1. [3 + 2 + 2 + 4 + 3 points] **A plate with the dimension of 100 x 100 x 5 mm³ formed from a polymeric material with the Young's modulus of 2.0 GPa is subjected to a biaxial load of 25 kN along the two directions on the largest plane. Answer the following questions.**

(a) Express the stress state in a [3 x 3] matrix form. Don't forget the unit.

   $\sigma_1 = \sigma_2 = (25 \times 10^3 \text{ N})/(100 \times 5 \times 10^{-6} \text{ m}^2) = 5 \times 10^7 \text{ N/m}^2 = 50 \text{ MPa}$

(b) To calculate the final dimension of the plate you need to assume that the polymer is isotropic. What other <u>assumption</u> do you need (about the behavior of the polymer)? Why?

   The polymer deforms in a (linearly) elastic manner when the strain is small (before it fails or yields). If not, the linear proportionality between stress and strain does not hold, and you cannot predict the strain from stress.
   Plane stress is a stress state, not a behavior. Small strain also cannot be an assumption.

(c) What additional <u>information</u> do you need to predict the final dimension of the plate? What would be the reasonable value for that?

   Poisson's ratio, 0.4 for polymers

(d) Estimate the final thickness of the plate using the value you gave in (c).

   Using the generalized Hooke's law, $e_3 = (1/E) [0 - (.4)(50+50)] = -.02$
   t = 4.9 mm

(e) Does the result of (d) justify the assumption you made in (b)? Explain.

   Yes, 2% strain is smaller than typical yield strain of polymers.

2. [3 x 3 points] **There are two mechanical models for linear viscoelasticity; Maxwell and Voigt models. They are composed of one spring element with the modulus E and one dashpot element with the viscosity η.**

(a) Show the constitutive equation for a creep test of a Maxwell model with the given stress of σ_0.

   $de/dt = \sigma_0/\eta$

(b) Show the constitutive equation for a stress relaxation test of a Voigt model with the given strain of e_0.

   $\sigma = E\, e_0$

(c) One of the problems of the models results from the fact that they have single <u>relaxation time</u> or <u>retardation time</u>, which is unrealistic. Fill the both blanks.

3. [4 x 3 points] **For the construction of 'master curve' on modulus vs time plot using the time-temperature superposition principle, answer the following questions.**

(a) Vertical shift (in modulus) can be ignored. Why?

   $E = \rho R T / M_e \rightarrow E \propto \rho T$
   % change in density and in temperature cancels each other out

(b) For horizontal shift (in time), does the results at a temperature higher than the reference temperature shift to a shorter or longer time frame? Why?

   to longer time, since a high temperature is equivalent to a long time

(c) When the reference temperature is set to the <u>glass transition</u> temperature, the amount of horizontal shift is given by the <u>WLF</u> equation. Fill the both blanks. For the second one, no need to spell out.

(d) What is the two constants in the equation of (c) called? Why are they called so?

   universal constants, since they can be used for t-T superposition of most polymers

4. [5 + 3 x 3 points] **For a polymer that follows pressure-dependent Tresca yield criterion and critical-strain craze criterion, answer the following questions.**

(a) It is observed that this polymer undergoes shear yielding at a stress of 60 MPa in uniaxial tension and at 70 MPa in uniaxial compression at the strain rate of 10 mm/min. It is also observed that the polymer undergoes crazing at a stress of 50 MPa in uniaxial tension and at 40 MPa in biaxial tension. Show the yield and craze locus on σ_1-σ_2 coordinate.

shown as the blue lines

(b) Is this polymer ductile or brittle upon biaxial compression? Explain your answer using your drawing of (a).

ductile: The (purple) biaxial compression line does not cross craze locus

(c) Show how the drawing changes when the strain rate increases to 100 mm/min.

Both of the loci move to higher stresses as shown as the green lines.

(d) Show by drawing that a ductile-brittle transition can occur with the increase in testing temperature.

Since yield stress increases more than craze stress does, yield stress can be lowered below craze stress as temperature increases, as shown as the red lines.

