1. <mark>As the absolute temperature is lowered below liquid-nitrogen temperatures</mark>, the phonon contribution to the energy transport becomes significant.
(77K~10K)
In this temperature range, the thermal <mark>conductivity becomes approximately proportional to T^{-2} for pure metals</mark>. The thermal conductivity($k_t \propto T^{-2}$) increases to a very high maximum as the temperature is lowered, until the mean free path($\lambda \downarrow, k_t \propto \lambda$ but small contribution) of the energy carriers becomes on the order of the dimensions of the material sample.

When this condition reached, the boundary of the material begins to introduce a resistance to the motion of the carriers, and the carrier mean free path becomes constant($\lambda = constant$) (approximately equal to the material thickness). Because the <mark>specific heat decreases to zero($c_v \downarrow, k_t \propto c_v$)</mark> as the absolute <mark>temperature approaches zero</mark>, from eqn. (2.5) we see that the <mark>thermal conductivity would also decreases with a decreases in temperature in this very low temperature region</mark>.

In disordered alloys and impure metals, the electronic contribution and the phonon contribution to energy transport are of the same order of magnitude. There is an additional scattering of the energy carriers due to the presence of impurity atoms in impure metals. This scattering effect is directly proportional to absolute temperature. <mark>Dislocations in the material provide a scattering that is proportional to T^2</mark>, and <mark>grain boundaries introduce a scattering that is proportional to T^3 at temperatures much lower than Debye temperature</mark>. All these effects combine to cause the thermal conductivity of alloys and impure metals to decreases as the temperature is decreased, and the high maximum in thermal conductivity is eliminated in alloys.

   1) Separate the problem into pure metal and alloy cases (3 points)
   2) Explain reasonably (7 points)

2. When heat is added to the powder in the apparatus shown in Fig.2.19, the increases in temperature tends to raise the concentration of normal fluid, and the superfluid rushes in to equalize the concentration. Normal fluid, because of its viscosity, cannot leave through the small openings between the find powder particles very rapidly. The amount of helium quickly builds up within the tube as a result of this inflow of superfluid, and finally liquid squirts out the open end of the capillary tube.

   1) Helium state changes (3 points)
   2) Viscosity and equilibrium (5 points)
   3) Pressure difference (2 points)

3.

Points	Description
2	Joule-Thomson coefficient $\mu_{JT} = \left(\dfrac{\partial T}{\partial P}\right)_h$ Using the calculus the following can be derived by chain rule, $\left(\dfrac{\partial h}{\partial P}\right)_T \left(\dfrac{\partial P}{\partial T}\right)_h \left(\dfrac{\partial T}{\partial h}\right)_P = -1$ Rearranging the terms, $\mu_{JT} = \left(\dfrac{\partial T}{\partial P}\right)_h = -\left(\dfrac{\partial T}{\partial h}\right)_P \left(\dfrac{\partial h}{\partial P}\right)_T$
2	Also, the entropy can be defined as the function of temperature and pressure. $s = f(T, p)$ Using the calculus, the following can be derived. $ds = \left(\dfrac{\partial s}{\partial T}\right)_P dT + \left(\dfrac{\partial s}{\partial P}\right)_T dP$, $Tds = T\left(\dfrac{\partial s}{\partial T}\right)_P dT + T\left(\dfrac{\partial s}{\partial P}\right)_T dP$ ----(1)

