Mechanics of Materials and Lab.

Final exam (Closed book), June 14th 10am – 12pm

Total 100 pts (5 Questions)

Name: Solution

Student id number:

Equations:

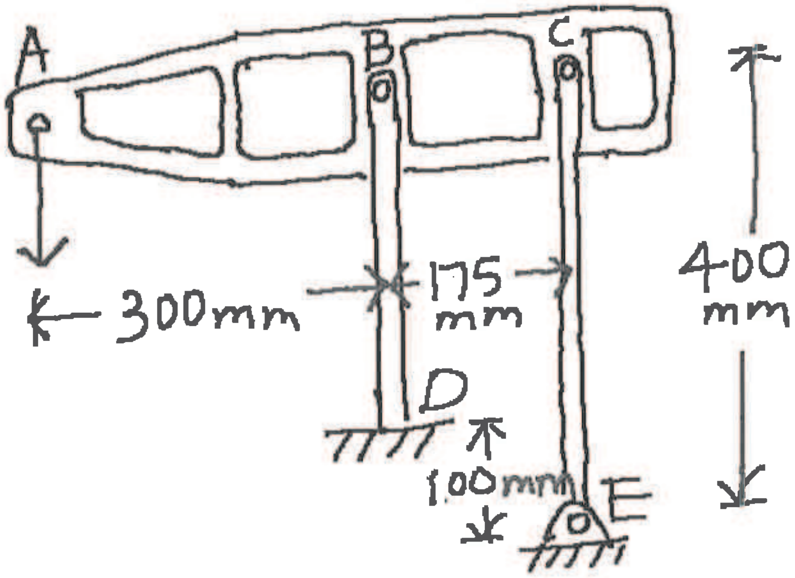
Critical buckling load for a pin-ended column: $P_{cr} = \dfrac{n^2 \pi^2 EI}{L^2}$

Shear stress: $\tau = \dfrac{VQ}{Ib}$; $Q(y_1) = \int y dA = \int_{y_1}^{h/2} yb\, dy = \dfrac{b}{2}\left(\dfrac{h^2}{4} - y_1^2\right)$ (rectangular section case)

Bending stress: $\sigma_x = -\dfrac{My}{I}$; $I = \int_A y^2 dA = \int_{-b/2}^{b/2} \int_{h/2}^{h/2} y^2 dy dx = \dfrac{bh^3}{12}$ (rectangular section case)

Problem # 1 (15 pts)

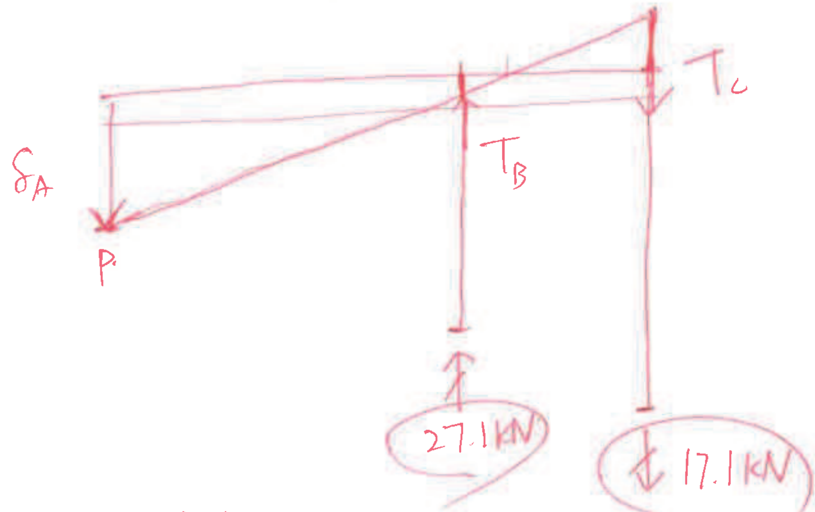
A device consists of a horizontal rigid beam ABC supported by two vertical bars BD and CE. Bar CE is pinned at both ends but bar BD is fixed to the foundation at its lower end. The distance from A to B is 300 mm and from B to C is 175 mm. Bars BD and CE have lengths of 300 mm and 400 mm, respectively, and their cross-sectional area is 300 mm². The bars are made of steel having a modulus of elasticity E = 100 GPa. If load P is 10 kN, calculate the displacement at point A. Use $\delta = \frac{PL}{EA}$.



