

# 2016년 2학기 항공기구조역학 중간고사

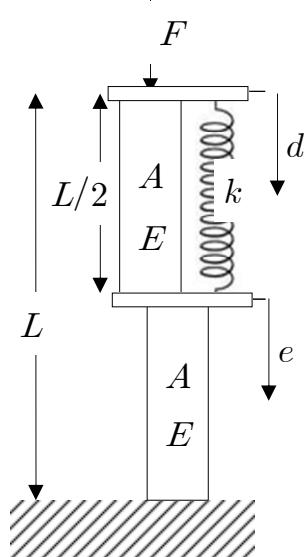
1. A landing gear with a hydraulic cylinder was modeled as Figure 1.

(a) By drawing free body diagram, determine the redundancy number  $N_R$ .

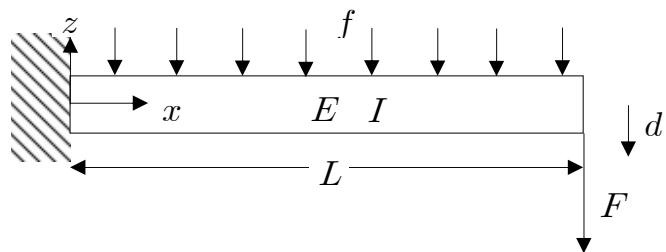
(b) Derive the total length decrement  $d$ . Select one method between force method and displacement method. (If you don't take the steps of those method, your point will be deducted a little.)

(c) Calculate the force loaded on hydraulic cylinder(spring).

$$L = 1.5 \text{ [m]}, E = 70 \text{ [GPa]}, A = 0.03 \text{ [m}^2\text{]}, F = 10^6 \text{ [N]}, k = 10^6 \text{ [N/m]}$$



2. A wing was modeled as a cantilevered beam with concentrated load  $F$  by the engine and distributed load  $f$  by the fuel.



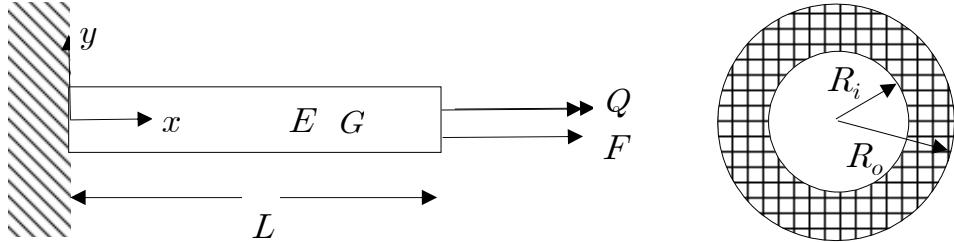
(a) Write the governing equation about the displacement to z-direction  $w(x)$  and the boundary conditions.

(b) Calculate the tip displacement  $d$  with those values.

$$E = 70 \text{ [GPa]}, I = 0.002 \text{ [m}^4\text{]}, L = 15 \text{ [m]}$$

$$f = 700 \text{ [N/m]}, F = 40 \text{ [kN]}$$

3. A turbine shaft is modeled as a hollow circular rod like in Figure 3. The thrust and the drag exert axial force  $F$  and torque  $Q$  on the shaft.



**Figure 3**

- (a) Express area polar moment  $J_h$  of the cross section with given symbols. And express another area polar moment  $J_s$  for a solid(not hollow), if the shaft was made with the same amount of material. Compare them.
- (b) From the result of (a), the wider the hollow area is, the more robust the shaft is. But if the wall becomes very thin, an unstable problem occurs. What is it?
- (c) Calculate the tip rotation angle  $\Theta$ .  
 $E = 70[\text{GPa}]$ ,  $G = 27[\text{GPa}]$ ,  $L = 10[\text{m}]$ ,  $R_i = 0.3[\text{m}]$ ,  $R_o = 0.5[\text{m}]$ ,  
 $F = 1000[\text{kN}]$ ,  $Q = 1000[\text{kNm}]$
- (d) On the surface of the shaft, calculate the principal stress, and draw the principal direction in axis-hoop orientation.

