5. Helicopter Aeroelasticity

Helicopter Aeroelasticity

- Single D.O.F. flutter

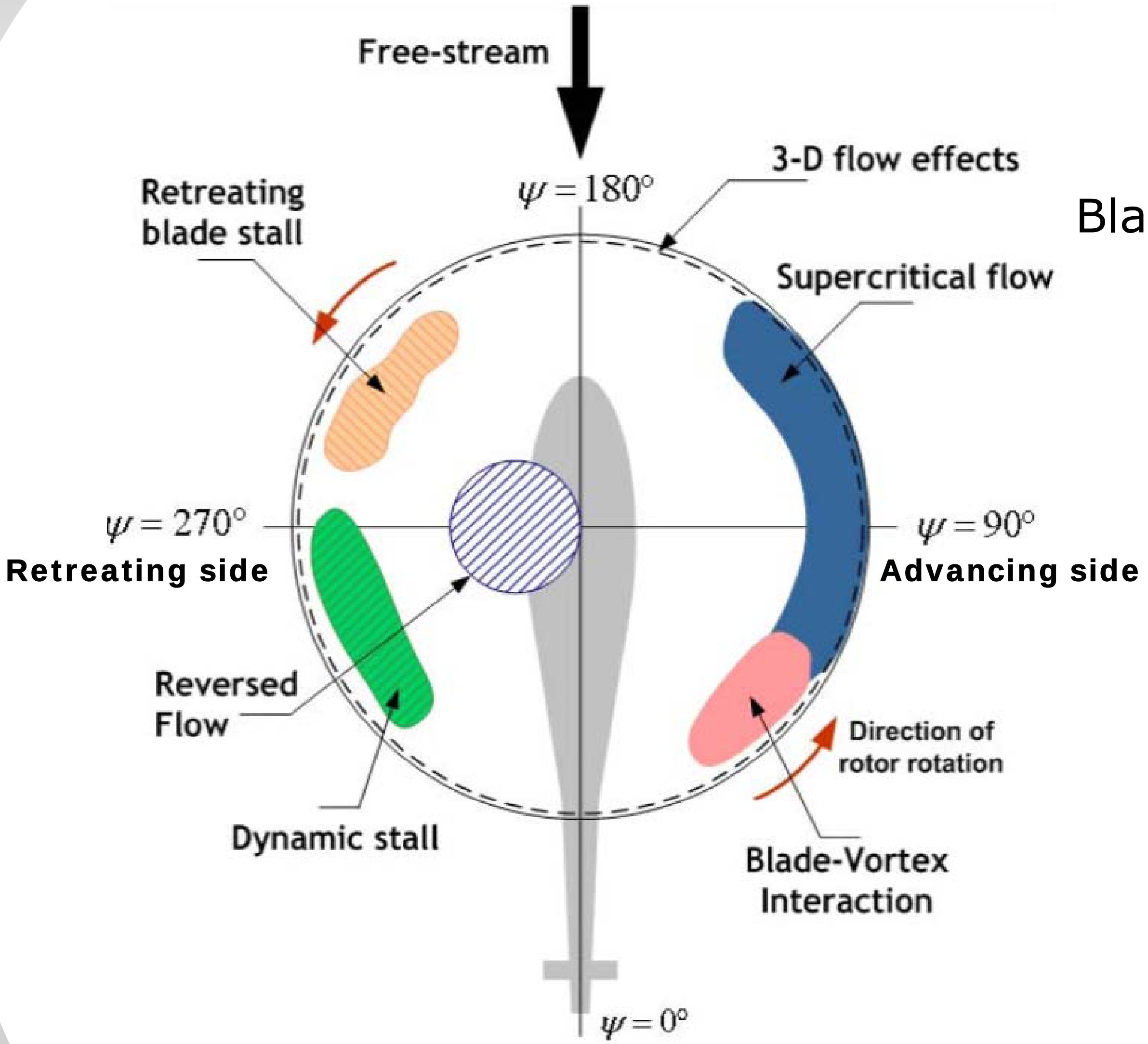
  blade motion (flap, lag, torsion) coupling

  hub

  divergence, flutter (freq. coalescence)