$A = 300\ mm^2$
$E = 100\ GPa$

$175 T_c = 300 \cdot 10\,kN$

$T_c = 17.1\ kN$



$$\delta_c = \frac{17.1\,kN \cdot 400\,mm}{100 \cdot 10^3 \frac{N}{mm^2} \cdot 300\,mm^2} = 0.228\ mm$$

$$\delta_B = \frac{27.1 \cdot 10^3 \cdot 300}{100 \cdot 10^3 \cdot 300} = 0.271\ mm$$

$$\delta_A = \delta_B + (\delta_B + \delta_c) \cdot \frac{300}{175} = 0.271 + 0.391 = \cancel{0.662\ mm}$$

1.126mm (↓)

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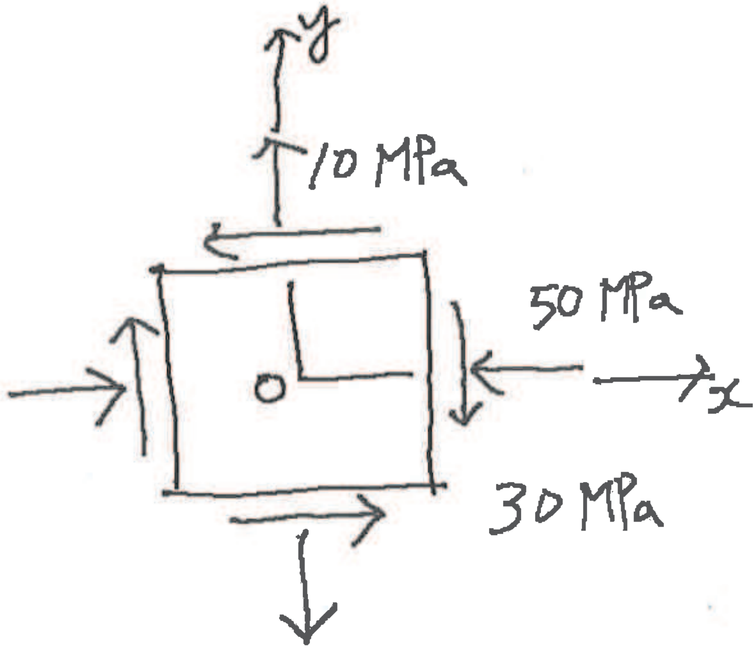
Problem # 2 (15 pts)

At a point on the surface of a generator shaft the stresses are $\sigma_x = -50MPa$, $\sigma_y = 10MPa$, and $\tau_{xy} = -30MPa$ as shown in below figure.

Using Mohr's circle, determine the following quantities: (a) the stresses acting on an element inclined at an angle $\theta = 40°$, (b) the principal stresses, and (c) the maximum shear stresses. Show all results on sketches of properly oriented elements.



