Micro Fluid Mechanics - Spring 2019

Problem Set 2

March 25, 2019 Prof. Ho-Young Kim

Problem 1

Figure 1 shows the type of thrust bearing used on the 200-inch telescope at Palomar. Oil is supplied at high pressure and flows radially outward between two parallel circular plates. We are interested in obtaining design formulas so that, given the total supported load, F, the supply gage pressure, p_0, and the desired gap, h, we can determine the necessary disk diameter, D, and the required volume flow rate, Q, for oil of known viscosity μ and density ρ, the hole diameter being d. In what follows, you may take for granted that $h \ll D$.

(a) Determine how the ratio of inertial forces to viscous forces acting in the fluid varies with radius r.

(b) Establish an order-of-magnitude criterion which shows in what range of radius the flow may be regarded as inertia-free. Express your criterion in terms of Q and h together with any other necessary quantities.

(c) Assuming that the criterion in (b) for inertia-free flow is satisfied, that streamlines are parallel to the plates except very near the injection hole, and that $d/D \ll 1$, show that F and Q are given by

$$F = \frac{\pi}{4} D^2 p_0 \frac{1 - (d/D)^2}{2\ln(D/d)}$$

$$Q = \frac{\pi h^3 p_0}{6\mu \ln(D/d)}$$

Problem 2

Figure 2 shows a circular bearing pad which rests on a flat base through the intermediary of a film of viscous liquid of instantaneous thickness $h(t)$. The load W causes the pad to sink slowly at the speed S, and this squeezes the liquid out from under the pad. Assume that $h \ll D$, that the viscosity is very high, and that the speed S is very small.

(a) Making approximation (state them precisely) consistent with these assumptions, show that the settling speed is

$$S = \frac{32}{3\pi} \frac{W h^3}{\mu D^4}$$

(b) An apparatus with two very flat plates of 0.3 m diameter carries a load of 100 kg on a film 0.003 cm thick. If the liquid is a heavy oil with a kinematic viscosity of 10 cm^2/s

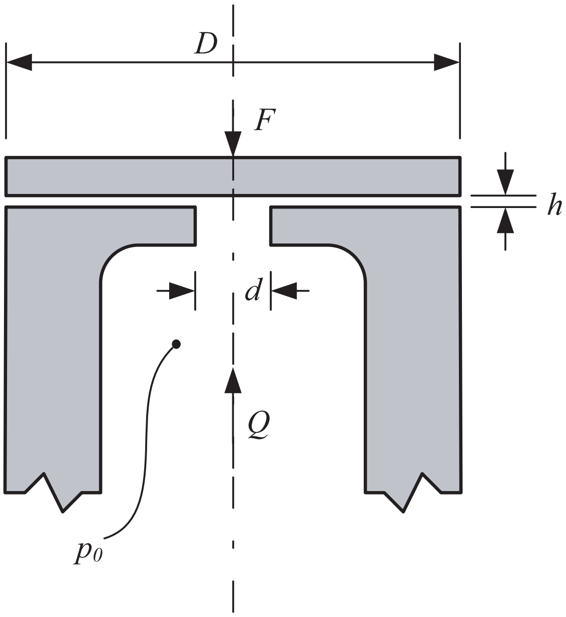

Figure 1: Problem 1

and a density of 0.93 g/cm^3, estimate the speed S.

(c) If the load W is constant, and the gap width is h_0 at time zero, show that the width h varies with time according to

$$\frac{h}{h_0} = \left(1 + \frac{64}{3\pi}\frac{Wh_0^2}{\mu D^4}t\right)^{-1/2}$$

(d) Suppose that the initial thickness is h_0 and that a constant upward force F pulls the disk *away* from the base. Show that the disk will be pulled away in a time

$$t_\infty = \frac{3\pi}{64}\frac{\mu D^4}{h_0^2 F}$$

Note: When h_0 is very small, the time t_∞ is very large. This is the basis of the phenomenon of viscous adhesion, e.g., adhesives such as Scotch tape, or the apparent adhesion of accurately-ground metal surfaces.

Problem 3

Consider uniform slow flow past a spherical bubble of radius a by modifying the analysis of the flow past a solid sphere accordingly, i.e. by replacing the no-slip condition on $r = a$ by the condition of no tangential stress ($\tau_\theta = 0$) on $r = a$. Show, in particular that

$$\Psi = \frac{1}{2}U(r^2 - ar)\sin^2\theta$$

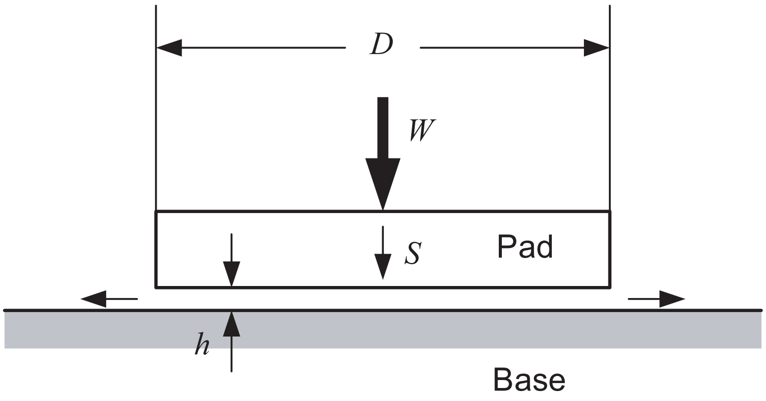

Figure 2: Problem 2

and that the normal component of stress on $r = a$ is $\tau_r = 3\mu U a^{-1} \cos\theta$. Hence show that the drag on the bubble is

$$D = 4\pi\mu U a$$

in the direction of the free stream.

Problem 4

Using the continuity equation ($\nabla \cdot \overline{u} = 0$) and the following vector identity,

$$\nabla^2 \overline{F} = \nabla(\nabla \cdot \overline{F}) - \nabla \times (\nabla \times \overline{F})$$

show that $\nabla p = \mu \nabla^2 \overline{u}$ can be converted to $\nabla^2(\nabla \times \overline{u}) = 0$.

Problem 5

In an in vitro motility gliding assay as shown in Fig. 3, a single kinesin molecule can move a microtubule of radius ~ 15 nm at a speed, v, of ~ 1 μm/s, independent of the microtubule's length (for lengths up to $L = 20$ μm).

(a) Assuming that the axis of a 10 μm-long microtubule is height $h = 25$ nm above the surface, calculate how much force must the motor molecule (kinesin) exert to move the microtubule against the drag force.

(b) How much should the viscosity be increased to slow the movement of the 10 μm-long microtubule to 0.25 μm/s ?

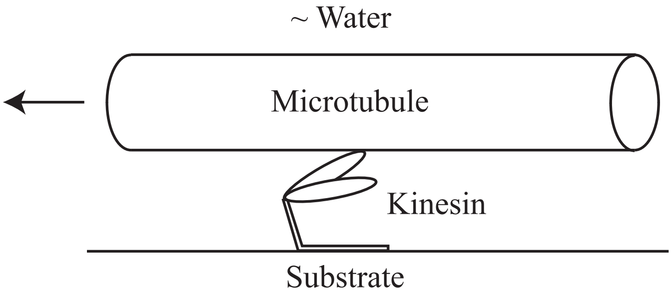

Figure 3: In vitro motility gliding assay