Viscous Flow, Final Exam
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- Write CLEARLY and describe ALL details of your work.
- Answers can be written in English or Korean.
- Without a proof, you may use the information provided together.

1. (20 points) Explain the followings as much as you can.

   (a) (10 points) Boussinesq approximation

   (b) (10 points) Rayleigh criterion for instability

2. (50 points) For a 2D flat-plate boundary-layer flow, by integrating the Prandtl's boundary-layer equation along the wall-normal direction (y), one can derive the following equation, known as a general momentum integral (GMI). Here, θ is momentum thickness, U_O is free-stream velocity, H is shape factor and c_f is wall-friction coefficient. Plate length is L.

$$\frac{d\theta}{dx}+(2+H)\frac{\theta}{U_O}\frac{dU_O}{dx}=\frac{c_f}{2}$$

   (a) (10 points) Starting with the Navier-Stokes equation, derive the Prandtl's boundary-layer equation. You need to provide assumptions that you used.

   (b) (10 points) Derive the GMI relation, given above.

   (c) (30 points) for the following approximate flat-plate boundary-layer profile, determine the displacement thickness, δ^*, momentum thickness, θ and shape factor H. Use the zero-pressure gradient boundary-integral equation to find $(\delta/x)Re_x^{1/2}$, $(\delta^*/x)Re_x^{1/2}$, $(\theta/x)Re_x^{1/2}$, $c_f Re_x^{1/2}$, and $C_D Re_L^{1/2}$. (C_D is the drag coefficient)

$$\frac{u}{u_\infty}=\begin{cases}\sin\left(\dfrac{\pi y}{2\delta}\right) & \text{for } 0\le y\le\delta \\ 1 & \text{for } y>\delta\end{cases}$$

3. (30 points) Let's consider the 2D laminar wake behind a body (length of L). Far downstream, the velocity defect (u_1) becomes small and has a similarity solution (see the figure below) such that

$$\frac{u_1}{U_O}=1-\frac{u(x,y)}{U_O}\ll 1$$

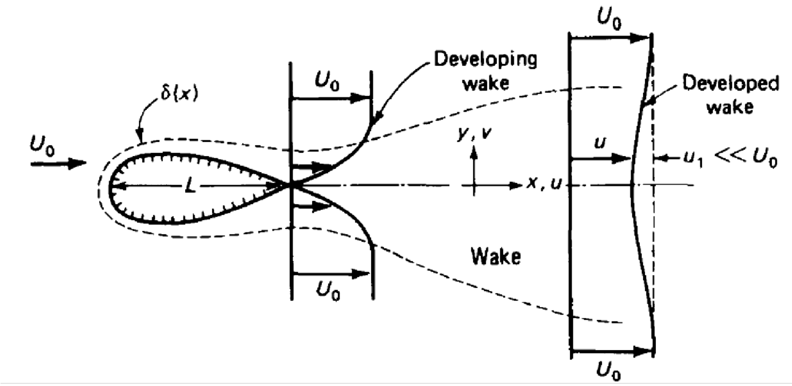
   (a) (5 points) Obtain the governing equation and associated boundary conditions in terms of u_1.

   (b) (5 points) Assuming the similarity solution (u_1) as $\frac{u_1}{U_O}=C\left(\frac{x}{L}\right)^{-\frac{1}{2}}g(\eta)$, obtain the governing equation and boundary conditions for $g(\eta)$. If necessary, find the similarity variable η.

(c) (5 points) Solve the equation in (b) to obtain $g(\eta)$ (Hint: Try to integrate the governing equation). How does the velocity defect scale with y?

(d) (5 points) How can you determine the constant C? (You don't need to calculate up to the final result.)

(e) (10 points) If the body in consideration is a flat plate (wetted on both upper and lower sides) which has a drag coefficient of $C_D = 2.7/\sqrt{\mathrm{Re}_L}$, where $\mathrm{Re}_L = U_O L/\nu$, what is the relationship between C_D and u_1?



4. (20 points) A simple realization of a temporal boundary layer involves the spinning fluid in a cylindrical container. Consider a viscous incompressible fluid (density = ρ, viscosity = μ) in a solid body rotation (rotational speed at Ω) in a cylindrical container of diameter d. Mean depth of the fluid is h. At time t = 0, the rotation stops. Define the time taken for the fluid to stop rotating as τ.

(a) (10 points) For h >> d, write a simple laminar-flow scaling law for τ, assuming that the velocity perturbation due to the no-slip condition on the container's sidewall must travel inward a distance d/2 by diffusion.

(b) (10 points) For h << d, write a simple laminar-flow scaling law for τ, assuming that the velocity perturbation due to the no-slip condition on the container's bottom wall must travel upward a distance h via diffusion.

5. (30 points) For a typical turbulent boundary layer, from the wall, three specific regimes are classified such as a wall layer, overlap layer and outer layer. For each regime, obtain the streamwise velocity distribution as a function of the distance (y) from the wall. You need to start from a proper set of variables and the final relation should be given as a non-dimensional form.

These might be useful….

Equations for incompressible flows in Cartesian coordinates (no body force)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0,\quad \begin{aligned} \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)&=-\frac{\partial p}{\partial x}+\mu\nabla^2 u\\ \rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)&=-\frac{\partial p}{\partial y}+\mu\nabla^2 v\\ \rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)&=-\frac{\partial p}{\partial z}+\mu\nabla^2 w \end{aligned}$$