omega L} I\sin\omega t$ ③ $p = \frac{1}{2} L \frac{di^2}{dt}$ ④ $E = \frac{1}{2} LI^2$

e) Which is *not* for the *relationship between sources and fields* ?

① $\nabla \times H = J + \partial D/\partial t = J + J_d$ ② $\nabla \times B = \mu_o (J + J_{mv} + J_d) = \mu_o J_t$

③ $\nabla \cdot D = \rho_v$ ④ $\nabla \cdot E = (\rho_v + \rho_{pv}) = \rho_t$

f) Which is *incorrect* for *a 1-D uniform plane wave,* $E(z,t) = \hat{x}\, Re[E_o\, j e^{j(\omega t - kz)}]$?

① This wave leads by $\pi/4$ ahead of a reference wave, $E_x(z,t) = E_o\cos(\omega t - kz)$.

② This wave satisfies the relation, $k \cdot E = 0$.

③ This wave is travelling backward along the z-direction with an amplitude of E_o.

④ The phase velocity u_p is found by $u_p = \omega/k$

g) Which is *incorrect* for the *circularly polarized waves,* which are given by $E(z,t) = \hat{x} E_{xo}\cos(\omega t - kz) + \hat{y} E_{yo}\cos(\omega t - kz + \delta)$?

① $E_{xo} = E_{yo}$ ② $E_x = E_o\cos\omega t$ & $E_y = \pm E_o \sin\omega t$ at $z = 0$

③ $\delta = \pm\pi$ ④ $jE_x/E_y = \mp 1$

h) Which is *incorrect* for the *TEM wave propagation in lossy media with good conductivity* ($\tan\delta_c \gg 10^2$)?

① $\alpha = \beta \cong \sqrt{\pi f \mu \sigma}$ ② Skin depth is inversely proportional to $\sqrt{f}$.

③ $H(z)$ lags behind $E(z)$ by $\pi/4$ ④ The wavelength λ is longer than that in air.

i) Which is *incorrect* for an *oblique incident* of plane electric wave *with parallel polarization*?

① There exists Brewster angle θ_B of no reflection.

② Both Snell's laws are applicable. ③ $1+\Gamma_{\parallel} = \tau_{\parallel}$

④ $\Gamma_{\parallel} = 0$ at certain incident angle for two nonmagnetic media whether $\epsilon_1 < \epsilon_2$ or $\epsilon_1 > \epsilon_2$.

j) Which is *not related with waves in plasmas* ?

① Plasma oscillations ($\omega^2 = \omega_p^2$) are electrostatic logitudianl waves.

② The refractive index of TEM waves in plasmas is larger than 1.

③ The TEM waves with frequency higher than the plasma frequency can only propagate through plasmas.

④ The phase velocity of TEM wave is always faster than c (speed of light).

(15점)

2. For *source-free($\rho_v = 0$)* ***lossy*** *($\sigma \neq 0$)* medium,

a) Derive the vector wave equation for $\boldsymbol{H}$ from Maxwell's equations and Ohm's law($\boldsymbol{j} = \sigma \boldsymbol{E}$).

b) Derive the Helmholtz's equation for time-harmonic $\boldsymbol{H}$ fields.

c) Find the electric phasor $\boldsymbol{E_s}$ in terms of $\boldsymbol{H_s}$ and intrinsic impedance η_c from Maxwell's equations.

(10점)

3. Given that the electric field intensity of a uniform plane wave in a nonconducting dielectric medium with $\epsilon = 9\epsilon_o$ and $\mu = \mu_o$ is

$$\boldsymbol{E}(z,t) = \hat{y} E_o \cos(\omega t - kz + \psi) \quad (\text{V/m}),$$

a) determine k, E_o and ψ by assuming that E_y has a frequency 1 GHz and a maximum value of 5 (V/m) at t=0 and z=1 (m), and

b) find the magnetic field intensity $\boldsymbol{H}(z,t)$.

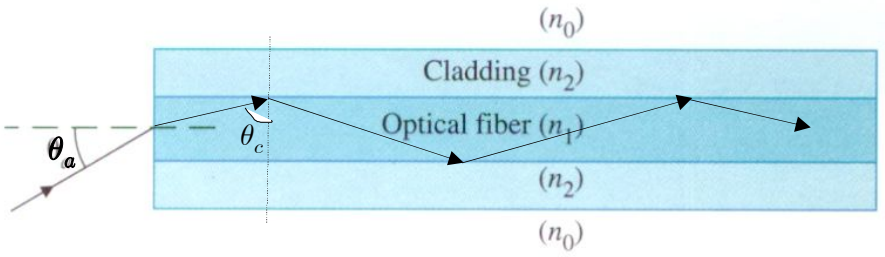
[*Note:* $\eta_o = \sqrt{\mu_o/\epsilon_o} = 120\pi$ (Ω)]

(20점)

4. A uniform plane wave propagates in the +z-direction into a conducting plate ($\epsilon_r = 36\pi,\ \mu_r = 1,\ \sigma = 20\ S/m$). The magnetic field at the conducting plate surface (z=0) is $\boldsymbol{H}(0,t) = \hat{\boldsymbol{y}}\, 10\cos(10^8 t)$ (A/m).
   a) Determine the skin depth ($\delta = 1/\sqrt{\pi f \mu \sigma}$) and the intrinsic impedance [$\eta_c \approx \sqrt{\pi f \mu/\sigma}\,(1+j)$] of the conducting plate.
   [*Note:* $\epsilon_o = 10^{-9}/36\pi$ (F/m) and $\mu_o = 4\pi\times 10^{-7}$ (H/m)]
   b) Find the expression of $\boldsymbol{H}(z,t)$ and $\boldsymbol{E}(z,t)$ in the conducting plate.
   c) Find the average power loss per unit area in the conducting plate as a function of z.

(10점)

5. For a cladded-core optical fiber as shown in the figure below ($n_2 < n_1$),



express the maximum angle of incidence $\boldsymbol{\theta_a}$ in terms of n_o, n_1, and n_2 for the rays incident on the fiber axis at the core's end face to be trapped inside the core by total internal reflection.

(20점)

6. Answer each of the following questions <u>in detail</u>.
   a) Explain the *relationships between electromagnetic potential functions and electromagnetic fields*.
   b) What is meant by *Poynting's theorem*? How can you derive this theorem from Maxwell's equations?
   c) Compare the 1-D homogeneous *Helmholtz's equations for lossless and lossy media*, and discuss the *characteristics of TEM wave propagation* by comparing the propagation constants for lossless and lossy media.
   d) Discuss the characteristics of TEM wave propagation *in plasmas* compared with those in air.