# 2016년 2학기 항공기구조역학 기말고사 풀이

문제 번호	소문제 번호	풀이	점수
	a	<p> <math>x_{2c} = 0</math> (symmetry)  <math>a = \sqrt{h^2 + (c/2)^2} = \frac{13}{5}h = 0.78[m]</math>  <math>S = \int EdA = Et(c+2a) = 10Eth = 2.1[GN]</math>  <math>S_3 = \int Ex_3 dA = Etha = \frac{13}{5}Eth^2 = 164[MN]</math>  <math>x_{3c} = S_3/S = \frac{ha}{c+2a} = \frac{13}{50}h = 0.078[m]</math> </p>	3
1		(0,0.078)	2
	b	$f(s) = c - \frac{Q_2(s)}{H_{22}^c}L$ $H_{22}^c = \int E(x_3 - x_{3c})^2 dA = Et \left\{ (c+2a)x_{3c}^2 - 2hax_{3c} + \frac{2}{3}h^2a \right\}$ $= \frac{Eth^2a(2c+a)}{3(c+2a)} = 25.62[MNm^2]$ $Q_2(s) = \int Et x_3(s) ds$ $Q_2(s_1) = -Et x_{3c} s_1 = -54.6[MN] \times s_1$ $Q_2(s_2) = Et \left( \frac{h}{2a} s_2^2 - x_{3c} s_2 \right)$ $= 135[MN/m] \times s_2^2 - 54.6[MN] \times s_2$ $Q_2(s_3) = Et \left\{ -\frac{h}{2a} s_3^2 + (h - x_{3c}) s_3 \right\}$ $= -135[MN/m] \times s_3^2 + 155[MN] \times s_3$	9

	$f_o(s_1) = c_1 + \frac{EtL}{H_{22}^c} x_{3c} s_1 = c_1 + 8.52 [MPa] \times s_1$ $f_o(s_2) = c_2 + \frac{EtL}{H_{22}^c} \left( -\frac{h}{2a} s_2^2 + x_{3c} s_2 \right)$ $= c_2 - 21.02 [MN/m^3] s_2^2 + 8.52 [MN/m^2] s_2$ $f_o(s_3) = c_3 + \frac{EtL}{H_{22}^c} \left\{ \frac{h}{2a} s_2^2 + (x_{3c} - h) s_2 \right\}$ $= c_3 + 21.02 [MN/m^3] s_2^2 - 24.3 [MN/m^2] s_2$	9
	$f_o(s_1) = \frac{EtL}{H_{22}^c} x_{3c} s_1 = 8.52 [MPa] \times s_1$ $f_o(s_2) = \frac{EtL}{H_{22}^c} \left( -\frac{h}{2a} s_2^2 + x_{3c} s_2 + x_{3c} c \right)$ $= -21.02 [MN/m^3] s_2^2 + 8.52 [MN/m^2] s_2 + 12.3 [MN/m]$ $f_o(s_3) = \frac{EtL}{H_{22}^c} \left\{ \frac{h}{2a} s_2^2 + (x_{3c} - h) s_2 - \frac{ha}{2} + (a+c)x_{3c} \right\}$ $= 21.02 [MN/m^3] s_2^2 - 24.3 [MN/m^2] s_2 + 6.14 [MN/m]$	2
c	$f_c = - \int_C \frac{f_o(s)}{Gt} ds / \int \frac{1}{Gt} ds$ $f_c = - \frac{EtL}{H_{22}^c} \frac{cha}{2(2a+c)} = -6.1377 [MN/m]$	8 2
d	<p>Shear center 위치는 <math>\vec{i}_3</math> 방향.</p> <p><math>L</math>의 moment arm은 <math>c/4</math>.</p> $M = M_{ac} - L \frac{c}{4} = 4.56 [MNm]$ $M = 2Af_m$ $f_m = \frac{M}{2A} = \frac{M}{hc}$ $f_m = 10.6 [MN/m]$	4 4 2

