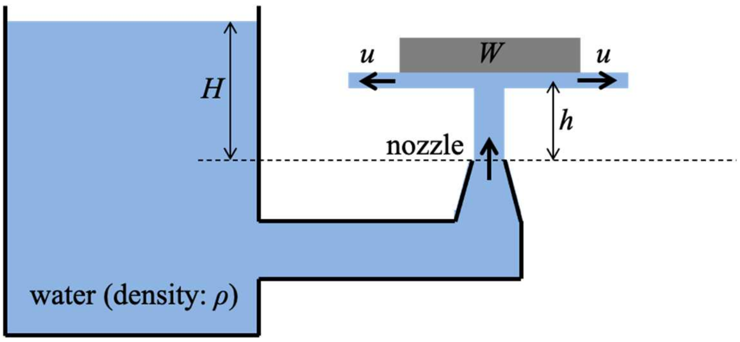
Midterm Exam #2 (Chapters 3 & 4)
(November 17, 2022, no calculator allowed)

<u>* Unless specified, you have to write down detailed procedure used to reach your answers.</u>

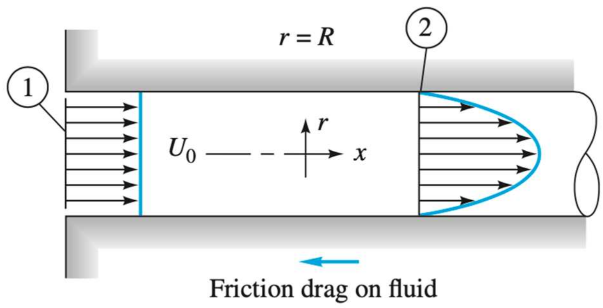
1. (20 points) A velocity field ($\bar{v}$) satisfies the following two conditions of $\nabla \cdot \bar{v} = 0$ and $\nabla \times \bar{v} = 0$.
   (a) (10 points) What does each of these conditions represent? Explain.
   (b) (10 points) Given a two-dimensional incompressible velocity potential function of $\phi(x, y) = xy + x^2 - y^2$, find the stream function, $\psi(x, y)$, of the corresponding flow.

2. (20 points) A water jet is issued vertically upward and impinges on a flat plate whose weight is *W*, as shown in the figure below. The source of the jet is the reservoir whose height above the nozzle is *H*, and the cross-sectional area of the jet nozzle is *A*. Assume two-dimensional and frictionless flow.
   (a) (5 points) Calculate the jet velocity and the volume flow rate at the exit of the nozzle.
   (b) (5 points) Calculate the jet velocity (u) at the height of h. Assume that there are no losses in this system.
   (c) (10 points) Calculate the weight *W* of the plate that can be held in place at the height of h. Assume that the plate does not move laterally.

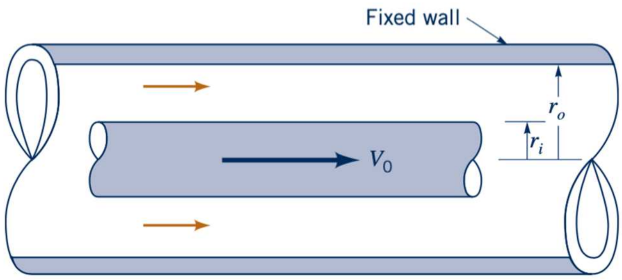


3. (20 points) Consider steady and incompressible flow in the entrance of a circular tube, as shown in the figure below. The inlet flow (section 1) is uniform as $u_1 = U_o$. The flow at section 2 is affected by the interaction between the pipe wall and fluid viscosity, such that the velocity field develops, as given below. For each laminar and turbulent condition, find the frictional drag force *F* acting on the wall as a function of pressure (p_1 and p_2), fluid density (ρ), inlet velocity (U_o), and pipe radius (R). <u>Use the control volume analysis and the maximum velocity (u_{max}) should be expressed in terms of U_o (do not include u_{max} in your final answers).</u>

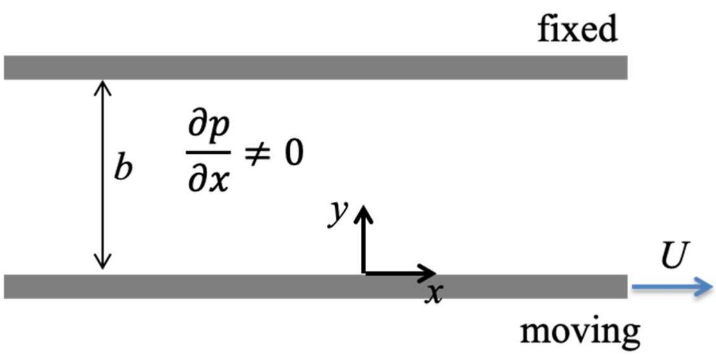
- Laminar flow: $u_2(r) = u_{max,L}(1 - r^2/R^2)$
- Turbulent flow: $u_2(r) = u_{max,T}(1 - r/R)^{1/7}$



4. (20 points) An incompressible Newtonian fluid flows steadily between two infinitely long, concentric cylinders as shown below. The outer cylinder is fixed, but the inner cylinder moves with a longitudinal velocity of V_o. The pressure gradient in the axial direction is $-\Delta p/l$. For what value of V_o will the viscous drag on the inner cylinder be zero? Assume that the flow is laminar, axisymmetric, and fully developed.



5. (20 points) The two-dimensional, viscous, incompressible flow between the parallel plates shown in the figure is driven by both the motion of the bottom plate (with a velocity of U) and a negative pressure gradient, $\partial p/\partial x$ (< 0). Starting from the Navier-Stokes equation, find the flow velocity, $u(y)$, as a function of fluid viscosity (μ), pressure gradient, and distance (b) between the plates. Also determine the location (y/b) where the maximum velocity is induced with $(b^2/2\mu U)(\partial p/\partial x) = -3$. Assume that the flow is fully developed and steady.



Appendix. Equations of motion of an incompressible Newtonian fluid in cylindrical coordinates (r, θ, z)

$$\frac{1}{\mathrm{r}}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta)+\frac{\partial}{\partial z}(v_z)=0$$ Continuity equation

$$\frac{\partial v_r}{\partial t}+(\bar{V}\cdot\nabla)v_r-\frac{1}{r}v_\theta^2=-\frac{1}{\rho}\frac{\partial p}{\partial r}+g_r+\nu\left(\nabla^2 v_r-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)$$ r-momentum equation

$$\frac{\partial v_\theta}{\partial t}+(\bar{V}\cdot\nabla)v_\theta+\frac{1}{r}v_r v_\theta$$
$$=-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}+g_\theta+\nu\left(\nabla^2 v_\theta-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right)$$ θ-momentum equation

$$\bar{V}\cdot\nabla= v_r\frac{\partial}{\partial r}+\frac{1}{r}v_\theta\frac{\partial}{\partial \theta}+v_z\frac{\partial}{\partial z}$$ Convective time derivative

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{\mathrm{r}^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}$$ Laplacian operator

$$v_r=\frac{1}{r}\frac{\partial \psi}{\partial \theta},\qquad v_\theta=-\frac{\partial \psi}{\partial r}$$ Stream function (r, θ)

$$\tau_{rz}=\mu\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right)$$ Shear stress, τ_{rz}