2	The definition of specific heat under constant pressure : $T\left(\dfrac{\partial s}{\partial T}\right)_P = c_P$ From the Maxwell's equation : $\left(\dfrac{\partial s}{\partial P}\right)_T = -\left(\dfrac{\partial v}{\partial T}\right)_P$ From above, equation (1) can be shown that $Tds = c_P dT - T\left(\dfrac{\partial v}{\partial T}\right)_P dP$ ---(2)
2	$Tds = dh - vdP$ From the above thermodynamic relation, it can be shown that $dh = c_P dT - \left[T\left(\dfrac{\partial v}{\partial T}\right)_P - v\right]dP$ ----(3)
2	Also, the enthalpy can be defined as the function of temperature and pressure $h = f(T, p)$ Using the calculus, the following can be derived $dh = \left(\dfrac{\partial h}{\partial T}\right)_P dT + \left(\dfrac{\partial h}{\partial P}\right)_T dP$ ----(4)
2	Comparison (3), (4), we can obtain the following $\mu_{JT} = \left(\dfrac{\partial T}{\partial P}\right)_h = \left(\dfrac{\partial T}{\partial h}\right)_P \left(\dfrac{\partial h}{\partial P}\right)_T = \dfrac{1}{c_P}\left[T\left(\dfrac{\partial v}{\partial T}\right)_P - v\right]$
1	From given virial equation of state, $P = \dfrac{RT}{v}\left(1 + \dfrac{B(T)}{v}\right)$ the following can be derived by partial differentiation. $0 = \dfrac{R}{v}\left(1 + \dfrac{B(T)}{v}\right) - \dfrac{R}{v^2}T\left(1 + \dfrac{B(T)}{v}\right)\left(\dfrac{\partial v}{\partial T}\right)_P + \dfrac{RT}{v}\dfrac{B'(T)}{v} + \dfrac{RT}{v}\left(-\dfrac{B(T)}{v^2}\right)\left(\dfrac{\partial v}{\partial T}\right)_P$ Arranging above equation, the following can be derived $T\left(\dfrac{\partial v}{\partial T}\right)_P = \dfrac{v + B(T) + TB'(T)}{1 + 2\dfrac{B(T)}{v}}$
1	Finally, calculating the Joule-Thomson coefficient $\mu_{JT} = \dfrac{1}{c_P}\left[T\left(\dfrac{\partial v}{\partial T}\right)_P - v\right] = \dfrac{1}{c_P}\left[\dfrac{v + B(T) + TB'(T)}{1 + 2\dfrac{B(T)}{v}} - v\right]$ $\mu_{JT} = \dfrac{1}{c_P}\left[\dfrac{-B(T) + TB'(T)}{1 + 2\dfrac{B(T)}{v}}\right]$
6	The isentropic expansion coefficient $\mu_s = \left(\dfrac{\partial T}{\partial P}\right)_s$ Using the calculus the following can be derived by chain rule, $\left(\dfrac{\partial s}{\partial P}\right)_T \left(\dfrac{\partial P}{\partial T}\right)_s \left(\dfrac{\partial T}{\partial s}\right)_P = -1$

Rearranging The terms,

$$\mu_s = \left(\frac{\partial T}{\partial P}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_P \left(\frac{\partial s}{\partial P}\right)_T$$

Also, the entropy can be defined as the function of temperature and pressure.

$$s = f(T, p)$$

Using the calculus, the following can be derived.

$$ds = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP \quad , \quad Tds = T\left(\frac{\partial s}{\partial T}\right)_P dT + T\left(\frac{\partial s}{\partial P}\right)_T dP \quad ----(1)$$

The definition of specific heat under constant pressure : $T\left(\frac{\partial s}{\partial T}\right)_P = c_P$

From the Maxwell's equation : $\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P$

From basic thermodynamics, it can be shown that $dh = c_P dT - \left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right]dP \quad ----(3)$

$$\mu_s = \left(\frac{\partial T}{\partial P}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_P \left(\frac{\partial s}{\partial P}\right)_T = \frac{T}{c_P}\left(\frac{\partial v}{\partial T}\right)_P$$

From given virial equation of state, $P = \frac{RT}{v}\left(1 + \frac{B(T)}{v}\right)$ the following can be derived by partial differentiation.

$$0 = \frac{R}{v}\left(1 + \frac{B(T)}{v}\right) - \frac{R}{v^2}T\left(1 + \frac{B(T)}{v}\right)\left(\frac{\partial v}{\partial T}\right)_P + \frac{RT}{v}\frac{B'(T)}{v} + \frac{RT}{v}\left(-\frac{B(T)}{v^2}\right)\left(\frac{\partial v}{\partial T}\right)_P$$
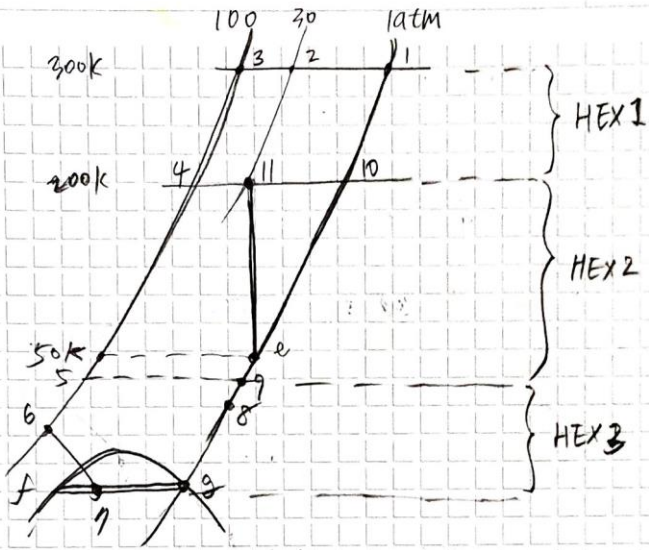
Arranging above equation, the following can be derived