  (e.a., c.g., whirl flutter)
- isolated blade instability (hover) ← collective pitch
- stall instability (forward flight) ← cyclic pitch longitudinal / lateral
- aeromechanical instability (rotor, fuselage)
- active control

Helicopter Aeroelasticity



Blade angle   Flapping

$\theta \rightarrow \beta$

"90^o phase lag"

⇩

Resonance system

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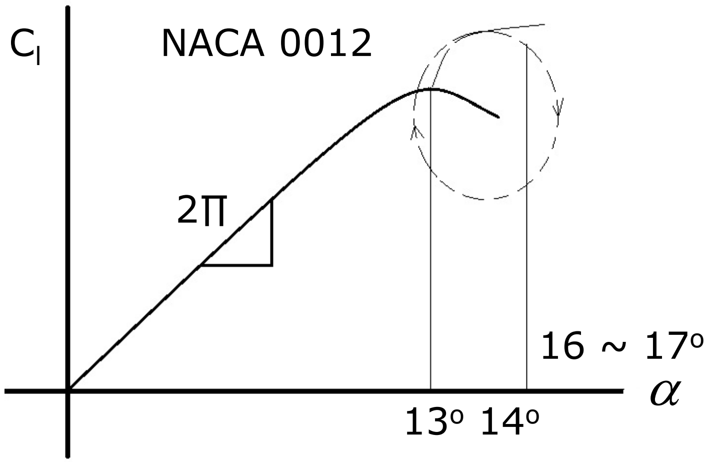
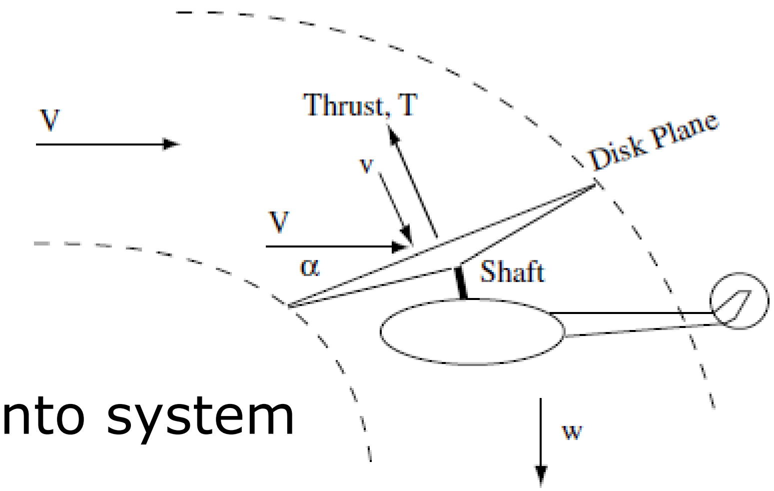




"hysteresis" → feed energy into system

- limit cyclic not destructive
- vibration load ↑

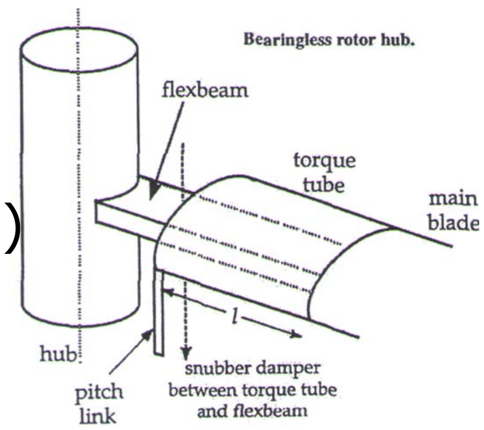
Rotor Type

a) Semi-articulated or teetering(2-bladed,bell)

