**Aeroelasticity M2795.005900**

**Final Examination**

Date: December 10 (Wednesday) 15:30 – 17:15

1. (30 Points) A two-dimensional airfoil model, which is mounted in a wind tunnel, is free to pitch about its mid-chord except for an elastic constraint provided by a linear spring, *K*, attached at the trailing edge. The mass moment of inertia is *I*, and the unsteady lift and moment at the quarter chord can be represented by:

 

and: *K* = 4.3 kN/m, *I* = 0.0145 kg-m, *b* = 0.30 m, *ρ* = 3.23 kg/m3

* 1. Derive the governing equation of motion.
  2. Compute the numerical value of the airspeed at the pitch **divergence** boundary.
  3. Compute the numerical value of the airspeed at the **flutter** boundary.

*U*

*L*

*M*

*b/2*

*b/2*

*b*

*K*

pivot

*α*

1. (5 Points each) In the aeroelastic formulation of turbomachinery, there exist three different formulations available. Explain briefly each formulation without the detailed mathematical derivation, and especially the advantage/disadvantage of each formulation.
   1. Traveling wave formulation
   2. Individual blade formulation
   3. Standing wave formulation

(Please add appropriate plots or figures for each function if they are necessary.)

1. (5 Points each) Explain briefly each one.
   1. In the turbomachinery, it turns out that the mass ratio is much larger than it is in the conventional fixed wings. Describe briefly about its advantage upon the flutter analysis procedure of the turbomachinery.
   2. Intentional mistuning on the turbine blade component may improve the flutter instability boundary of turbomachinery. Describe briefly about its procedure and draw a representative diagram.
   3. Campbell diagram is used in the forced vibration and the high-cycle fatigue analysis in the turbomachinery. Describe briefly its procedure and the usage of the Campbell diagram.
2. (10 Points) Explain why it is necessary to use an approximate method such as Pade Approximation for unsteady aerodynamics, even if there are a few available closed-form analytical formulations such as Theodorsen’s.
3. (30 Points) Find an analytical solution for the natural mode shapes (continuous) and frequencies of the simply supported uniform beam with end loads P as shown below. Show (plot) how the first three natural frequencies change as a function of the applied preload **P** (the pre-tension/compression in the beam).

Length = L, Flexural stiffness = EI (const.), Mass/length = m (const.), Area = A (const.)

**P**

w

**P**