Mechanics of Materials and Lab.

Midterm exam (Closed book), May 8th 10am – 12pm

Total 100 pts (5 Questions)

Name: Solution

Student id number:

Equations:

Normal: $\sigma = \frac{P}{A}$; $\varepsilon = \frac{\delta}{L}$; $\sigma = E\varepsilon$;

Shear: $\tau = G\gamma$; $G = \frac{E}{2(1+\nu)}$

Deformation: $\delta = \frac{PL}{EA}$; $\delta_T = \varepsilon_T L = \alpha(\Delta T)L$

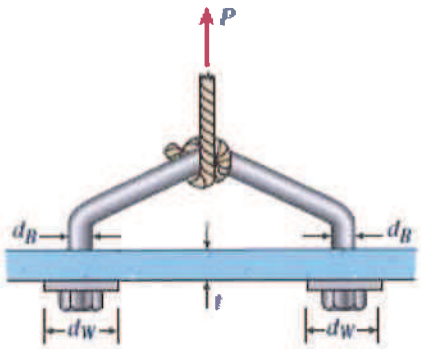
Stress on inclined section: $\sigma_\theta = \frac{P}{A}\cos^2\theta$; $\tau_\theta = -\frac{P}{A}\sin\theta\cos\theta$

Torsion: $\theta = \frac{d\phi}{dx}$; $\phi = \frac{TL}{GI_P}$; $I_P = \int_A \rho^2 dA = \frac{\pi d^4}{32}$ (circular bar case)

Bending: $\varepsilon = -\frac{y}{\rho} = -\kappa y$; $\sigma_x = E\varepsilon_x = -\frac{Ey}{\rho} = -E\kappa y$

Problem # 1 (15 pts)

A tie-down on the deck of a sailboat consists of a bent bar bolted at both ends, as shown in the figure. The diameter d_B of the bar is 20 mm, the diameter d_W of the washers is 50 mm, and the thickness t of the fiberglass deck is 40 mm.



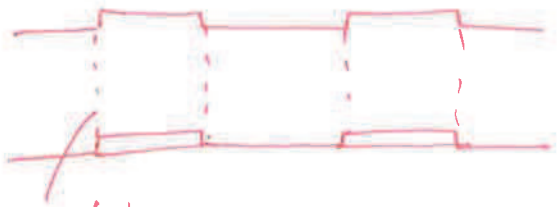
1) Draw possible area under shear stress.

2) Draw possible failure shape due to shear failure.

3) If the allowable shear stress in the fiberglass is 2 MPa, and the allowable bearing pressure between the washer and the fiberglass is 4 MPa, what is the allowable load P_{allow} on the tie-down?

1)

2)



shear failure area.

3) $P_{allow.\,shear} = 2 \cdot 2 \cdot 10^6\,pa \cdot \pi \cdot 0.05 \cdot 0.04 = 25\ kN$

$P_{allow.\,normal} = 2 \cdot 4 \cdot 10^6\,pa \cdot \frac{\pi}{4}(0.05^2 - 0.02^2) = 13\ kN$

$\Rightarrow P_{allow} = 13\ kN.$

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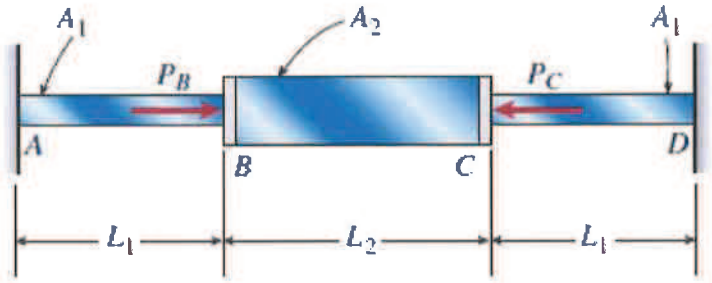
Problem #2. (25 pts)

The fixed-end bar ***ABCD*** consists of three prismatic segments, as shown in the figure. The end segments have cross-sectional area A_1 (diameter of 30 mm) and length L_1 = 200 mm. The middle segment has cross-sectional area A_2 (diameter of 50 mm) and length L_2 = 250 mm. Loads P_B and P_C are equal to 25 kN and 18 kN, respectively. The Young's modulus of AB and CD segment is 20 GPa while that of BC segment is 15 GPa.



1) Determine the reactions and its directions of R_A and R_D at the fixed supports.
2) Determine the compressive axial forces of all segments.
3) Determine whether the applied axial force is tension or compression for each segments.
4) Calculate maximum shear stress for each segments.
5) Draw potential failure shape due to shear stress for each segments.