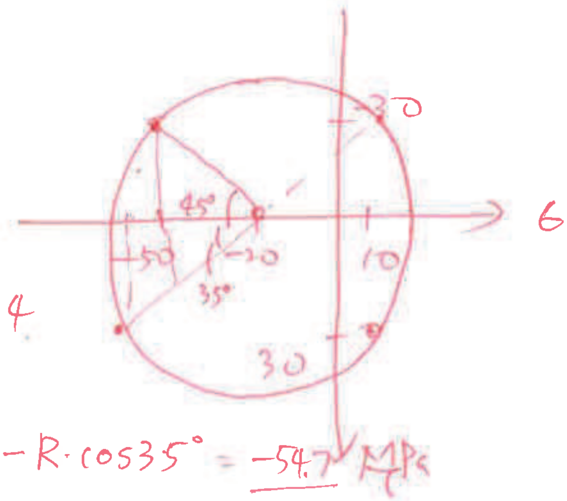
$\sigma_x = -50$

$\sigma_y = 10$

$\tau_{xy} = -30.$

$\sigma_{avg} = -20$

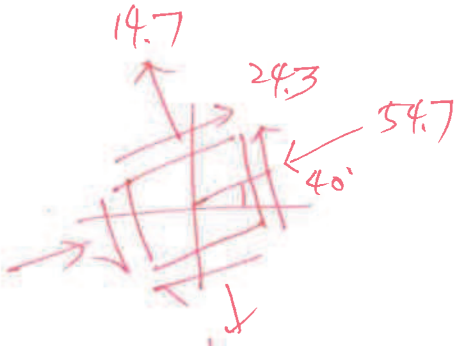
$R = 30\sqrt{2} = 42.4$



(a) $\sigma_{x_1} = -20 - R\cdot\cos 35° = \underline{-54.7}$ MPa

$\sigma_{y_1} = -20 + R\cdot\cos 35° = \underline{14.7\ MPa}$

$\tau_{x_1y_1} = R\sin 35 = \underline{24.3\ MPa}$

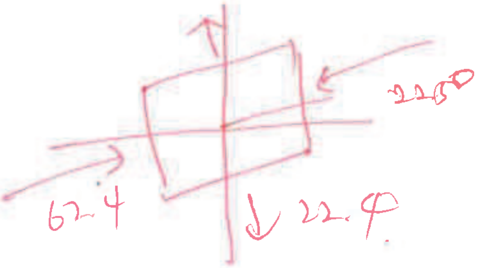


(b) $\sigma_1 = -20 \pm 42.4 = \underline{-62.4\ MPa}$

or $\sigma_2 = -20 + 42.4 = \underline{22.4\ MPa}$

$\underline{22.5°}$ //

22.4
-62.4
-67.5°



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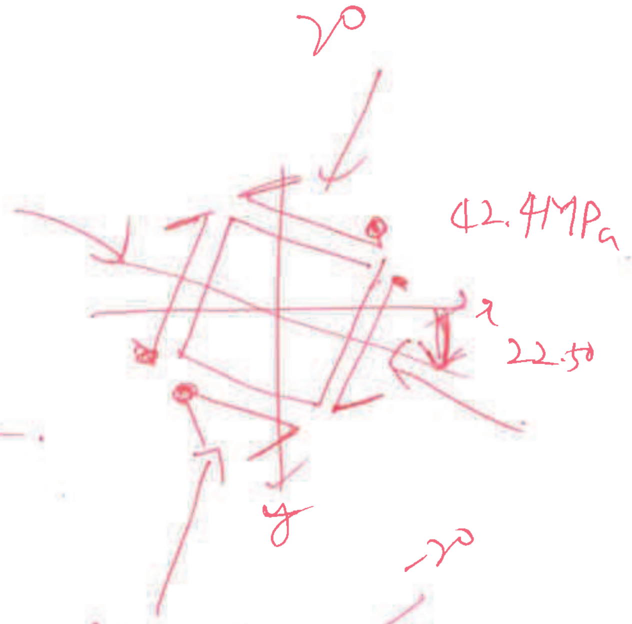
(c) $-22.5°$

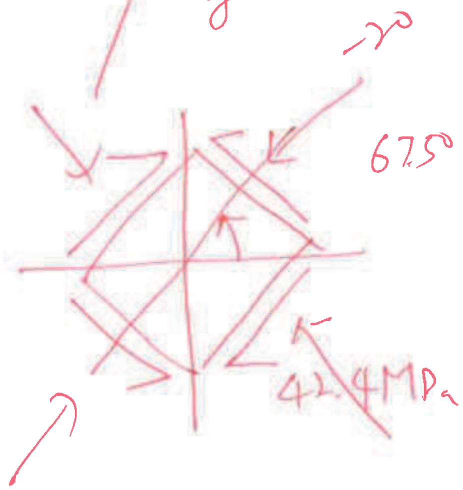
$\tau_{max} = -42.4$ MPa



or

$67.5°$

$\tau_{max} = +42.4$ MPa

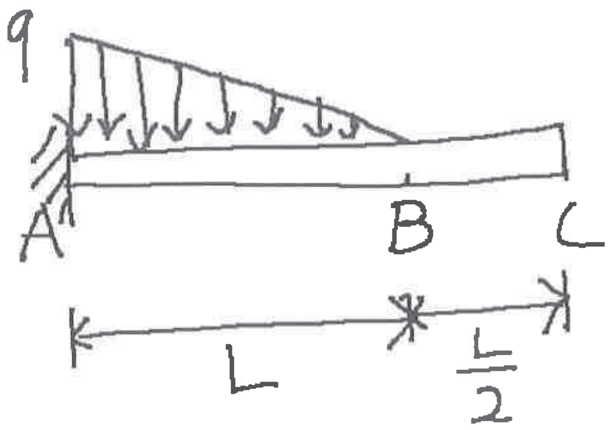


$\sigma_x = -20$

$\sigma_y = -20$

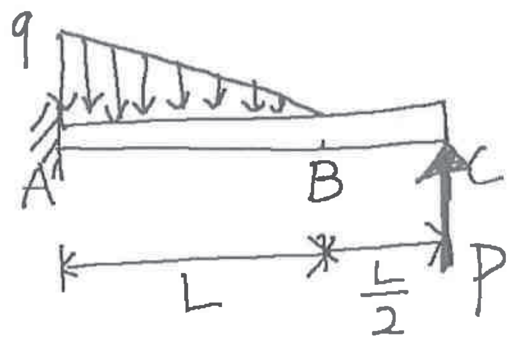
Problem # 3 (35 pts)

(1) Determine the angle of rotation θ_B and the deflection δ_B at point of B in a cantilever beam ABC subjected to a linearly varying load as shown in below figure. Note that the beam has constant flexural rigidity of EI. First, determine the equation of the deflection curve for AB part.



