2016-2학기 항공기구조역학 기말고사

1. (50pts) A cross section of a wing is modeled as thin-walled closed triangle, as shown in Fig. 1. Aerodynamic center(AC) is located at 1/4 chord. Lift($\boldsymbol{L}$) and aerodynamic pitching moment($\boldsymbol{M_{ac}}$) are applied at AC.

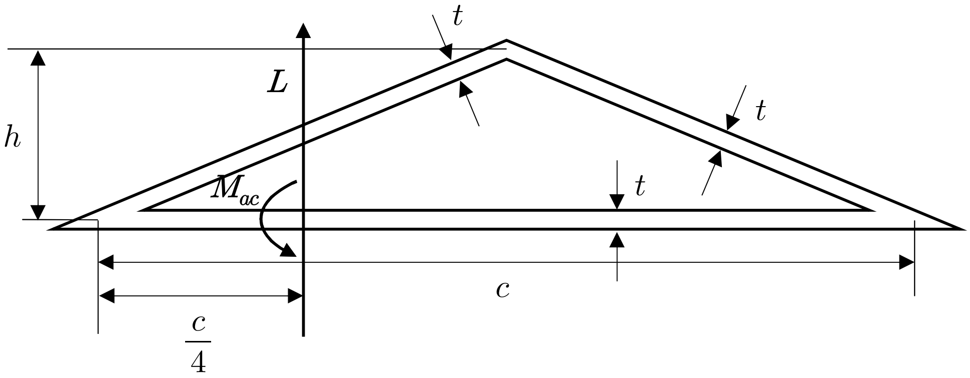


Figure 1. Wing cross section

$$E = 70[GPa],\ h = 0.3[m],\ c = \frac{24}{5}h,\ t = 0.01[m],\ L = 4[MN],\ M_{ac} = 6[MNm].$$

a) (5pts) Select origin $\boldsymbol{O}$ at any location that you wish. Then, find the location of the centroid $\boldsymbol{C}$.

b) (25pts) Create a cut at any location that you wish and derive the expression of the open section shear flow $f_o(s)$ generated by lift($\boldsymbol{L}$).

$$\text{Hint: } f_o(s) = c + \frac{Q_3(s)H_{23}^c - Q_2(s)H_{33}^c}{\Delta H} V_3 - \frac{Q_3(s)H_{22}^c - Q_2(s)H_{23}^c}{\Delta H} V_2$$

$$\Delta H = H_{22}^c H_{33}^c - (H_{23}^c)^2$$

c) (10pts) Estimate the closing shear flow f_c for $f_o(s)$.

d) (10pts) Find the shear flow f_m generated by the moment due to both lift($\boldsymbol{L}$) and aerodynamic moment($\boldsymbol{M_{ac}}$).

2. (15pts) A lever-spring system consists of a rigid bar ABC and a spring(stiffness k) as depicted in Fig. 2. The spring is un-stretched when angle $\theta = 0$. Use the principle of virtual work to determine the equilibrium relation of the system. (Find relation between P and θ for equilibrium)

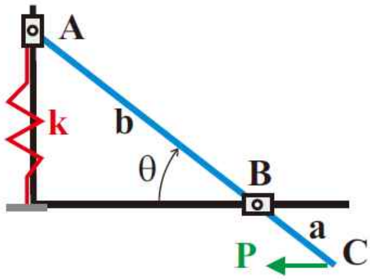


Figure 2. Lever and spring system

3. (35pts) One side of a wing spar on landing gears is modeled as a beam depicted in Fig. 3. The left end of the beam is restricted to rotate but allowed to move vertically by slider and spring(k). The right end is free conditioned. Pin support is located at $L/4$. The weights of the fuselage and wing exert vertical load W and P, respectively.

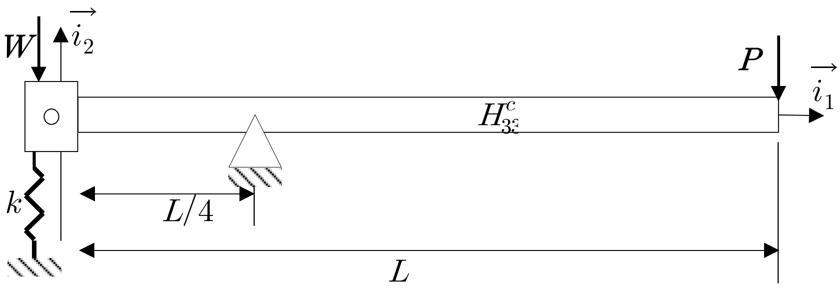


Figure 3. Beam model of wing and landing gears

$$L = 20[m],\ \ W = 3.5[MN],\ \ P = 0.5[MN],\ \ H_{33}^c = 60[GNm^2],\ \ k = 30[GN/m]$$

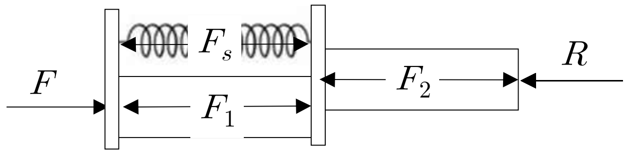
a) (10pts) Formulate the equilibrium equations with reaction forces. Select redundant force(s) as you wish and express the other forces by the redundant force(s).

