Aeroelasticity M2795.005900
Final Examination

Date: December 13 (Tuesday) 14:00 – 16:00

1. (10 Points) In the derivation of Theodorsen's formulation about the unsteady lift and aerodynamic pitching moment, explain **<u>where</u>** and **<u>why</u>** the lift deficiency factor, $C(k)$, expressed below, appears in the formulation.

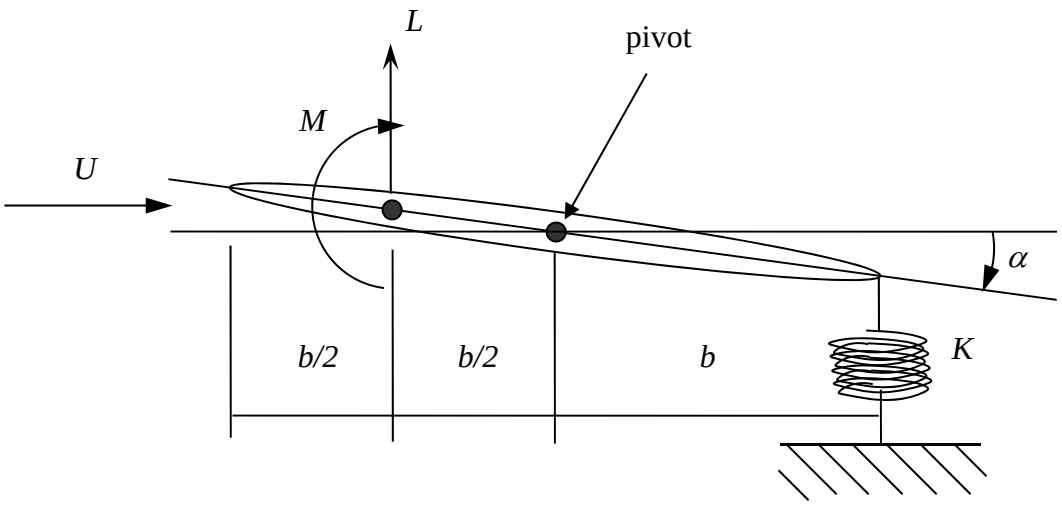
$$C(k) = \frac{\int_1^\infty \frac{\xi}{\sqrt{\xi^2 - 1}} e^{-ik\xi} d\xi}{\int_1^\infty \frac{\xi + 1}{\sqrt{\xi^2 - 1}} e^{-ik\xi} d\xi}$$

2. (30 Points) A two-dimensional airfoil model, which is mounted in a wind tunnel, is free to pitch about its mid-chord except for an elastic constraint provided by a linear spring, K, attached at the trailing edge. The mass moment of inertia is I, and the unsteady lift and moment at the quarter chord can be represented by:

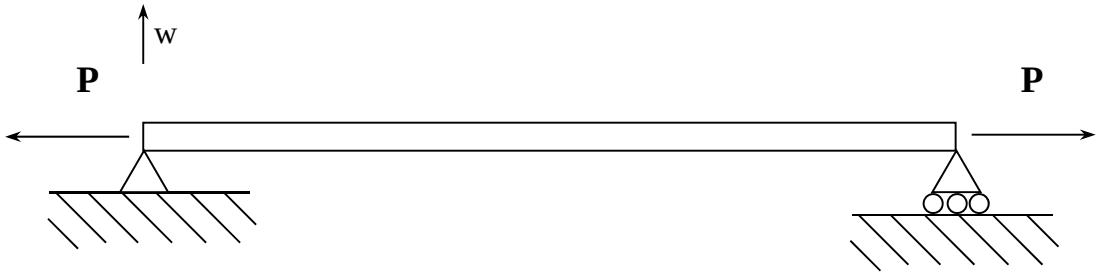
$$L(k) = \pi\rho U^2 bk\left[\frac{2}{k} - 2i\right]\alpha \qquad M(k) = \pi\rho U^2 b^2 k^2\left[\frac{1}{8} - i\frac{1}{2k}\right]\alpha$$

and: K = 4.3 kN/m, I = 0.0145 kg-m, b = 0.30 m, ρ = 3.23 kg/m^3

(a) Derive the governing equation of motion.
(b) Compute the numerical value of the airspeed at the pitch **<u>divergence</u>** boundary.
(c) Compute the numerical value of the airspeed at the **<u>flutter</u>** boundary.



3. (15 Points) Find an analytical solution for the natural mode shapes (continuous) and frequencies of the simply supported uniform beam with end loads P as shown below. Show (plot) how the first **three natural frequencies** change as a function of the applied preload **P** (the pre-tension/compression in the beam).



Length = L, Flexural stiffness = EI (const.),
Mass/length = m (const.), Area = A (const.)

4. (15 Points)
   (a) In the turbomachinery, it turns out that the mass ratio is much larger than it is in the conventional fixed wings. Describe briefly about its advantage upon the flutter analysis procedure of the turbomachinery.
   (b) Intentional mistuning on the turbine blade component may improve the flutter instability boundary of turbomachinery. Describe briefly about its procedure and draw a representative diagram.
   (c) Campbell diagram is used in the forced vibration and the high-cycle fatigue analysis in the turbomachinery. Describe briefly its procedure and the usage of the Campbell diagram.

5. (30 Points) The three major concerns regarding the rotorcraft aeroelasticity are listed as follows. Describe briefly about each concern and draw a representative diagram or figure describing each phenomenon.
   (a) Isolated blade instability
   (b) Stall flutter
   (c) Aeromechanical instability (Ground and air resonance)