2016-1 Computational Structural Analysis Midterm Exam and Solution

※ 풀이과정을 빠짐없이 적으시오. 모든 과정은 부분점수가 있습니다.

1. Explain without any numerical value. (30 pts)

(1) Explain why the structure with finite element is made artificially stiffer than the real structure (**15 pts)**

the structure with finite element is limited its deformation to be the linear combination of a finite number of arbitrarily preselected deformation mode shapes. In reality, structure is able to deform in an infinite number of deformation shapes. Moreover, the shape functions are required only to satisfy the geometric boundary conditions.

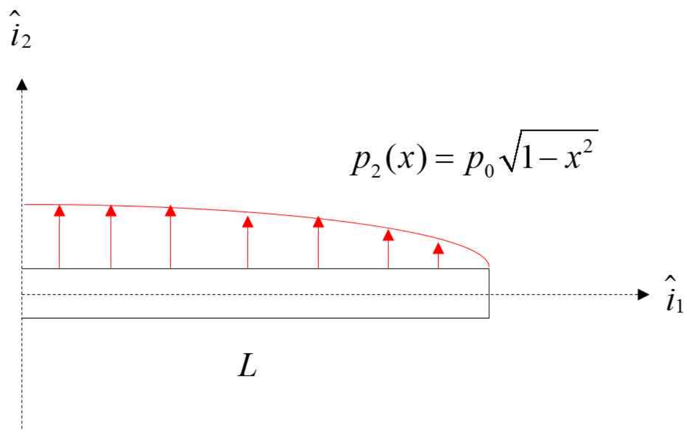
[limit, linear combination, infinite, finite, only geomteric boundary 3점씩]

(2) Explain why the convergence rates of the bending moment and shear force predictions are far slower than those observed for the transverse displacement.
(15 pts)

Internal forces(shear forces) and bending moments are obtained by taking derivatives of the approximated displacement field. Hence, the accuracy of the predictions decreases as the order of the derivative increases.

[approximated displacement 7점, derivatives 8점]

2. Solve the problem (30 pts)



PB2. Cantilevered beam with eliptical pressure load

(1) Develop a one-term approximate solution and compare the tip deflection **(10 pts)**

$$u_2(\eta) = q_1\eta^2,\ \frac{d^2u_2}{d\eta^2} = \frac{2q_1}{L^2}$$

$$A = \frac{1}{2}L\int_0^1 H_{33}^c\left(\frac{2q_1}{L^2}\right)^2 d\eta = \frac{2q_1^2}{L^3}H_{33}^c$$

$$\Phi = -L\int_0^1 p_2(\eta)u_2(\eta)d\eta = -Lp_oq_1\int_0^1 \eta^2\sqrt{1-\eta^2}\,d\eta = -Lp_oq_1\int_0^{\frac{\pi}{2}} \cos^2x - \cos^4x\,dx = -\frac{\pi}{16}Lp_oq_1$$

$$\Pi = A + \Phi = \frac{2q_1^2}{L^3}H_{33}^c - \frac{\pi}{16}Lp_oq_1$$

$$\frac{\partial\Pi}{\partial q_1} = \frac{4q_1^2}{L^3}H_{33}^c$$

[각 항목마다 2점씩 총 10점]

(2) Develop a two-term approximate solution and compare the tip deflection **(20 pts)**

$$u_2(\eta) = q_1\eta^2 + q_2\eta^3,\ \frac{d^2u_2}{d\eta^2} = \frac{1}{L^2}(2q_1 + 6\eta q_2)$$

$$A = \frac{1}{2}L\int_0^1 H_{33}^c\left(\frac{4q_1^2 + 24\eta q_1q_2 + 36\eta^2q_2^2}{L^4}\right)^2 d\eta = \frac{H_{33}^c}{L^3}[2q_1^2 + 6q_1q_2 + 6q_2^2]$$

$$\Phi = -L\int_0^1 p_2(\eta)u_2(\eta)d\eta = -Lp_o\int_0^1 (q_1\eta^2 + q_2\eta^3)\sqrt{1-\eta^2}\,d\eta = -\frac{Lp_o}{240}[15\pi q_1 + 32q_2]$$