b) Fully-articulated { flapping, pitch, lag / elastomeric bearing

c) Hingeless → feathering hinge only

d) Bearingless



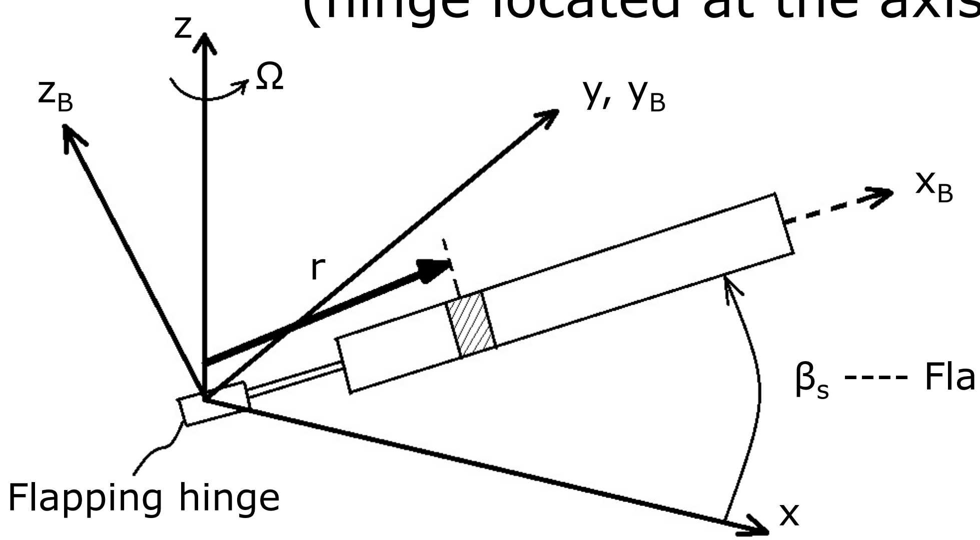
Helicopter Aeroelasticity

- *Isolated Blade Dynamics*

  - Articulated Rigid Blade Motion

  First let's consider a single blade with only flapping motion

  (hinge located at the axis of rotation)



$x_B\ y_B\ z_B$ ---- rotating axis (Ω)
x, y, z ---- β_s inclination

- *Rotor blade structural blade dynamics modeling*

  - rigid blade + spring concentrated at hub
  - elastic blade + geometric coupling

  → box-beam

Helicopter Aeroelasticity

- complete cross-section shape

Assumption $\begin{cases} \text{flapping, pitch, lag} \\ \text{elastomeric bearing} \end{cases}$

From dynamics

$$\frac{d^I \overrightarrow{H_P}}{dt} = \overrightarrow{M}$$

I : Inertia $\quad \overrightarrow{H_P}$: Angular momentum

$$\dot{\overrightarrow{H_P}} + \overrightarrow{\Omega_P} \times \overrightarrow{H_P} = \int_0^R \vec{r} \times d\overrightarrow{F_A}$$

The blade is slender, assume

$$\begin{cases} I_B \approx I_{y_B} \approx I_{z_B} \\ I_{x_B} = 0 \end{cases}$$

(rotational inertia)

If $\overrightarrow{\Omega_B} = p_B \hat{i}_B + q_B \hat{j}_B + r_B \hat{k}_B$

$$\Rightarrow \overrightarrow{H_P} = \vec{\vec{I}} \cdot \overrightarrow{\Omega_B} = \left(I_B q_B \right) \hat{j}_B + \left(I_B r_B \right) \hat{k}_B$$

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From dynamics

(dyadic)

$$\vec{\vec{I}} = \hat{i}\,\cancelto{0}{I_{11}}\,\hat{i} + \hat{i}\,\cancelto{0}{I_{12}}\,\hat{j} + \hat{i}\,\cancelto{0}{I_{13}}\,\hat{k} + \hat{j}\,\cancelto{0}{I_{21}}\,\hat{i} + \underbrace{\hat{j}\,I_{22}\,\hat{j}}_{I_{y_B}} + \hat{j}\,\cancelto{0}{I_{23}}\,\hat{k} + \ldots$$

$$\Rightarrow \quad I_B(\dot{q}_B - p_B r_B)\,\hat{j}_B + I_B(\dot{r}_B + p_B q_B)\,\hat{k}_B = \int_0^R \vec{r} \times d\vec{F}_A$$