$$T\left(\frac{\partial v}{\partial T}\right)_P = \frac{v + B(T) + TB'(T)}{1 + 2\frac{B(T)}{v}}$$
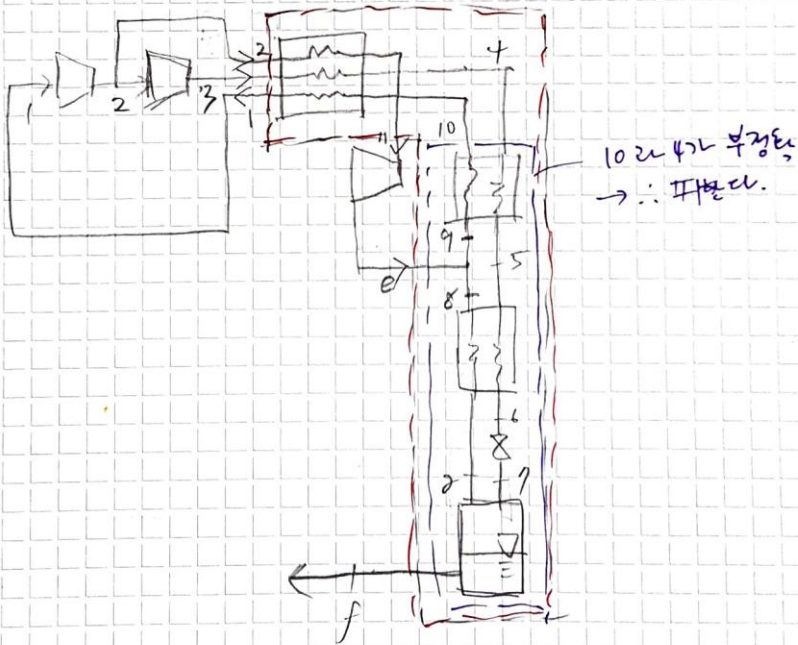
Finally, calculating the isentropic expansion coefficient

$$\mu_s = \frac{T}{c_P}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{c_P}\frac{v + B(T) + TB'(T)}{1 + 2\frac{B(T)}{v}}$$

4.

(a)

100 30 1atm

300k

3 2 1

} HEX 1

200k 4 11 10

} HEX 2

50k 5 e

6 7 8

} HEX 3

f 9

(b), (c)

2 4

2 3

11 10

3

9 5

e 8

2

6

2 7

f

10과 4가 부정통
→ ∴ 파틀다.

$$\dot{Q} - \dot{W} = 0$$

$$= (\dot{m}h)_{in} - (\dot{m}h)_{out}$$

$$= (\textcircled{1} + \textcircled{11} + \textcircled{f}) - (\textcircled{2} + \textcircled{3} + \textcircled{e})$$

$$= [(\dot{m} - \dot{m}_f)h_1 + \dot{m}_e h_{11} + \dot{m}_f h_f]$$

$$- [\dot{m}_e h_2 + (\dot{m} - \dot{m}_e)h_3 + \dot{m}_e h_e]$$

$$\Rightarrow \quad 0 = [(1 - y)h_1 + x h_{11} + y h_f]$$

$$- [x h_2 + (1 - x)h_3 + x h_e]$$

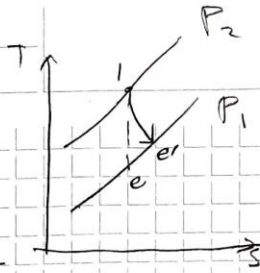$$\Rightarrow \quad y(h_1 - h_f) = (h_1 - h_3) + x(h_3 - h_2) + x(h_{11} - h_e)$$

$$\Rightarrow \quad y = \frac{h_1 - h_3}{h_1 - h_f} + x\frac{h_3 - h_2}{h_1 - h_f} + x\frac{h_{11} - h_e}{h_1 - h_f}$$

$$= \frac{372 - 375}{372 - \cancel{\textcircled{0}}} + 0.6\frac{375 - 373}{372 - \cancel{\textcircled{0}}} + 0.6\frac{270 - 115}{372 - \cancel{\textcircled{0}}}$$

$$= \quad 0 \quad + \quad 0 \quad + \quad \cancel{0.33} \; 0.25$$

$$= \cancel{0.33} = 0.25$$

(d) $\quad -\dot{W}/\dot{m} = -\dot{W}_c/\dot{m} - \dot{W}_e/\dot{m}$

$$= [T_1(s_1 - s_2) - (h_1 - h_2)] - x(h_{11} - h_e)$$

$$= [300(5.5 - 4.5) - (372 - 373)] - 0.6(270 - 115)$$

$$= 300 - 93$$

$$= 207 \; [kJ/kg]$$

$T_0 = 5 = (h_1 - h_2)P$

(e) $y = \dfrac{h_1 - h_3}{h_1 - h_f} + x \dfrac{h_3 - h_2}{h_1 - h_f} + x \dfrac{h_{11} - h_e}{h_1 - h_f}$

$\rightarrow y = \dfrac{h_1' - h_3}{h_1' - h_f} + x \dfrac{h_3 - h_2}{h_1 - h_f} + x \dfrac{h_{11} - h_{e'}}{h_1 - h_f}$