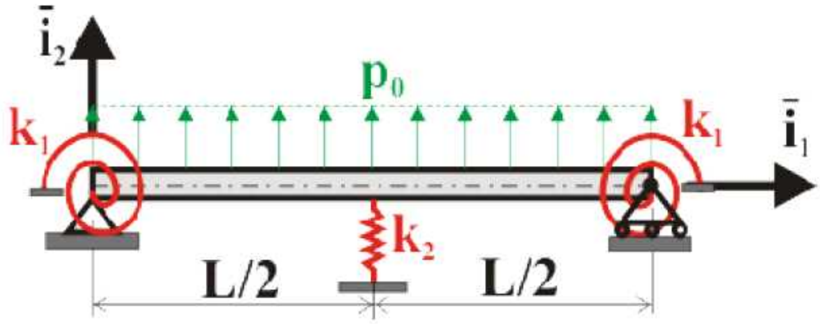
$$\Pi = A + \Phi = \frac{H_{33}^c}{L^3}[2q_1^2 + 6q_1q_2 + 6q_2^2] - Lp_o$$

$$\frac{\partial\pi}{\partial q_1} = \frac{H_{33}^c}{L^3}[4q_1 + 6q_2] - \frac{\pi}{16}Lp_o, \qquad \frac{\partial\pi}{\partial q_2} = \frac{H_{33}^c}{L^3}[6q_1 + 12q_2] - \frac{2}{15}Lp_o$$

$$\begin{bmatrix}4 & 6\\ 6 & 12\end{bmatrix}\begin{Bmatrix}q_1\\ q_2\end{Bmatrix} = \frac{L^4p_0}{H_{33}^c}\begin{bmatrix}\frac{\pi}{16}\\ \frac{2}{15}\end{bmatrix}, \qquad \begin{Bmatrix}q_1\\ q_2\end{Bmatrix} = \begin{bmatrix}\frac{\pi}{16} - \frac{1}{15}\\ -\frac{\pi}{32} + \frac{2}{45}\end{bmatrix}\frac{L^4p_0}{H_{33}^c}$$

[각 항목마다 4점씩 총 20점]

3. Solve the problem (40 pts)



Pb3. Simply supported beam with end point

(1) establish the governing equation and its boundary condition for exact solution **(15 pts)**

L/2을 기준으로 좌측을 $u_a(x)$, 우측을 $u_b(x)$로 설정하면, Governing ODE는 다음과 같다

$$H_{33}\frac{d^4u_a}{dx^4}=p_0, \qquad H_{33}\frac{d^4u_b}{dx^4}=p_o$$

또한 Governing Equation을 풀기위한 Boundary Condition은 다음과 같다.

$$B.C1)\, H_{33}\frac{d^2u_a}{dx^2}|_{x=0}-k_1\frac{du_a}{dx}|_{x=0}=0$$

$$B.C2)-H_{33}\frac{d^2u_a}{dx^2}|_{x=\frac{L}{2}}+H_{33}\frac{d^2u_b}{dx^2}|_{x=\frac{L}{2}}=0$$

$$B.C3)\, H_{33}\frac{d^2u_b}{dx^2}|_{x=L}+k_1\frac{du_b}{dx}|_{x=L}=0$$

[각 항목마다 7.5점씩 총 15점]

(2) Construct a 2-term trigonometric approximate solution by using defined [H].
Describe [K]{q} = [Q], do not perform the inverse [K] matrix. **(25 pts)**

$$[H]=\begin{bmatrix}\sin\pi\eta\\ \sin3\pi\eta\end{bmatrix}$$

for 1st derivatives of [H]

$$[B_s] = \frac{1}{L}\frac{dH}{\mathrm{det}a}$$

$$= \begin{bmatrix} \frac{\pi}{L}\cos\pi\eta \\ \frac{3\pi}{L}\cos 3\pi\eta \end{bmatrix}$$

for 2nd derivatives of [H]

$$[B] = \frac{1}{L^2}\frac{d^2H}{d\eta^2}$$

$$= \begin{bmatrix} -\frac{\pi^2}{L^2}\sin\pi\eta \\ -\frac{9\pi^2}{L^2}\sin 3\pi\eta \end{bmatrix}$$