문제 번호	소문제 번호	풀이	점수
2		$\vec{u}_C = \vec{OC} = (a+b)\cos\theta \vec{i}_1 - a\sin\theta \vec{i}_2$ $\vec{u}_A = \vec{OA} = b\sin\theta \vec{i}_2$ $\vec{P} = -P\vec{i}_1$ $\vec{F}_s = -F_s \vec{i}_2 = -kb\sin\theta \vec{i}_2$	5
		$\vec{\delta u}_C = \{-(a+b)\sin\theta \vec{i}_1 - a\cos\theta \vec{i}_2\} \delta\theta$ $\vec{\delta u}_A = \{b\cos\theta \vec{i}_2\} \delta\theta$	3
		$\delta W = \vec{\delta u}_C \cdot \vec{P} + \vec{\delta u}_A \cdot \vec{F}_s$ $= P(a+b)\sin\theta - kb^2\sin\theta\cos\theta = 0$	5
		$P = \frac{kb^2\cos\theta}{a+b}$	2

문제 번호	소문제 번호	풀이	점수
3	a		4
		$\vec{i}_2$ force equilibrium : $F_s + R_2 - W - P = 0$ $\vec{i}_3$ moment equilibrium : $M_R + R_2 \frac{L}{4} - PL = 0$	
		마지수 $F_s, R_2, M_R$ 3개 식 2개	2
		redundant force $R_2$	
		$F_s = -R_2 + W + P$	4
		$M_R = -\frac{L}{4}R_2 + LP$	
	b	$0 \leq x_1 < L/4,$ $M_3(x_1) = (F_s - W)x_1 - M_R = (P - R_2)x_1 - LP + \frac{L}{4}R_2$	5
	c	$L/4 \leq x_1 \leq L,$ $M_3(x_1) = Px_1 - LP$	5
		$A' = \int_0^L \frac{\{M_3(x_1)\}^2}{2H_{33}^c} dx_1 + \frac{F_s^2}{2k}$ $= \frac{1}{2H_{33}^c} \left\{ \frac{(P - R_2)^2}{3} \left( \frac{L}{4} \right)^3 - \frac{2(P - R_2)(LP - \frac{L}{4}R_2)}{2} \left( \frac{L}{4} \right)^2 + (LP - \frac{L}{4}R_2)^2 \left( \frac{L}{4} \right) \right\}$ $+ \frac{1}{2H_{33}^c} \left\{ \frac{P^2}{3} \frac{63}{64} L^3 - \frac{2LP^2}{2} \frac{15}{16} L^2 + L^2 P^2 \frac{3}{4} L \right\} + \frac{(R_2 - W - P)^2}{2k}$	6
		$\frac{\delta A'}{\delta R_1} = \frac{1}{2H_{33}^c} \left[ -\frac{2}{3}(P - R_2) \left( \frac{L}{4} \right)^3 + \left( LP - \frac{L}{4}R_2 + \frac{L}{4}(P - R_2) \right) \left( \frac{L}{4} \right)^2 - \frac{L}{4} 2(LP - \frac{L}{4}R_2) \frac{L}{4} \right]$ $+ \frac{R_2 - W - P}{k} = 0$	6

		$R_2 = \frac{W + \left\{ \frac{11k}{6H_{33}^c} \left( \frac{L}{4} \right)^3 + 1 \right\} P}{\frac{k}{3H_{33}^c} \left( \frac{L}{4} \right)^3 + 1} = 2.8 [MN]$ $F_s = 1.19 [MN]$ $M_R = -4.04 [MNm]$	3
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