① $\varepsilon = \dfrac{C_c (T_1' - T_3)}{C_{min}(T_3 - T_3)} \simeq \dfrac{h_1' - h_3}{h_1 - h_3}$

$\rightarrow h_1' - h_3 = \varepsilon (h_1 - h_3) \qquad h_1' = h_3 + \varepsilon(h_1 - h_3)$

② $\eta_e = \dfrac{h_{11} - h_{e'}}{h_{11} - h_e}$

$\rightarrow h_{11} - h_{e'} = \eta_e (h_{11} - h_e)$

$\rightarrow y = \dfrac{h_1 - h_2 - (1-\varepsilon)(h_1 - h_3)}{h_1 - h_f - (1-\varepsilon)(h_1 - h_3)}$

$\qquad + x \dfrac{h_3 - h_2}{h_1 - h_f} + x \eta_e \dfrac{h_{11} - h_e}{h_1 - h_f}$

$\dfrac{h_3 + \varepsilon(h_1 - h_3) - h_3}{h_3 + \varepsilon(h_1 - h_3) - h_f}$

$\dfrac{h_1 - h_3 - h_1 + h_3 + \varepsilon(h_1 - h_3)}{h_1 - h_f - h_1 + h_3 + \varepsilon(h_1 - h_3)}$

$\dfrac{h_1 - h_3 - (1-\varepsilon)(h_1 - h_3)}{h_1 - h_f - (1-\varepsilon)(h_1 - h_3)}$

(f) heat leak

$$Q - W = [(\dot{m} - \dot{m}_f)h_1 + \dot{m}_e h_{11} + \dot{m}_f h_f]$$
$$- [\dot{m}_e h_2 + (\dot{m} - \dot{m}_e) h_3 + (\dot{m}_o h_e)] - \dot{Q}_{inleak}$$

$$0 = [(1-y)h_1 + x h_{11} + y h_f] - [x h_2 + (1-x)h_3 + x h_e] - \frac{\dot{Q}_{inleak}}{\dot{m}}$$

$$\boxed{y} = \frac{h_1 - h_3}{h_1 - h_f} + x \frac{h_3 - h_2}{h_1 - h_f} + x \frac{h_{11} - h_e}{h_1 - h_f} - \frac{\dot{Q}_{inleak}}{\dot{m}}$$

↓ by heat in-leak

(g) More expanders, heat exchanger effectiveness↑, expander efficiency↑

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4. (a) (5 points)
    (b) (5 points)
    (c) (5 points)
    (d) (10 points)
    (e) (5 points)
    (f) (5 points)
    (g) (5 points)

5. (20 points)

points	Descriptions
10	$$C_R = 0$$ $$d\dot{Q} = -C_h dT_h$$ $$d\dot{Q} = -C_c dT_c$$ $$d\dot{Q} = UdA(T_h - T_c)$$ $$C = \dot{m}c_p$$ $$dT_h - dT_c = d(T_h - T_c) = -d\dot{Q}\left(\frac{1}{C_h} - \frac{1}{C_c}\right) = -UdA(T_h - T_c)\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$ $$\frac{d(T_h - T_c)}{T_h - T_c} = -\left(1 - \frac{C_h}{C_c}\right)\frac{U}{C_h}dA$$ $$\int_{T_{h1}-T_{c2}}^{T_{h2}-T_{c1}} \frac{d(T_h - T_c)}{T_h - T_c} = -\left(1 - \frac{C_h}{C_c}\right)\frac{U}{C_h}\int_0^A dA$$ $$\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}} = \exp\left[-\frac{UA}{C_h}\left(1 - \frac{C_h}{C_c}\right)\right]$$
5	Assume $C_h = C_{min}, C_c = C_{max}$ $$\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}} = \exp\left[-\frac{UA}{C_{min}}\left(1 - \frac{C_h}{C_c}\right)\right]$$
5	Meanwhile, $\dot{Q}_{max} = C_{min}(T_{h1} - T_{c1}), \dot{Q}_{actual} = C_c(T_{c2} - T_{c1})$ Therefore, $\epsilon = \frac{C_c(T_{c2} - T_{c1})}{C_{min}(T_{h1} - T_{c1})}$ Also, $(T_{h2} - T_{c1})/(T_{h1} - T_{c2}) = (1 - \epsilon)/(1 - C_R\epsilon)$ $$\frac{1 - \epsilon}{1 - C_R\epsilon} = \exp\left[-NTU\left(1 - \frac{C_{min}}{C_{max}}\right)\right] \quad \left(\because \frac{UA}{C_{min}} = NTU\right)$$ $$\epsilon = \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R\exp[-NTU(1 - C_R)]}$$ $$\epsilon = 1 - \exp(-NTU) \quad (\because C_R = 0)$$