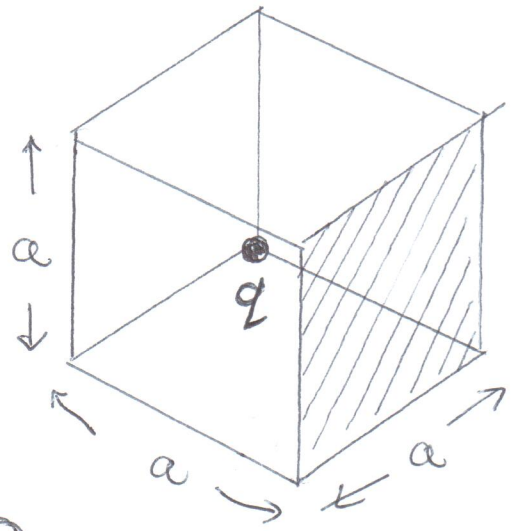
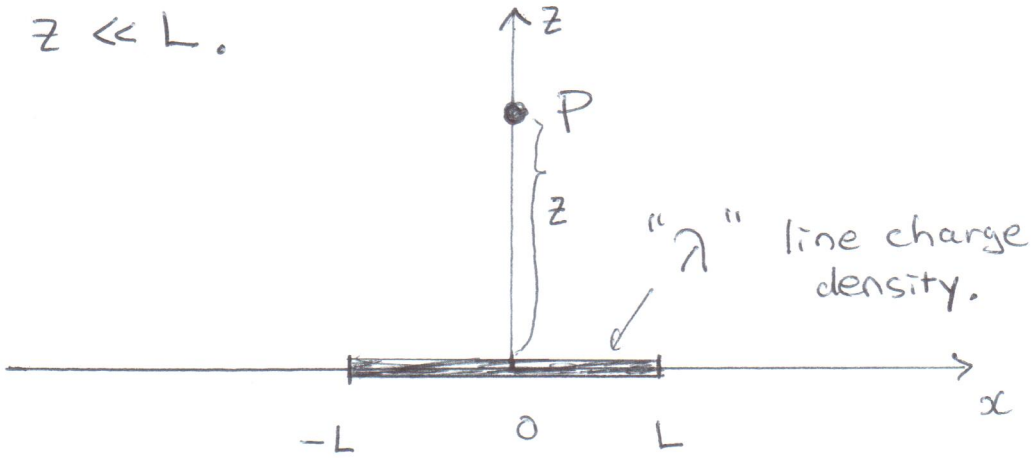
# Fundamentals of Engineering Physics 2019

## 1st Mid term Exam

(I). 10pts A charge "q" sits at the lower back left corner of a cube as shown in the figure. What is the flux of $\vec{E}$ through the shaded side? Explain your reasoning.




(II). 20pts a) Find an electro-static potential $\Phi$ at a distance "z" above the mid-point of a straight line segment of length "2L" which carries a uniform line charge density "$\lambda$". b) Then, calculate the electric field $\vec{E}$ at the same point. c) Discuss limiting cases of $z \gg L$, and $z \ll L$.



III.

40pts.

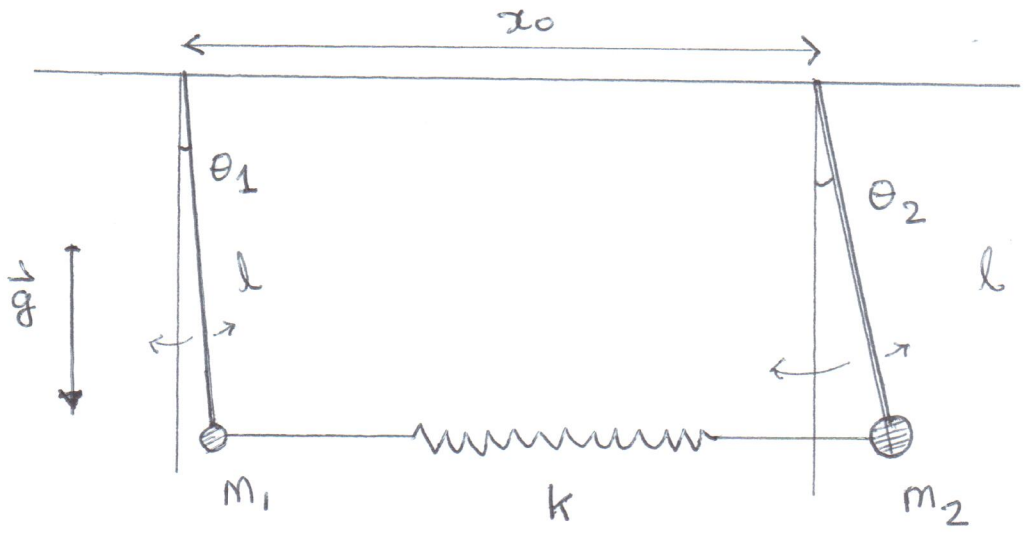



Fig. 1.

Consider a system consisting of two pendulums of the same length "$l$" and mass "$m_1$" and "$m_2$" in a downward gravitational field $\vec{g}$. The stiff pendulums which are allowed to move on the same plane (NOT into or out of the page) are connected by weightless spring with a spring constant "$k$" and equilibrium length "$x_0$" which is the same as the distance between the points of suspension. The angles of inclination of the pendulums are "$\theta_1$" and "$\theta_2$" respectively.