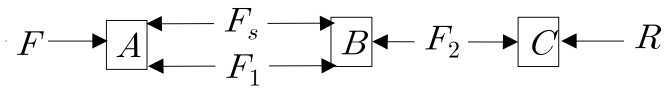
b) (10pts) Find the moment distribution $M_3(x_1)$

c) (15pts) Derive the expression of the complementary strain energy A'. And by using principle of least work, estimate the reaction forces.

2016년 2학기 항공기구조역학 중간고사 풀이

<table>
<tr><th>문제 번호</th><th>소문제 번호</th><th colspan="2">풀이</th><th>점수</th></tr>
<tr><td rowspan="3">1</td><td rowspan="2">a</td><td colspan="2">





</td><td>3</td></tr>
<tr><td colspan="2">

$$\begin{cases} \boxed{A}\ F = F_s + F_1 \\ \boxed{B}\ F_s + F_1 = F_2 \\ \boxed{C}\ F_2 = R \end{cases}$$

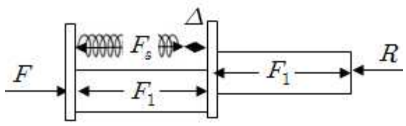
미지수 F_s, F_1, F_2, R 4개

식 3개

$N_R = 4 - 3 = 1$

</td><td>2</td></tr>
<tr><td>b</td><td>

Force method



redundant force F_1

$$\begin{cases} F_s = F - F_1 \\ F_2 = F \\ R = F \end{cases}$$

$$d_1 = \frac{LF_1}{2EA}$$

$$d_2 = \frac{LF_2}{2EA} = \frac{LF}{2EA}$$

$$d_s = \frac{F_s}{k} = \frac{F - F_1}{k}$$

$$\Delta = d_s - d_1 = \frac{F - F_1}{k} - \frac{LF_1}{2EA}$$

</td><td>

Displacement method

$$F_1 = \frac{2EA}{L} d_1$$

$$F_2 = \frac{2EA}{L} d_2$$

$$F_s = k d_s$$

$$d_1 = d_s = d - e$$

$$d_2 = e$$

$$F_1 = \frac{2EA}{L}(d - e)$$

$$F_2 = \frac{2EA}{L} e$$

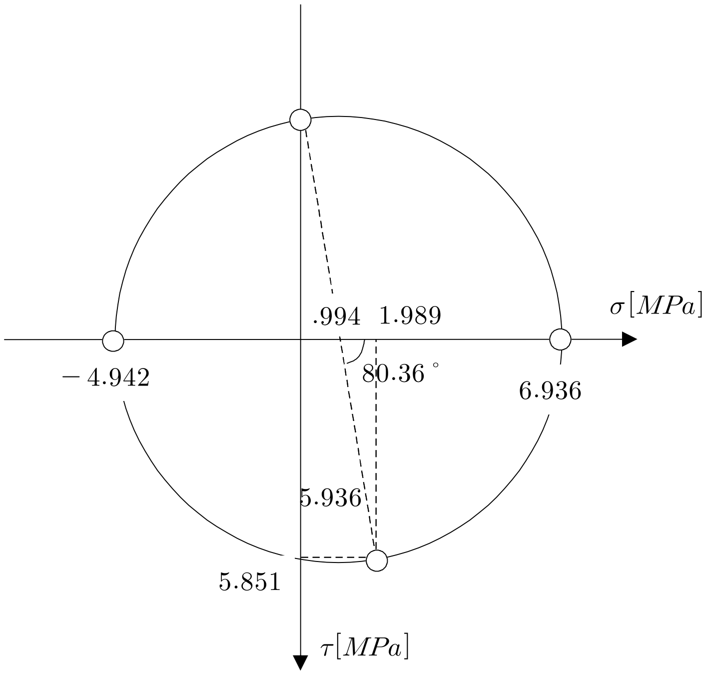
$$F_s = k(d - e)$$

</td><td>10</td></tr>
</table>

		$\varDelta = 0$일 때, $\dfrac{F - F_1}{k} = \dfrac{LF_1}{2EA}$ $F_1 = \dfrac{2EAF}{(2EA + kL)}$ $d_1 = \dfrac{LF}{2EA + kL}$ $d_2 = \dfrac{LF}{2EA}$ $d_s = \dfrac{LF}{2EA + kL}$ $d = d_1 + d_2 = \dfrac{(4EA + kL)LF}{2EA(2EA + kL)}$	$\begin{cases} F = k(d-e) + \dfrac{2EA}{L}(d-e) \\ k(d-e) + \dfrac{2EA}{L}(d-e) = \dfrac{2EA}{L}e \\ \dfrac{2EA}{L}e = R \end{cases}$ $e = \dfrac{LF}{2EA}$ $d = \dfrac{4EA + kL}{2EA + kL}e$ $= \dfrac{(4EA + kL)LF}{2EA(2EA + kL)}$	
	c	$d_s = d - e = \dfrac{LF}{2EA + kL}$ $F_s = kd_s = \dfrac{kLF}{2EA + kL} = 357[N]$		10

문제 번호	소문제 번호	풀이	점수