1) $R_A = -8.48\ kN\ (\leftarrow)$

$R_D = 1.48\ kN\ (\rightarrow)$

2) $P_{AB} = 8.48\ kN$

$P_{BC} = -16.52\ kN$

$P_{CD} = 1.48\ kN$

3) AB tension

BC compression

CD tension

4) $\tau_{AB} = \dfrac{8.48 \times \frac{1}{2}}{\frac{\pi}{4}(0.03)^2} =$ ~~4.25~~ 6 MPa

$\tau_{BC} = 4.2\ MPa$

$\tau_{CD} = 1.0\ MPa$

5)

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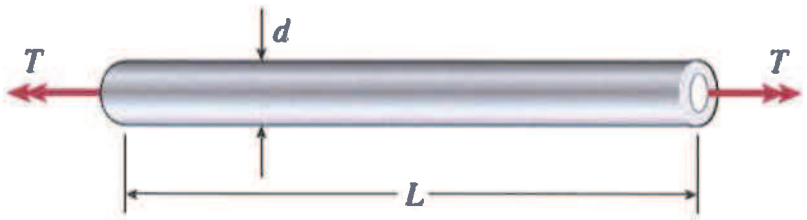
Problem #3. (20 pts)

A propeller shaft for a small yacht is made of a solid steel bar 150 mm in diameter with a hole of 80 mm diameter. The allowable stress in shear is 40 MPa and the allowable rate of twist is 2.0° in 3.5 meters.



Assuming that the shear modulus of elasticity is G = 80 GPa, determine the maximum torque T_{max} that can be applied to the shaft.

$\tau_{allow} = 40\,MPa$

$T_{max} = \frac{40 \cdot 10^6\,Pa}{\frac{0.15}{2}} \cdot I_p = \frac{40 \cdot 10^6\,Pa}{\frac{0.15}{2}} \cdot \frac{\pi}{32}(0.15^4 - 0.08^4)$

$= 24.3\ kN \cdot m$

$T_{max} = 80 \cdot 10^9\,Pa \cdot I_p \cdot \frac{\pi/90}{3.5} = 36.4\ kN \cdot m$

$\Rightarrow T_{max} = 24.3\ kN \cdot m$

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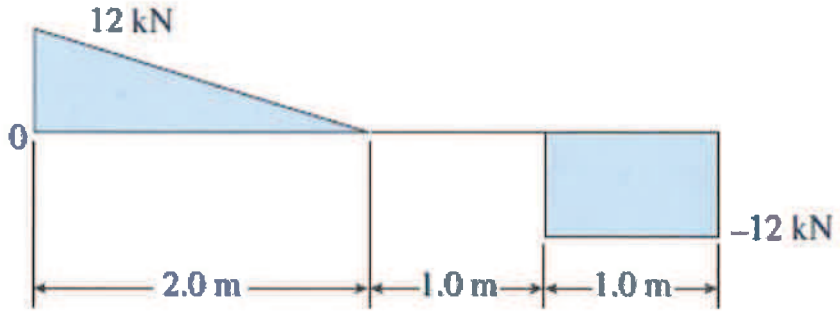
Problem #4. (15 pts)

The shear-force diagram for a simple beam is shown in below figure.

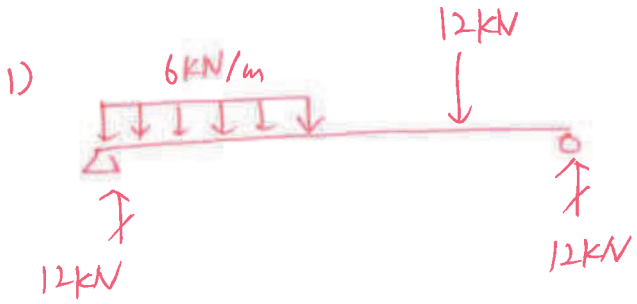


1) Determine the loading on the beam

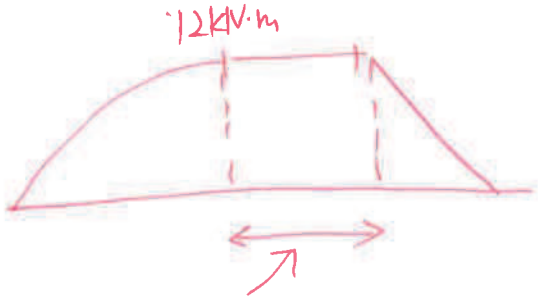
2) Draw the bending-moment diagram, assuming that no couples act as loads on the beam.

3) Compute maximum moment and mark its position(s).