(2) In the same loading condition, determine the angle of rotation θ_C and the deflection δ_C at point of C.

(3) Determine the vertical force of P as shown below figure to make the δ_C = 0.



(4) With the calculated vertical force of P from above question of (3), determine the new angle of rotation θ_C at the point of C.

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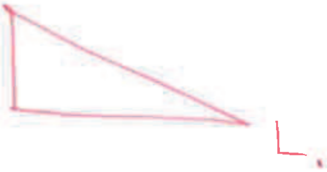
(1).



$$p = q - \frac{x}{L} q = -EI \frac{d^4 v}{dx^4}$$

$$V = \frac{x^2}{2L} q - qx + C_1$$

$$V = \frac{x^2}{2L} q - qx + \frac{1}{2} qL \qquad V(L)=0$$

$$M = \frac{x^3}{6L} q - \frac{q}{2} x^2 + \frac{1}{2} qLx + C_2$$

$$M = \frac{x^3}{6L} q - \frac{q}{2} x^2 + \frac{L}{2} qx - \frac{1}{6} qL^2 \qquad M(L)=0$$

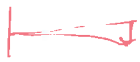
$$\theta = \frac{1}{EI}\left(\frac{q}{24L} x^4 - \frac{q}{6} x^3 + \frac{qL}{4} x^2 - \frac{qL^2}{6} x \right) \qquad \theta(0)=0$$

$$= \frac{qx}{24LEI}\left(x^3 - 4Lx^2 + 6L^2 x - 4L^3 \right) \qquad v(0)=0$$

$$v = \frac{qx^2}{120LEI}\left(x^3 - 5Lx^2 + 10L^2 x - 10L^3 \right)$$

$$\theta_B = -\theta_{x=L} = -\frac{qL}{24LEI}\left(L^3 - 4L^3 + 6L^3 - 4L^3 \right) = \boxed{\frac{qL^3}{24EI}}$$

$$\delta_B = -v_{x=L} = -\frac{qL^2}{120LEI}\left(L^3 - 5L^3 + 10L^3 - 10L^3 \right) = \boxed{\frac{qL^4}{30EI}}$$



(2)

$$\theta_C = \theta_B = \boxed{\frac{qL^3}{24EI}}$$

$$\delta_C = \delta_B + \left(\frac{1}{2}L\right) \cdot \theta_B$$

$$= \frac{qL^4}{30EI} + \frac{L}{2} \cdot \frac{qL^3}{24EI}$$

$$= \boxed{\frac{13qL^4}{240EI}}$$

$$\frac{27pL^3}{24EI}$$

(3)

$$\frac{p\left(\frac{3}{2}\right)^3 L^3}{3EI} = \boxed{\frac{13qL^4}{240EI}}$$

$$p = \frac{13qL^4}{240EI} \cdot \frac{8 \cdot 3EI}{27L^3}$$

$$= \boxed{\frac{13qL}{270}}$$

$$\left(\frac{10qL^3}{240EI} - \frac{13qL^3}{240EI} \right)$$

(4) $\theta_B - \frac{P}{2EI}\left(\frac{3}{2}L\right)^2 = \frac{qL^3}{80EI}$ (↺)

