Fluid Mechanics
Fall 2022
Hyungmin Park

Final Exam (Chapters 5 & 6)
(December 15, 2022, no calculator allowed)

* Unless specified, you have to write down detailed procedure used to reach your answers.

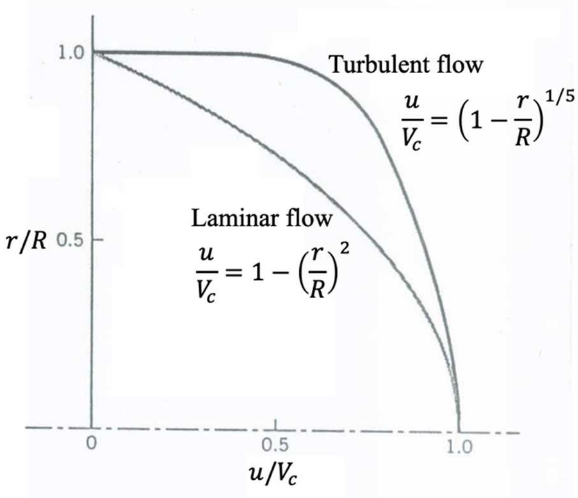
1. (30 points) Explain the followings.
   (a) (15 points) Moody chart
   (b) (15 points) Reynolds-averaged Navier-Stokes equation and turbulent stress

2. (20 points) Some fluids behave as a non-Newtonian power-law fluid characterized by the shear stress of $\tau(r) = C(du/dr)^n$, where n = 1, 3, 5 and so on, and C is a constant (if n = 1, the fluid is customary Newtonian fluid). For the circular pipe flow (D, diameter; l, length; Δp, pressure difference), obtain the velocity profile, $u(r)$ in terms of n.

3. (20 points) Many flying and swimming animals as well as human-engineered vehicles rely on some type of undulating (flapping) motion of wings or fins for propulsion. For this problem, assume the average travel speed (U) depends on the flapping frequency (f), the characteristic length of the body (L), the gravitational acceleration (g), the density of the body (ρ_o), the fluid density (ρ), and the fluid viscosity (μ).
   (a) (10 points) Perform the dimensional analysis on the speed (U) using pi theorem, and discuss the meaning of the results.
   (b) (5 points) Simplify your answer for part (a) for much higher Reynolds numbers where μ is no longer a parameter, and discuss the result.
   (c) (5 points) For the moving body that is neutrally buoyant ($\rho_o = \rho$), simplify your answer for part (b), and discuss the result.

4. (20 points) In the overlap layer of the turbulent boundary layer, the viscous shear stress is comparable to the turbulent stress, by which one may approximate the velocity gradient as $du/dy = f(y, \tau_w, \rho)$. Here, y is the distance from the wall, τ_w is the wall shear stress and ρ is the fluid density. Using the dimensional analysis, show that this functional form leads to the logarithmic velocity profiles in the overlap layer.

5. (25 points) As shown in the figure below, the velocity profile for a laminar flow in a circular pipe is quite different from that of a turbulent flow. With a laminar flow, the velocity profile is parabolic; with a turbulent flow it can be approximated by the power-law profile, as shown in the figure. For laminar and turbulent flow, determine at what radial location, you would place a Pitot tube to measure the bulk velocity in the pipe. In the velocity profiles, V_c is the centerline velocity.



6. (35 points) Consider the fully-developed, two-dimensional, incompressible laminar channel flow between two parallel plates, separated by $2h$, whose streamwise length is L. The channel is aligned to be horizontal and you may neglect the effect of gravity and assume the steady flow.

(a) (10 points) Obtain the velocity profile inside the channel.
(b) (5 points) Obtain the Darcy friction factor (f) in terms of the Reynolds number (based on h).
(c) (5 points) What is the hydraulic diameter (D_h) for this geometry? Obtain the friction factor in terms of the Reynolds number based on D_h.
(d) (15 points) Find the expression for friction factor in the case a turbulent flow. Assume that the log law in turbulent boundary layer can be used to approximate the whole velocity profile inside the channel.