1)



2)



3) $M_{max} = 12\,kN \cdot m$

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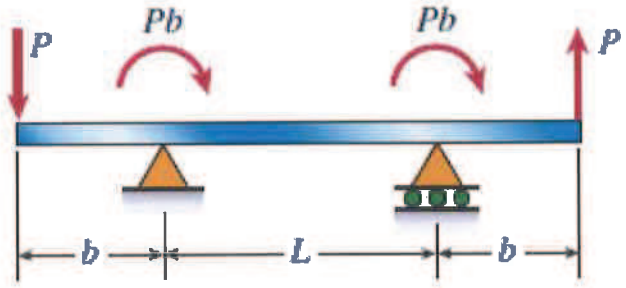
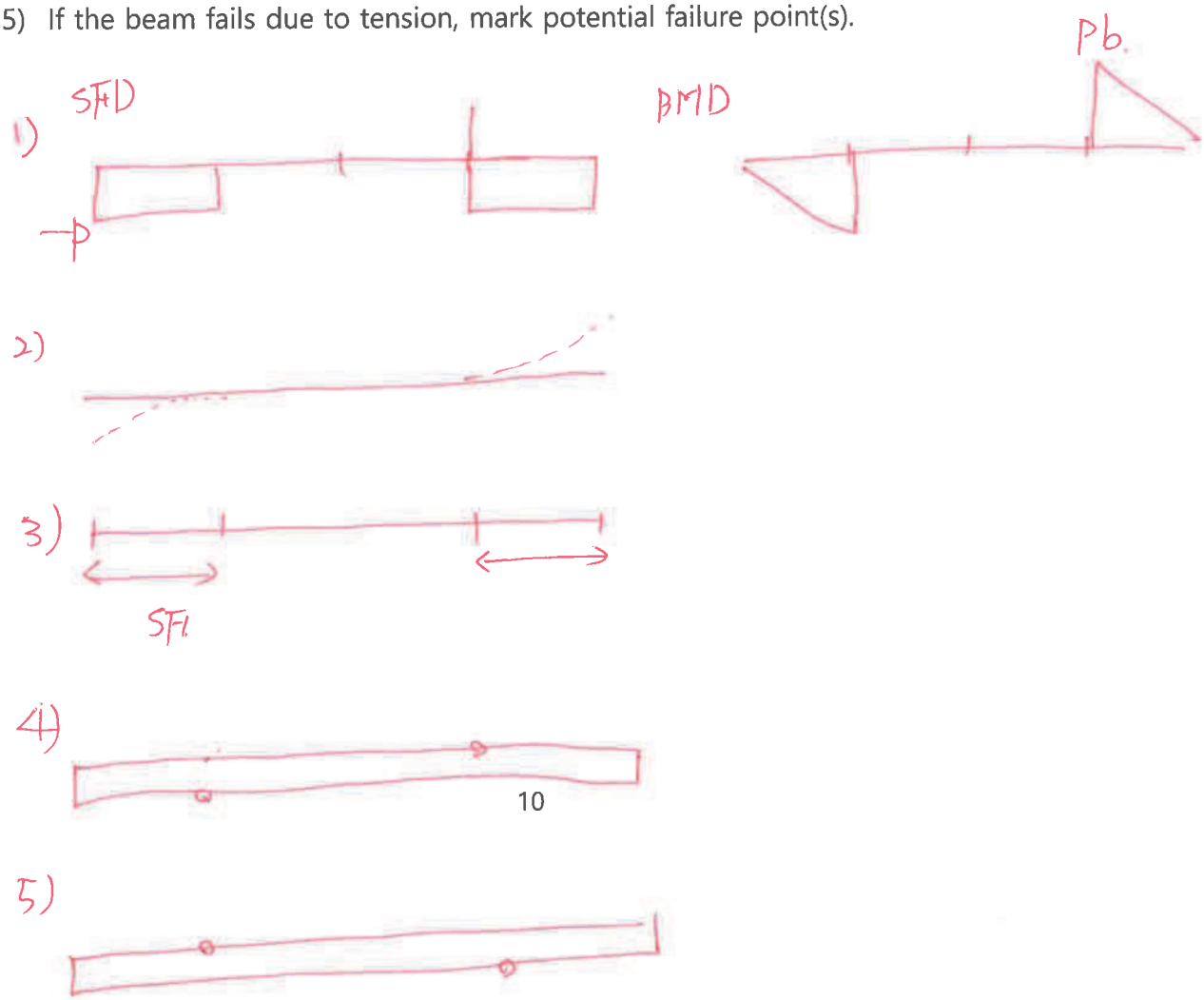
Problem #5. (25 pts)

A simple beam with two overhangs has one load downward (***P***) and the other upward (***P***), and clockwise moments ***Pb*** as shown in below figure. The beam has rectangular steel section.



1) Draw shear force diagram (SFD) and bending moment diagram (BMD).
2) Based on the BMD, draw possible deflection shape of the beam.
3) If the beam fails due to shear, mark possible failure section(s) or region(s).
4) If the beam fails due to compression, mark potential failure point(s).
5) If the beam fails due to tension, mark potential failure point(s).



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