

Theory of Fusion Plasmas 2 (2019)

Final Exam

Please read and follow the instruction:

- Answer the questions **in English** if possible.
- **Write Clearly.** Extremely bad or thin or vague writing, or in small letters may lead to deductions in points.
- Use **black** or **blue** color only.
- Answer each problem **continuously**. Don't answer one problem partially and change to another problem and return.
- Clean up your **eraser** (지우개) **residues** completely before submitting!

Final Exam .

(I.)

A uniform plasma has ions that are initially at rest, but its electrons are streaming through

the ions with velocity \vec{u} . Neglecting both

- a) ion and electron thermal effects, show that the dispersion relation for electrostatic oscillations involving both ions and electrons is

$$D(k, \omega) \equiv 1 - \frac{m_e}{M_i} \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - ku)^2} = 0.$$

- b) Show that, for any streaming speed u , the plasma is always unstable to modes with sufficiently long wavelengths.

- c) Show that the typical growth rate " γ " has magnitude $\gamma \sim \left(\frac{m_e}{M_i}\right)^{1/3} \omega_{pe}$.

(II.)

Drift-kinetic equation in uniform magnetic field

60pts In the absence of equilibrium electric total field is given by.

$$\frac{\partial f_\alpha}{\partial t} + \frac{e\vec{E} \times \vec{B}}{B^2} \cdot \vec{\nabla} f_\alpha + v_z \frac{\partial f_\alpha}{\partial z} + \frac{q_\alpha e}{m_\alpha} E_z \frac{\partial f_\alpha}{\partial z} = 0 \quad (1)$$

for each species " α ".

For equilibrium distribution for each species, take bi-Maxwellian " $F_{0\alpha}$ " with with a "flat" density. i.e., $\frac{\partial n_{0\alpha}}{\partial x} = 0$.

$$F_{0\alpha} = n_{0\alpha} \left(\frac{m_\alpha}{2\pi T_{0\alpha}^{1/3} T_{0\alpha}^{1/3}} \right)^{3/2} e^{-\frac{v_{\perp\alpha}^2}{2T_{0\alpha}}} e^{-\frac{v_{\parallel\alpha}^2}{2T_{0\alpha}}}$$

with

$$v_{T_{0\alpha}}^2 = \frac{T_{0\alpha}}{M_\alpha}, \quad v_{T_{0\alpha}}^2 = \frac{T_{0\alpha}}{M_\alpha}.$$

Don't forget $T_{0\alpha}$ and $T_{0\alpha}$ are functions of "x".

Perturbed electric field is given by

$$\delta \vec{E} = -\vec{\nabla} \delta \phi(\vec{x}, t),$$

$$\delta \phi(\vec{x}, t) \propto e^{-i\omega t + i\vec{k} \cdot \vec{x}}$$

3.

A. Linearize the equation (1)
10pts

B. Identify the terms in the answer for A
5pts which are involved in wave-particle resonance.

Write the resonant condition in terms of relevant speeds.

C. Derive the perturbed electron density
10pts. response $\delta n_e/n_{e0}$ satisfying

$$\frac{\omega}{k_z} \ll v_{Te+}, v_{Te\parallel}$$

Write down the leading order term only.

D. Consider an electrostatic perturbation
15pts. satisfying $\frac{\omega}{k_z} \gg v_{Ti\perp}, v_{Ti\parallel}$ in addition to the conditions given in C.

From linearized drift-kinetic equation,
find the perturbed ion density response
 $\delta n_i/n_{i0}$ up to the nontrivial leading order.
you can take $k_E C_s \sim \omega \sim \omega_{Ti\perp}$ where

4.

$$\omega_{\text{ITG}+} = -k_y \rho_i \frac{v_{T_i+}}{L_{T_i+}} \quad \text{and}$$

$$\omega_{\text{ITG}\parallel} = -k_y \rho_i \frac{v_{T_i\parallel}}{L_{T_i\parallel}} \quad \text{are comparable.}$$

E. From the answers for parts C and D,
 5pts. find a dispersion relation for the flat density
 ion temperature gradient instability.

F. Discuss the physical mechanism for
 5pts. that instability.

G. In the course of deriving the answer
 10pts. in part E, a cubic equation for " ω "
 must have appeared. Discuss a possibility
 of making another root (other than ITG
 instability)
 unstable by including a contribution
 from wave-electron resonant interaction.