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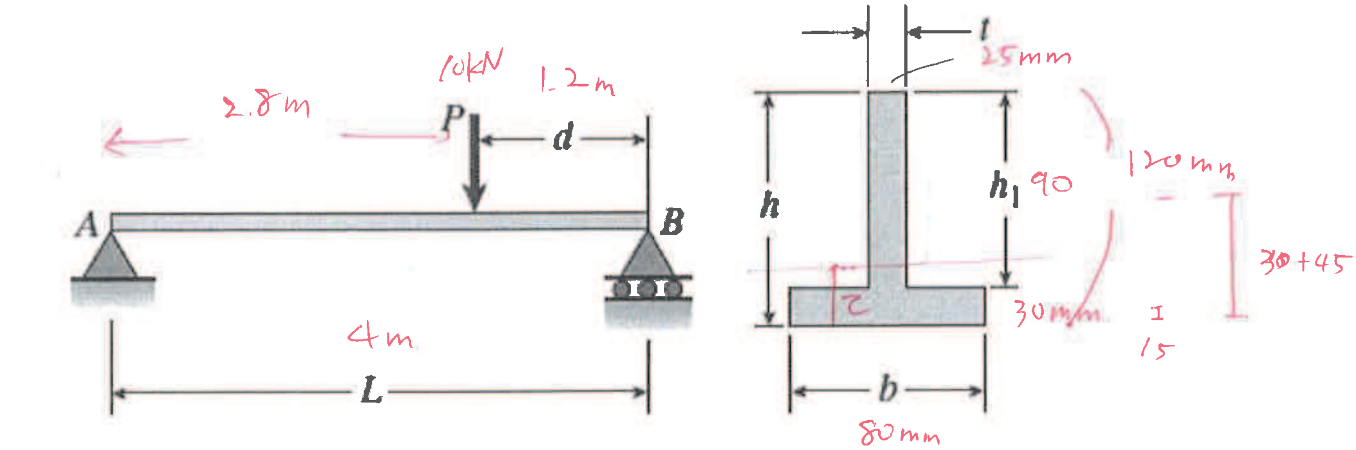
Problem # 4 (20 pts)

Determine the maximum tensile stress σ_t and maximum compressive stress σ_c due to the load P acting on the simple beam AB. In addition, determine the maximum shear stress τ_{max} and minimum shear stress τ_{min} in the web of the beam.

Use data as follows: P = 10 kN. L = 4 m, d = 1.2 m, b = 80 mm, t = 25 mm, h = 120 mm, and h_1=90 mm.



$$\bar{z} = \frac{30\cdot 80\cdot 15 + 90\cdot 25\cdot(30+45)}{30\cdot 80 + 90\cdot 25}$$

2400
2250

$$= \frac{36000 + 168750}{4650}$$

$= 44\ mm$

$$I = \frac{80\cdot 30^3}{12} + 80\cdot 30\cdot(44-15)^2 + \frac{25\cdot 90^3}{12} + 25\cdot 90\cdot(120-44-45)^2$$

$= 5,879,400\ mm^4$

$$\max \sigma_t = \frac{8.4\ kN\cdot m\cdot 44\ mm}{I} = 62.8\ MPa\ (\text{bottom})$$ (5pt)

$$\max \sigma_c = \frac{8.4\ kN\cdot m\cdot(120-44)}{I} = 108.6\ MPa\ (\text{Top})$$ (5pt)

$$\max \tau_{web} = \frac{7\ kN\cdot Q}{I\cdot 25} = \frac{7\ kN}{I\cdot 25\ mm}\cdot\{25(90-14)^2/2\} = 3.4\ MPa$$ (5pt)

$\min \tau_{web} = 0$ (5pt)

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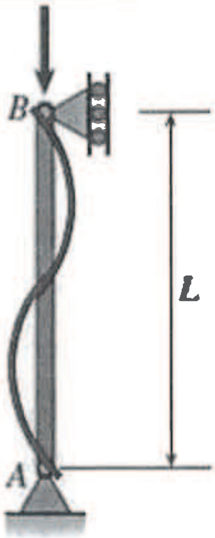
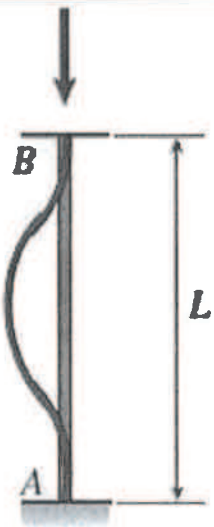
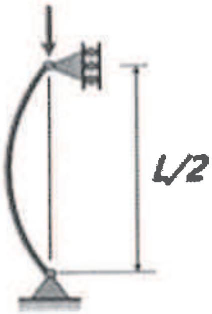
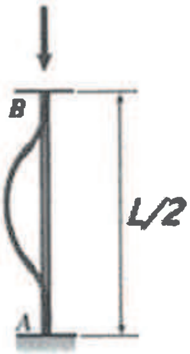
Problem # 5 (15 pts)

Determine the critical buckling load for below each cases. All columns have identical section **(Same I)** and made of a same material **(Same E)**.

Case (a)	 $\frac{4\pi^2 EI}{L^2}$ (3 pt)
Case (b)	 $\frac{4\pi^2 EI}{L^2}$ (3 pt)

Case (c)		$\frac{4\pi^2 EI}{L^2}$ (3pt)
Case (d)		$\frac{16\pi^2 EI}{L^2}$ (3pt)
Case (e)		$\frac{16\pi^2 EI}{L^2}$ (3pt)