Micro Fluid Mechanics - Spring 2019

Problem Set 3

April 9, 2019 Prof. Ho-Young Kim

Problem 1 Determine the shape of the surface of a liquid rising between two parallel vertical flat plates (Fig. 1). The static contact angle is θ.

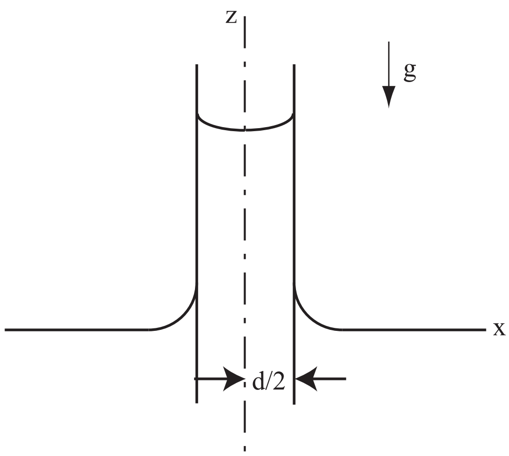

Figure 1: Capillary rise between parallel plates

Problem 2 Self-propelling slugs

Read Bico & Quéré's paper (*J. Fluid Mech.* **467**, 101-127). Especially follow the equations from (1) to (23).

Problem 3 Consider a superhydrophobic textured surface as shown in Fig. 2. When the pressure difference between inside and outside of the liquid is ΔP, determine the criteria that the post height b and the interpost pitch d should satisfy to prevent the liquid meniscus from touching (or invading) the bottom.

Problem 4 Scaling law for the Landau-Levich problem

Consider a plate being pulled out of a pool of wetting liquid. The plate drags a liquid film along with it as shown in Fig. 3. The upper part of the meniscus (shown as a dotted line) finds itself perturbed by the liquid film dragged along by the plate. The junction between the static meniscus and the film being dragged along is referred to as the *dynamical meniscus*, and its length is denoted l.

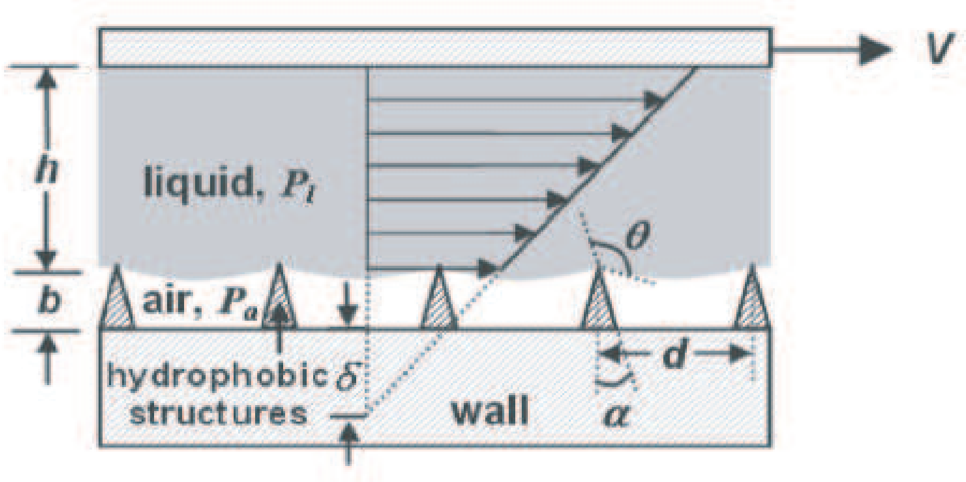

Figure 2: Superhydrophobic textured surface. From Choi & Kim, *Phys. Rev. Lett.* **96**, 066001 (2006)

(a) Show that the balance between the viscous force and the capillary force in the dynamical meniscus region is written as

$$\frac{\mu U}{e^2} \sim \frac{\sigma}{l_c l}$$

where μ is the viscosity, U the plate velocity, e the film thickness, l_c the capillary length.
(b) Show that matching the curvature of the static meniscus and that of the dynamical meniscus leads to

$$\frac{e}{l^2} \sim \frac{1}{l_c}$$

(c) Now finally obtain the Landau-Levich law:

$$e \propto l_c Ca^{2/3}$$

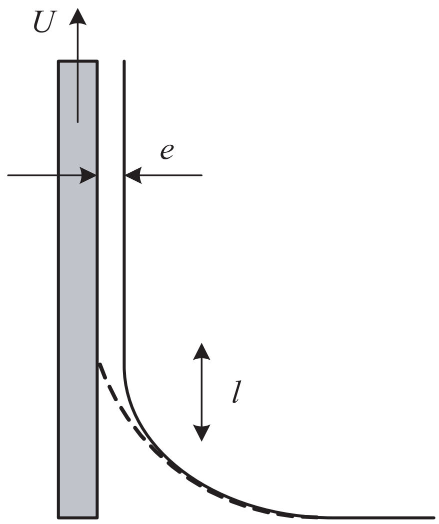

Figure 3: Landau-Levich problem