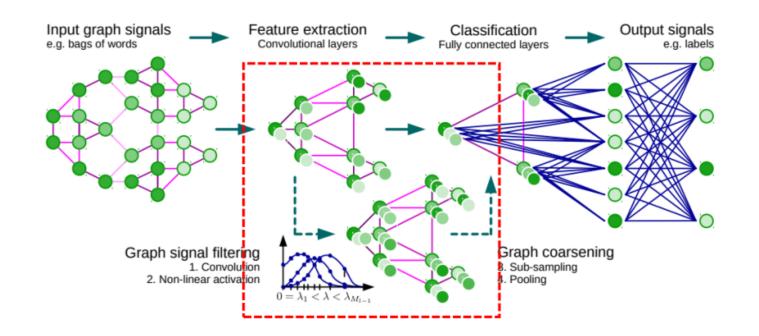
- Explain the spectral filtering of a graph signal
- Decompose the graph signal f with the Fourier Transform, and multiply the filter \hat{g} to filter the transformed signal in the spectral domain. Then, reconstruct the filtered signal in the spatial/temporal domain with the inverse Fourier Transform.

$$f \xrightarrow{\mathbf{GFT}} \hat{f}_{i} = \mathbf{u}_{i}^{T} \mathbf{f} \xrightarrow{\hat{g}(\lambda_{i})} \widehat{g}(\lambda_{i}) \cdot \mathbf{u}_{i}^{T} \mathbf{f} \xrightarrow{\mathbf{IGFT}} \sum_{i} \widehat{g}(\lambda_{i}) \mathbf{u}_{i}^{T} \mathbf{f} \mathbf{u}_{i}$$
Filter $\widehat{g}(\lambda_{i}) \cdot \mathbf{u}_{i}^{T} \mathbf{f} \mathbf{u}_{i}$
Filtered coefficients Filtered signal

- How to design the spectral filter for GCN?
- The spectral filters are learned in a data-driven way. For end-to-end learning, such filters are parametrized.



- What is the channel in a graph signal?
- Letting N and d₁ denote the number of nodes and the feature dimension in each node respectively, the graph signal is represented by a N × d₁ matrix. Then, each column of the matrix refers to 'channel'.

$$\boldsymbol{F}_{in} \in \mathbb{R}^{N \times d_1} \to \boldsymbol{F}_{out} \in \mathbb{R}^{N \times d_2}.$$

- How to deal with multi-channel signals in GCN?
- To create the *n*'th output channel, we transform each input channel with a different filter and add up the resulting signals. Thus, we need to learn $d_1 \times d_2$ many filters.

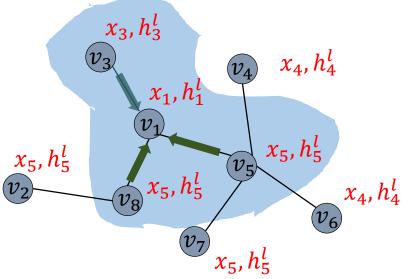
$$\boldsymbol{F}_{O}[:,n] = \sum_{m=1}^{d_{1}} \widehat{g}_{nm}(\boldsymbol{L}) \boldsymbol{F}_{I}[:,m] \quad n = 1, \dots d_{2}$$

Filter for input channel Learn $d_{2} \times d_{1}$ filter

GCN(G)

Geometric deep learning via graph convolution ... (continue)

$$h_i^{(l+1)} = \sum_{v_j \in N(v_i)} f\left(x_i, w_{ij}, h_j^{(l)}, x_j\right)$$



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