Advanced Theory of Helicopters – Mid-Term Exam

April 26 (Thurs) 11:00 – 12:30

1. There exist four possible formulations for the flexible beam model to analyze the rotating rotor blades as follows. Explain the characteristics of each formulation and difference between the formulations briefly. (Each 15 points)

a) $$\frac{\partial^2}{\partial r^2}\left(EI\frac{\partial^2 w}{\partial r^2}\right)+m\frac{\partial^2 w}{\partial t^2}-\frac{\partial}{\partial r}\left(T\frac{\partial w}{\partial r}\right)=f_z(r,t)$$

b) $$m\ddot{w}+\left(EIw''\right)''-\left(EAu'w'\right)'=f_z(r,t)$$

c) $$\left\{\begin{array}{c} m\ddot{w}+\left(EIw''\right)''-\left(Tw'\right)'=f_z(r,t) \\ T=\int_r^R m\Omega^2\rho d\rho \end{array}\right\}$$

d) $$\left\{\begin{array}{c} m\ddot{w}+\left(EIw''\right)''-\left(EAu'w'\right)'=f_z(r,t) \\ \left[EA\left(u'+\frac{1}{2}w'^2\right)\right]'=m\ddot{u}-m\Omega^2(r+u)-f_h \end{array}\right\}$$

2. As it is shown above in Question 1, the governing equation for the rotating flexible rotor blade is usually simultaneous PDE's coupled between the flap and axial deformation. Then, how do you approach to solve such governing equations by using any available approximate solution methods, such as Galerkin, Rayleigh-Ritz, or Finite Element Methods, for the following purpose? (15 points)

   a) Natural frequencies and mode shapes for the free vibration of rotating rotors

3. Describe the reason why Lock number is required to prescribe in order to obtain the similarity between small-scaled wind-tunnel experimental model and the full-scale

helicopter blade. (15 Points)

4. Fourier coordinate transformation is one possible way to express the rotor motion degrees of freedom, in a fixed frame. The, compared with the expression in a rotating frame, which advantages will be obtained? Or for which purpose, such degrees of freedom expressed in the non-rotating frame must be used in the analysis of the helicopter rotors, instead of the ones expressed in the rotating frame? (10 Points)