Fluid Mechanics
Fall 2022
Hyungmin Park

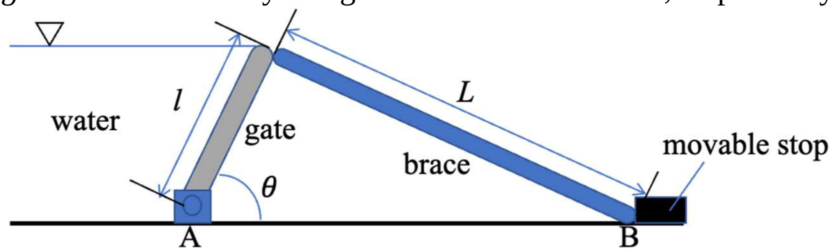
Midterm Exam #1
(October 11, 2022; 11:00-12:30)

*You need to write down detailed procedure used to reach your answers.

1. (15 points) Explain the followings.
(a) (5 points) Reynolds number
(b) (10 points) Newtonian fluids vs. non-Newtonian fluids

2. (15 points) Consider a steady two-dimensional flow field in the Cartesian coordinate system (x, y), of which the velocity components are given as $(u, v) = (2ax, -ay)$. Here, a is a positive constant.
   (a) (10 points) Determine and sketch the streamlines of the flow.
   (b) (5 points) Calculate the divergence of the field, that is $\nabla \cdot \bar{V}$, where $\bar{V}$ is the velocity vector. What can you say about the compressibility (capability of volume change) of the flow field?

3. (20 points) A homogeneous gate (weight of W_g) of width of b and length of l is hinged at point A and held in place by a brace (length L), as shown in the figure. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace. Specific weight of water is $\gamma = \rho_w g \ (N/m^3)$, where ρ_w and g is the water density and gravitational acceleration, respectively.



   (a) (15 points) Find an expression for the force (F_B) exerted on the gate by the brace as a function of the angle of the gate, θ.
   (b) (5 points) Repeat (a) for the case when the gate weight can be negligible. Give a comment on the result as $\theta \rightarrow 0$.

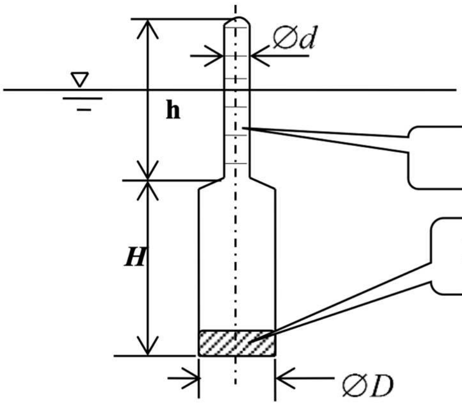
4. (20 points) A rigid cylindrical container of diameter $2R$ and height H contains equal volumes of two immiscible liquids of densities ρ_1 and ρ_2 such that $\rho_1 > \rho_2$. The container and its contents are rotating at a constant angular velocity of Ω in the presence of gravity. Let us determine the equation for shape of the interface $h(r)$ between the two liquids. Assume that the pressure at the top center is zero.
   (a) (10 points) Determine the pressure fields in each of the liquids. Leave the center of interface location H_O as a free parameter, for now.

(b) (5 points) Obtain the equation for the interface shape, $h(r)$, by using the continuity of pressure at the interface. Leave the center of interface location H_O as a free parameter, for now.

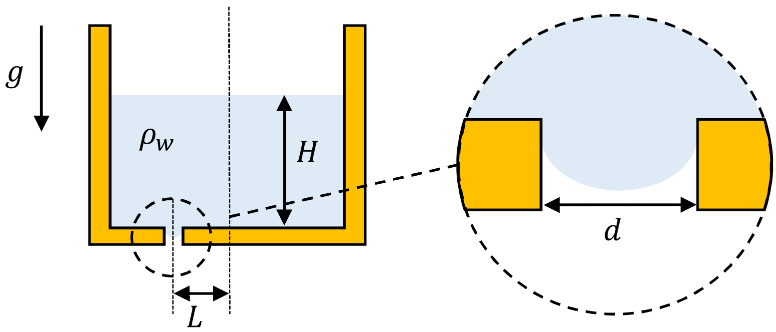
(c) (5 points) Determine H_O and complete the expression of $h(r)$.

(d) (5 points) What would the interface shape be if the gravity is neglected?

5. (15 points) A hydrometer whose dimensions are given in the figure is used to measure the density of varying fluids. The mass of the apparatus, not including the mass of the mercury at the bottom, is m_{body}. If the hydrometer is immersed in water whose density is ρ_w, then half of the upper cylinder is immersed in water (the scale $h/2$ is at the water level). If a lighter fluid of density $\eta\rho_w$, with $\eta < 1$ is used, the entire body is immersed, yet buoyant. Find the mass m_{Hg} of the mercury at the bottom of the hydrometer and the height H of the lower cylinder, respectively.



6. (15 points) Consider a bucket containing water of height H. It has a small cylindrical hole in the bottom. Because all surfaces of the bucket, including the vicinity of the hole, are hydrophobic, water exhibits a high contact angle (larger than 90°) as shown in the figure below. The hole is located at a distance L from the center of the bucket and its diameter (d) is much smaller than L. Atmospheric pressure can be negligible.



(a) (5 points) Find the height at which the contained water begins to overcome the surface tension and leak through the hole. Surface tension coefficient is σ (N/m).

(b) (10 points) Now let's consider that we accelerate this bucket to the right. When the water accelerates with a constant acceleration of a, its free surface will tilt by the angle of θ. When the height of water is H before it is accelerated, find the acceleration a at which the water begins to overcome the surface tension and leak through the hole. Water does not overflow out of the bucket.