2	a	$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right)=-f$ B.C : $w(0)=w'(0)=0$ $-\frac{d}{dx}\left(EI\frac{d^2w}{dx^2}\right)\Big\vert_{x=L}=-F$ $\left\vert EI\frac{d^2w}{dx^2}\right\vert_{x=L}=0$	10
	b	$\frac{d}{dx}\left(EI\frac{d^2w}{dx^2}\right)=-fx+C_3$ $EI\frac{d^2w}{dx^2}=-\frac{f}{2}x^2+C_3x+C_2$ $\frac{dw}{dx}=-\frac{f}{6EI}x^3+\frac{C_3}{2EI}x^2+\frac{C_2}{EI}x+\frac{C_1}{EI}$ $w=-\frac{f}{24EI}x^4+\frac{C_3}{6EI}x^3+\frac{C_2}{2EI}x^2+\frac{C_1}{EI}x+C_0$ $w(0)=w'(0)=0$에 의해 $C_0=C_1=0.$ $\frac{d}{dx}\left(EI\frac{d^2w}{dx^2}\right)\Big\vert_{x=L}=F$에 의해 $C_3=F+fL.$ $\left\vert EI\frac{d^2w}{dx^2}\right\vert_{x=L}=0$에 의해 $C_2=-FL-\frac{fL^2}{2}.$ 따라서 $w(x)=-\frac{f}{24EI}x^4+\frac{F+fL}{6EI}x^3-\frac{2FL+fL^2}{4EI}x^2$	15
		$d=-w(L)=\frac{8FL^3+3fL^4}{24EI}=0.353[m]$	5

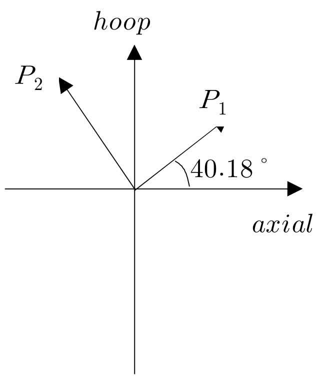
문제 번호	소문제 번호	풀이	점수
3	a	$J_h = \int_{A_h} r^2 dA = \int_0^{2\pi} \int_{R_i}^{R_o} r^2 r dr d\theta$ $= \frac{\pi(R_o^4 - R_i^4)}{2}$ $A_h = \pi(R_o^2 - R_i^2)$ $A_s = \pi R_s^2$ $A_h = A_s$가 만족하려면 $R_s^2 = R_o^2 - R_i^2$ $J_s = \int_{A_s} r^2 dA = \int_0^{2\pi} \int_0^{R_s} r^2 r dr d\theta$ $= \frac{\pi R_s^4}{2} = \frac{\pi(R_o^2 - R_i^2)^2}{2}$ $\frac{J_h}{J_s} = \frac{R_o^4 - R_i^4}{(R_o^2 - R_i^2)^2} = \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} > 1$이므로 $J_h > J_s$	10
	b	Torsional buckling	10
	c	$\frac{d}{dx}\left(GJ_h \frac{d\theta}{dx}\right) = 0$ B.C $\theta(0) = 0,\ GJ_h \frac{d\theta}{dx}\Big\vert_{x=L} = Q$ $\theta(x) = \frac{Q}{GJ_h} x$	5
		$\Theta = \theta(L) = \frac{QL}{GJ_h} = 0.004334[rad] = 0.24[\deg]$	5
	d	$\sigma_{xx} = \frac{F}{A_h} = 1.989[MPa]$ $\tau_{x\theta} = GR_o \frac{d\theta}{dx} = \frac{QR_o}{J_h} = 5.851[MPa]$ $\sigma_{\theta\theta} = 0$	15



Mohr's circle

$\sigma_{P1}=6.936[MPa]$

$\sigma_{P2}=-4.942[MPa]$



Principal direction in axial-hoop orientation

※ Eigenvalue problem으로 풀어도 됨