(a) blue
(c) at a higher strain rate [green]
(b) (balanced) biaxial compression

(d) red line at a higher temperature
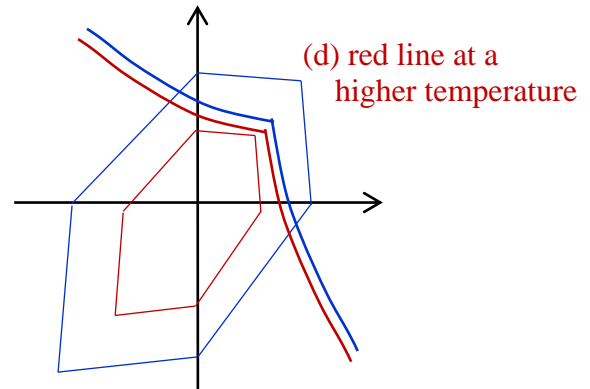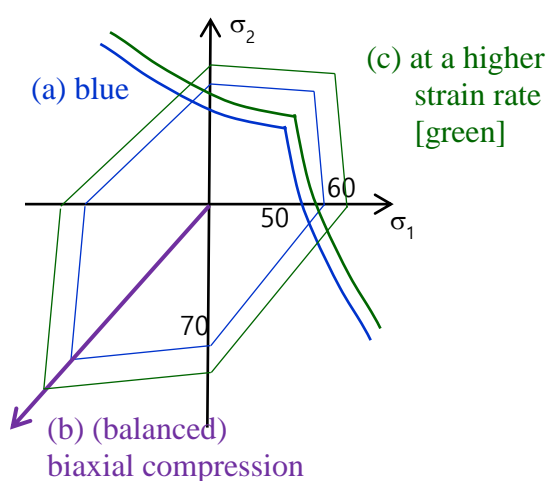
5. [4 x 3 points] **Referring to the figure on the right, answer the following questions.**

(a) As the average molar mass of an amorphous polymer increases, would the width (in temperature) of rubbery plateau increase, decrease, or remain the same? Why?

increases. As the avg MM increases, the number of entanglements that keep chains from translating or flowing increases.

(b) As the average molar mass of an amorphous polymer increases, would the level (in modulus) of rubbery plateau increase, decrease, or remain the same? Why?

remains the same. The plateau modulus is dependent upon the molar mass between the entanglement not on the total molar mass, as the motion is limited between entanglements.

(c) As the crystallinity of a semi-crystalline polymer increases, would the level (in modulus) of rubbery plateau increase, decrease, or remain the same? Why?

increases. Since the modulus of crystalline phase or crystallite is (much) larger than that of
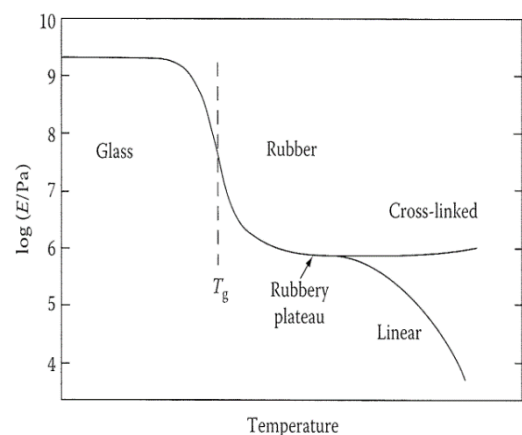
rubbery phase, modulus of semicrystalline polymer increases with crystallinity.
(d) As the average molar mass between the crosslinks of a rubber increases, would the level (in modulus) of rubbery plateau increase, decrease, or remain the same? Why?
decreases. As the avg MM between the crosslinks increases, segmental motion is less and less interrupted by the crosslinks.

6. [9 x 2 points] **Fill the blanks in the description of the fracture stress [σ_f] of polymers.**
(a) The theoretical fracture stress of a material is one-tenth of its (a1) modulus. However, the measured σ_f is much smaller than this due to the existence of flaw or crack, which results in (a2) stress concentration and (a3) plastic constraint.
(b) In front of a crack, exceedingly high stress resulted from (a2) is cut off by (b1) yield stress of the polymer, and the material undergo (b2) plastic deformation or (shear) yielding. This is the reason why σ_f measured is much larger than that estimated based on the (b3) surface energy that are created by crack propagation.
(c) Since both of the low σ_f of (a) and the high σ_f of (b) are due to the existence of crack, it is called (c) Griffith('s) paradox, following the name of the scientist who suggested the both.
(d) (a3) in front of a crack tip is due to the (d1) triaxial stress state, which make the (b2) occur at a higher (b1). (a3) can explain the ductile-brittle transitions with the change in (d2)[temperature, strain-rate, surface crack, specimen thickness] (Choose two).

7. [6 x 3 points] **Answer the following questions in one sentence. No drawing or equation.**
(a) Why is a rubber called 'entropy spring'?
The retracting [spring] force is the recovering of the decreased entropy by stretching.
(b) Why is a polymer melt 'pseudo-plastic', while some colloids or suspensions 'dilatant'?
The decreasing viscosity of polymer melt comes with aligning polymer chains with increasing shear rate, while increasing viscosity of colloids with interactions of particles.
(c) Why is a PRP not as efficient in reinforcing [enhancing modulus] as an FRP with the same matrix and reinforcement materials?
Due to the low aspect ratio and low level of stress transfer, the particles carry the low stress similar to the matrix.
(d) Why is the dielectric constant of a polymer related to its solubility parameter?
Both of dielectric constant and solubility parameter are affected by the polarizability and dipole moment of the material.
(e) How is the thermal stability different from the heat resistance?
Thermal stability is the resistance to thermal degradation, while heat resistance is the resistance to softening.
(f) Why is the flame resistance of a polymer related to its thermal stability?
The first step of burning is pyrolysis, which is the decomposition or degradation by heat.