Unless it's instructed otherwise, fill in empty boxes with appropriate word(s) or formula.

The kinetic energy of $m_1$ is given by [a) formula] 1pt and that of $m_2$ is given by [b) formula] 1pt.

The potential energy due to the gravitational field is [c) formula] 1pt for $m_1$ [d) formula] 1pt for $m_2$.

In addition, there is a potential energy due to interaction of $m_1$ and $m_2$ via the spring.

This part of potential energy is given by "$\frac{1}{2}\alpha(\theta_2-\theta_1)^2$" when $\theta_1 \ll 1$ and $\theta_2 \ll 1$.

e) 3pts. Express "$\alpha$" in terms of relevant parameters given in the problem. Then, for $\theta_1 \ll 1$, $\theta_2 \ll 1$, the Lagrangian of the system is given by

[f) formula] 3pts. (Hint: $\sin\theta \approx \theta$, $\cos\theta \approx 1-\theta^2/2$ for $\theta \ll 1$)

The form of Lagrangian given in f) suggests a coordinate transform which manifests [g) word] 1pt of the system.

h),i) Find a set of transformations

$q_1 =$ [h) formula] 1pt "a linear combination of $\theta_1$ and $\theta_2$"

$q_2 =$ [i) formula] 1pt "another "

This leads to a Lagrangian:

$$\mathcal{L} = \frac{1}{2} l^2 (\mu \dot{q}_1^2 + M \dot{q}_2^2) - \frac{1}{2} \alpha q_1^2 - \frac{1}{2} \mu g l q_1^2 - \frac{1}{2} M g l q_2^2 ,$$

where $M = m_1 + m_2$ and $\mu =$ [j) formula] 1 pt.

This Lagrangian exhibits that the original problem involving two pendulums is now reduced to a set of two [k) word] 1pt problems involving a single pendulum respectively as described by the figures below.

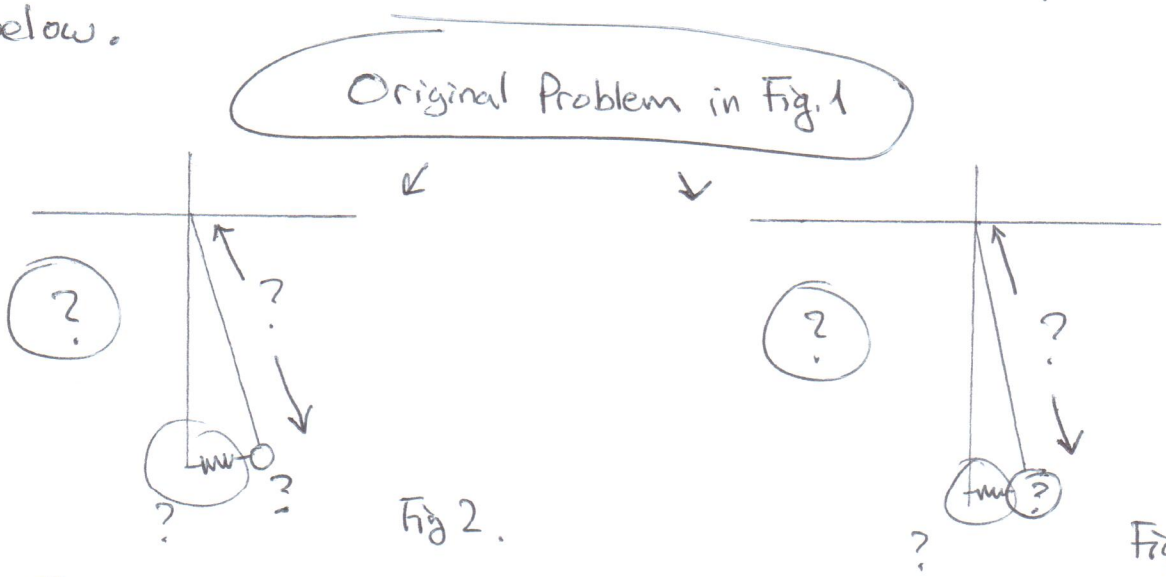



Fig 2. Fig 3.

l) Fill in relevant parameters (angle, length, mass, spring constant, ...) and more objects if necessary in Figs 2 and 3. 4pts.

Draw the figures clearly!

m) When $\vec{g} = 0$, find a cyclic coordinate, and the expression for the generalized momentum conjugate to it. 4pts

Discuss the physical meaning of that quantity.

o) Now consider the gravity again ($\vec{g} \neq 0$).

5pts Find the expression of Hamiltonian, "$H(P_1, P_2, q_1, q_2)$" starting from the Lagrangian given in ⊛.

Here, $P_1$ and $P_2$ are the generalized momenta conjugate to $q_1$ and $q_2$ respectively.

p) 4pts Write down the relevant Hamilton's equations for the Hamiltonian in o).

q, r) When the total Energy of the system is "E", find the two characteristic frequencies of oscillations of the [ q) word ] 1pt. described by those Hamilton's equations.

r) 5pts. Give a derivation of the frequencies.

s) 2pts Explain why these two frequencies are different.

Note that you can answer some problems based on the Lagrangian in ⊛ even if you could not answer the problems which appeared earlier.