for bending stiffness matrices

$$K_B = H_{33}\int_0^1 BB^T L deta = LH_{33}\int_0^1 \begin{bmatrix} -\frac{\pi^2}{L^2}\sin\pi\eta \\ \frac{9\pi^2}{L^2}\sin 3\pi\eta \end{bmatrix} \begin{bmatrix} -\frac{\pi^2}{L^2}\sin\pi\eta & \frac{9\pi^2}{L^2}\sin 3\pi\eta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{H_{33}\pi^4}{2L^3} & 0 \\ 0 & \frac{81H_{33}\pi^4}{2L^3} \end{bmatrix}$$

for spring stiffness matrices

$$K_s = K_{s1} + K_{s2} + K_{s3}$$

$$K_{s1} = k_1 B_s B_s^T|_{\eta=0} = \frac{\overline{k_1}}{L}\begin{bmatrix} \frac{\pi}{L}\cos\pi\eta \\ \frac{3\pi}{L}\cos 3\pi\eta \end{bmatrix}\begin{bmatrix} \frac{\pi}{L}\cos\pi\eta & \frac{3\pi}{L}\cos 3\pi\eta \end{bmatrix}|_{\eta=0}$$

$$= \begin{bmatrix} \frac{\overline{k_1}}{L^3}\pi^2 & \frac{3\overline{k_1}}{L^3}\pi^2 \\ \frac{3\overline{k_1}}{L^3}\pi^2 & \frac{9\overline{k_1}}{L^3}\pi^2 \end{bmatrix}$$

$$K_{s3} = k_1 B_s B_s^T|_{\eta=1} = \frac{\overline{k_1}}{L}\begin{bmatrix} \frac{\pi}{L}\cos\pi\eta \\ \frac{3\pi}{L}\cos 3\pi\eta \end{bmatrix}\begin{bmatrix} \frac{\pi}{L}\cos\pi\eta & \frac{3\pi}{L}\cos 3\pi\eta \end{bmatrix}|_{\eta=1}$$

$$= \begin{bmatrix} \frac{\overline{k_1}}{L^3}\pi^2 & \frac{3\overline{k_1}}{L^3}\pi^2 \\ \frac{3\overline{k_1}}{L^3}\pi^2 & \frac{9\overline{k_1}}{L^3}\pi^2 \end{bmatrix}$$

$$K_{s2} = k_2 H H^T|_{\eta=\frac{1}{2}} = \frac{H_{33}\overline{k}_2}{L^3}\begin{bmatrix} \sin\pi\eta \\ \sin 3\pi\eta \end{bmatrix}[\sin\pi\eta \;\; \sin 3\pi\eta]|_{\eta=\frac{1}{2}}$$

$$= \frac{H_{33}\overline{k}_2}{L^3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K = K_B + K_s = \begin{bmatrix} \frac{2\overline{k}_1}{L^3}\pi^2 + \frac{\overline{k}_2}{L^3} & \frac{6\overline{k}_1}{L^3}\pi^2 - \frac{\overline{k}_2}{L^3} \\ \frac{6\overline{k}_1}{L^3}\pi^2 - \frac{\overline{k}_2}{L^3} & \frac{18\overline{k}_1}{L^3}\pi^2 + \frac{\overline{k}_2}{L^3} \end{bmatrix} H_{33}$$

for load

$$Q = p_o \int_0^1 [H] L deta = p_o \int_0^1 \begin{bmatrix} \sin\pi\eta \\ \sin 3\pi\eta \end{bmatrix} L deta$$

$$= p_o L \begin{bmatrix} \frac{2}{\pi} \\ \frac{2}{3\pi} \end{bmatrix}$$

final equation is

$$[K]q = [Q] \;\; \leqq > \;\; \begin{bmatrix} \frac{2\overline{k}_1}{L^3}\pi^2 + \frac{\overline{k}_2}{L^3} & \frac{6\overline{k}_1}{L^3}\pi^2 - \frac{\overline{k}_2}{L^3} \\ \frac{6\overline{k}_1}{L^3}\pi^2 - \frac{\overline{k}_2}{L^3} & \frac{18\overline{k}_1}{L^3}\pi^2 + \frac{\overline{k}_2}{L^3} \end{bmatrix} H_{33}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = p_o L \begin{bmatrix} \frac{2}{\pi} \\ \frac{2}{3\pi} \end{bmatrix}$$

[Bs, B 모두 맞아야 5점, bending stiffness matrices 5점, spring stiffness matrices 5점]

[load 5점, final equation 5점]