Micro Fluid Mechanics - Spring 2019

Problem Set 4

June 7, 2019 Prof. Ho-Young Kim

Problem 1

A long cylindrical microtube of radius $a = 500$ μm and length $L = 0.01$ m is closed at both ends by electrodes, across which a voltage drop $\Delta\phi = 10$ V is applied. The tube contains an ideal dilute aqueous solution of a fully dissociated doubly charged symmetrical binary salt ($z = 2$) at a concentration $c_0 = 1$ mol/m^3. The tube wall has a fixed surface potential $\zeta = 1.43 \times 10^{-3}$ V. The temperature $T = 25$°C, the permittivity $\epsilon = 7 \times 10^{-10}$ C/V/m, and the viscosity $\mu = 10^{-3}$ Pa s.

1-a Determine the Debye length λ_D at the tube wall.

1-b Find the radial distance at which the water velocity is zero.

1-c Find the water velocity at the tube center.

1-d Find the water velocity at a distance λ_D from the wall.

1-e Sketch the velocity profile in the tube down to the wall as a function of the radial distance r.

1-f Assume that spherical particles of radius R and charge q are introduced with the number of particles small enough that the suspension is dilute. What is the particle velocity at the radius where the water velocity is zero, assuming $R = 10^{-6}$ m and $q = 26.3 \times 10^{-16}$ C.

Problem 2

Consider a glass microchannel (cleaned with HCl, NaOH and deionized (DI) water for 5 minutes, respectively) of width $a = 500$ μm, height $h = 30$ μm and length $L = 5$ mm. Across the channel, a voltage drop of 10 V is applied. The channel is connected to a reservoir that contains an ideal dilute aqueous solution of a fully dissociated NaCl at a concentration $c_0 = 1$ mol/m^3. The temperature $T = 25$°C, the permittivity $\epsilon = 7 \times 10^{-10}$ C/V/m, and the viscosity $\mu = 10^{-3}$ Pa s. Assume that the zeta potential of the glass was measured to be -110.68 mV for this solution.

2-a When the chambers connected at the ends of the channel are open to the atmosphere so that no pressure gradient is applied, find the volume flow rate of water.

2-b List the methods (both electrokinetic and other means) that can increase the volume flow rate twice the value obtained above.

Problem 3

Assume that an electrophoretic separation chip is built with the same material and dimensions as given in Problem 2. An optical detector is located 4 mm down the injection point. The solution (same as Prob 2) contains two kinds of spherical particles. The detection results are such that particle A is detected 10 seconds after the injection is made.

3-a What is the zeta potential of particle A?

3-b If particle B reaches the detection point in 30 seconds, what is the zeta potential of particle B?

3-c If you want to reduce the difference of the arrival times of A and B down to 8 seconds, what should be done?

Problem 4

Read Li & Bashir, Sensors and Actuators B **86**, 215-221 (2002). Explain why the cells are collected at different locations as shown in Figs. 2-4.

Problem 5

Consider a vertical cylinder whose tip is exposed to the atmosphere while the rest of it is immersed in a liquid, as shown in Fig. 1. If the cylinder is long enough, it is prone to buckling because of the compressive force arising from the surface tension of the liquid. Estimate the critical length of the cylinder, over which the buckling is likely to occur (scaling analysis is acceptable). Assume a nearly zero contact angle.

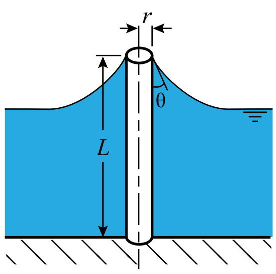

Figure 1: Problem 5