Engineering Economic Analysis

Spring 2019

Final Exam.

2019.06.18

1. Prove or disprove rigorously (by using mathematical representation) the following statements
   (a) (5pts.) When a company has a technology exhibiting constant returns to scale, it is impossible to enjoy a positive level of maximized profit with a finite amount of inputs under the given input prices.
   (b) (5pts.) The supply curve of a firm in a competitive market is always increasing with respect to the price
   (c) (10pts.) At the optimal third-degree price discrimination in two markets by a monopolist, the marginal revenue in each market must be increasing more rapidly than the marginal cost for the output as a whole.

2. An industry consists of a large number of firms, each of which has a cost function of the firm
$$c(w_1, w_2, y) = (y^2 + 1)w_1 + (y^2 + 2)w_2$$
   (a) (5pts.) Find the average cost curve of a form and describe how it shifts as the factor price w_1 / w_2 changes.
   (b) (5pts.) Find the short-run supply curve of an individual firm.
   (c) (5pts.) Find the long-run industry supply curve.

3. Suppose that a firm owns two plants, each producing the same good. Every plant *j*'s average cost is given by $AC_j = \alpha + \beta_j q_j$ for $q_j \geq 0,$ where $j = 1, 2.$ Note that coefficient β_j may differ from pant to plant. Now you are asked to determine the cost-minimizing distribution of aggregate output $Q = q_1 + q_2$, among the two plant.
   (a) (10pts.) If $\beta_j > 0$ for every plant *j*, find the optimal amount of output for each plant when the firm should produce *Q* in total.
   (b) (5pts.) If $\beta_1 < 0$ and $\beta_2 > 0$ for every plant *j*, find the optimal amount of output for each plant when the firm should produce *Q* in total.
   (c) (10pts.) Suppose that the firm has constructed one more plant with each plant's average cost as $AC_j = \alpha + \beta_j q_j$ for $q_j \geq 0,$ where $j = 1, 2, 3.$ If $\beta_j > 0$ for every plant *j*, find the optimal amount of output for each plant when the firm should produce *Q* in total.

4. Consider *N* symmetric firms compete with outputs, each with cost function $c(q) = F + cq,$ where $F, c > 0.$ Assume that the inverse demand is given by $p(Q) = a - bQ,$ where $a > c, b > 0,$ and where *Q* denotes aggregate output.

(a) (10pts.) If exit and entry is not possible in the industry, find the equilibrium output for each firm and the market price.

(b) (10pts.) Suppose that firms now behave as a perfectly competitive firm with the given market price as the equilibrium price found in (a). If firms have enough time to enter or to exit the industry, find the equilibrium number of firms in this perfectly competitive market.

5. Consider a market which consists of upstream and downstream market. In the upstream market, two firms sell inputs to the downstream firms by determining its price, c_i, $i=1,2$, to maximize own profits. Using these inputs, the downstream firms produce final goods and sell them to final consumers. We assume that the downstream market is a duopoly in which two firms compete with the prices of their final goods, p_i, $i=1,2$, which are heterogeneous (See Figure (a)). Assume that to produce one unit of final good needs one unit of input and the marginal costs are zero. Assume that the demand function of final good for firm *i* is given by $q_i(p_i, p_j) = a - p_i + bp_j$ where $b > 0$.

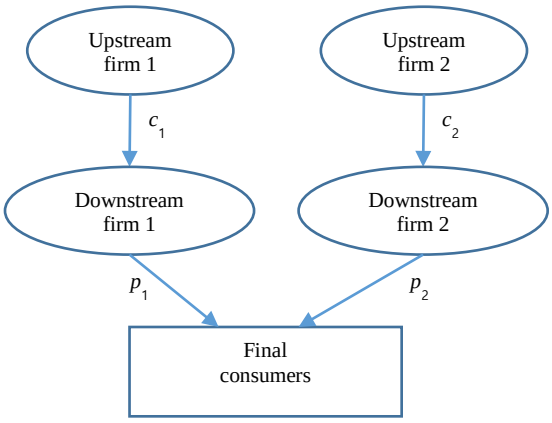

Figure (a). Non-integrated market

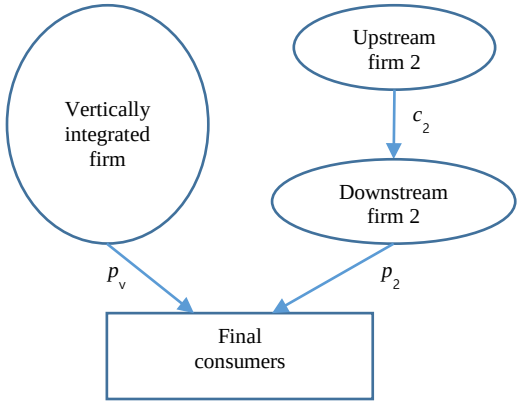

Figure (b) Vertically integrated market

(a) (10pts.) Find the optimal input prices, c_i, $i=1,2$, in the non-integrated market.

(b) (10pts.) Now the upstream firm 1 and downstream firm 1 decided to integrate vertically as one company each other (See Figure (b)). Thus the vertically integrated firm produces final good as well as input by itself and sells final good with price p_v. Find the condition that the equilibrium price of vertically integrated firm is less than the equilibrium price of downstream firm in the non-integrated market.