Micro Fluid Mechanics, Spring 2019

Problem Set 1

March 19, 2019

Problem 1. Continuum hypothesis

Judge whether the following flow fields should be classified as continuum or not. Assume that the surrounding gas is composed entirely of oxygen atoms and that T=300K for all flows. Diameter of O atom is 0.364 nm.

(a) The flow field around a probe (1 cm in diameter) in a vacuum chamber at 1 Torr.

(b) The flow around a micrometeorite (1 mm diameter) at an altitude of 17 km. Use the following approximation for the density of the atmosphere:

$$\frac{\rho}{\rho_0} = \exp\left(-\alpha H\right),$$

where ρ_0 is the sea-level density 1.23 kg/m^3, $\alpha = 4.25 \times 10^{-5}\,/\,\mathrm{ft}$, and H is the altitude. Tip: what is the relationship between the mass density ρ and the number density n?

Problem 2. Fluctuations in the continuum

The validity of the continuum assumption rests in part on the very large number of molecules or atoms that make up everyday objects. A key notion is the idea of small statistical fluctuations in samples of matter. Consider a small subvolume δV within a larger volume V that contains a total of N molecules. The probability that an individual molecule will be found in the subvolume is just $P = \delta V / V$. Simple considerations suggest that for very small volumes, the probability $p(k)$ that k molecules will be found within the subvolume at any instant is given by the *Poisson* distribution,

$$p(k) = \frac{n^k \exp(-n)}{k!}$$

where $n = NP$.

(a) Compute the expected number of molecules $<k>$ in terms of the parameter n. The expectation of a function $< f(k) >$ is given by

$$< f(k) >= \sum_{k=0}^{\infty} f(k) p(k)$$

(b) Compute the variance $\mathrm{var}\{k\} =< (k- < k >)^2 >$.

(c) Estimate the size of fluctuations in the number of molecules within a cube 100 nm on a side in water at 300 K and 1 bar. A standard method of measuring fluctuations is to compute $\sqrt{\mathrm{var}\{k\}} / < k >$, the relative root mean square (rms) deviation.

(d) How small must the sample volume δV be before the relative rms deviations reach significant levels, i.e. 10%?

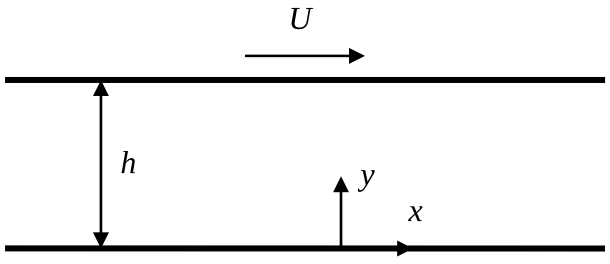
(e) For fun What is the probability that in your lifetime you will breathe some of the same molecules exhaled by General Sunshin Yi in his moment of dying?

Problem 3. Gas flow in microchannel

Consider a circular channel through which oxygen gas passes. Assuming that T=300 K and channel diameter is 50 μm, below which gas pressure should the continuum hypothesis start to break down?

Problem 4. Couette flow with slip boundary condition

Consider a flow between a pair of parallel plates (spacing h), one of which moves with speed U. Obtain the steady velocity profile when the Navier slip boundary condition is applied for the stationary plate.



Problem 5. Enhanced flow in carbon nanotubes

Read Majumder et al. Nature 44, **438** (2005). Explain how they obtained the slip length (units in micrometers not in millimeters) as listed in Table 1. (Use only one case)