Engineering Economic Analysis

Fall 2019

Mid-term Exam.

2019.04.30

1. Consider a consumer with utility function $u(x_t) = \ln x_t$ for the cash for consumption x_t he expenses in period t. Assume that he receives no income in the first two periods (t = 0,1), but receives an income $w > 0$ in the second period (t=2) only. In addition, this consumer discounts his consumption stream (x_0, x_1, x_2) according to the following utility function: $u(x_0, x_1, x_2) = \ln x_0 + \beta(\delta \ln x_1 + \delta^2 \ln x_2)$, where $\delta \in (0,1)$ denotes his discount rate factor, and $\beta \leq 1$ is the measure of his present bias. Assume that, once the individual makes plans in period 0, he does not revise these plans in the future.
   (a) (15 pts.) Assume that he borrows the amount of cash he needs during period 0 and 1, and in period 2 he uses w to payback his debt. For simplicity, assume that it is enough to payback $(1+r)(x_0 + x_1)$ in period 2, where $r \in (0,1)$ is a borrowing interest rate. Find his optimal consumption plan for (x_0, x_1, x_2).
   (b) (10 pts.) We can say that the individual is a person without *present bias* when $\beta = 1$, otherwise he has a *present-bias* character. Prove or disprove the following statement; *"The consumer with present-bias character would borrow less debt than one without present bias."*

2. (15 pts.) When consumer's utility function is $u(x_1, x_2) = x_1^2 + x_2^2$, perform the comparative statics analysis of the demand of good 1 with respect to the changes of its own price

3. Consider a consumer with quasilinear utility function $u(x_1, x_2) = 2x_1^{\alpha} + x_2$, where $\alpha \neq 1$.
   (a) (10pts.) Find the consumer's Marshallian demand functions.
   (b) (10pts.) Find the consumer's Hicksian demand functions.
   (c) (10pts.) Assume that the consumer's income is \$10, and the prices for both commodities are same with \$2. When $\alpha = 0.5$, if the price of good 1 decreases by 50%, find three types of welfare changes such as (i) changes in consumer's surplus, (ii) compensating variations, and (iii) equivalent variations

4. Consider an economy with only two goods x and y. Suppose that there are 100 consumers with A-type utility function $u_A(x, y) = \min\{x, 2y\}$, and 200 consumers with B-type utility function

$u_B(x, y) = \sqrt{xy}$. And all consumers will have a same amount of income as \$3,600.

(a) (10pts.) Find a market demand function for the good *x*.

(b) (10pts.) Now the price of good *x* is \$20 and the price of good *y* is \$10. Prove or disprove the following statement using the elasticity of own price; "*If a producer of good x wants to make more revenue, the producer should raise the price of good x.*"

(c) (10pts.) The government plans a substantial investment in R&D project to improve the productivity of good *x*, and as a result it is anticipated that the price of good *x* will be decreased to \$10 after the investment. Calculate the maximum lump-sum tax that each type of consumer would be willing to pay for financing this project.