To relate $\vec{\Omega}_B$ with Ω, $\dot{\beta}_s$

$$\begin{Bmatrix} p_B \\ q_B \\ r_B \end{Bmatrix} = \begin{bmatrix} \cos\beta_s & 0 & \sin\beta_s \\ 0 & 1 & 0 \\ -\sin\beta_s & 0 & \cos\beta_s \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \Omega \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\dot{\beta}_s \\ 0 \end{Bmatrix}$$

opposite to y_B

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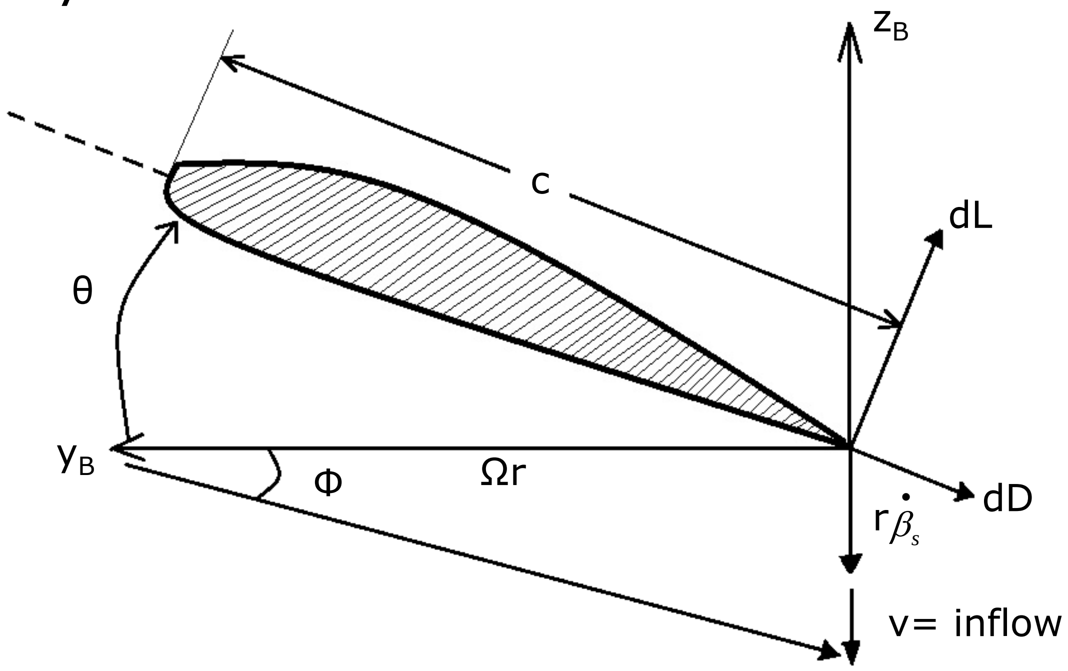
$$p_B = \Omega \sin \beta_s ; \quad q_B = -\dot{\beta}_s ; \quad r_B = \Omega \cos \beta_s$$

and the equation of motion

$$I_B(-\ddot{\beta}_s - \Omega^2 \cos \beta_s \sin \beta_s)\,\hat{i}_B + I_B(-2\Omega \sin \beta_s\, \dot{\beta}_s)\,\hat{j}_B = \int_0^R \vec{r} \times d\vec{F}_A$$

For aerodynamics "Hover"



Strip theory ⟹ "blade element theory"

Inflow ⟵ Momentum theory

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$$d\overrightarrow{F_A} = dL\,\overrightarrow{k_B} + (-dD - \varphi\, dL)\,\overrightarrow{j_B}$$ ϕ ≈ small

$$dL = \frac{1}{2}\rho(\Omega r)^2 (c\cdot dr) C_{L_\alpha} (\theta - \varphi)$$

$$dD = \frac{1}{2}\rho(\Omega r)^2 (c\cdot dr) C_{d0}$$

C_{d0} → Profile drag co-efficient

$$\phi = \frac{r\dot{\beta}_s + v}{\Omega r}$$

Introduce non-dimensional variables

$$x \in \frac{r}{R}$$

$$\lambda = -\frac{v}{\Omega R}$$: inflow parameter

$$^*\gamma = \frac{\rho C_{L\alpha}\, c\, R^4}{I_B}$$: Lock no. …. blade motion sensitivity

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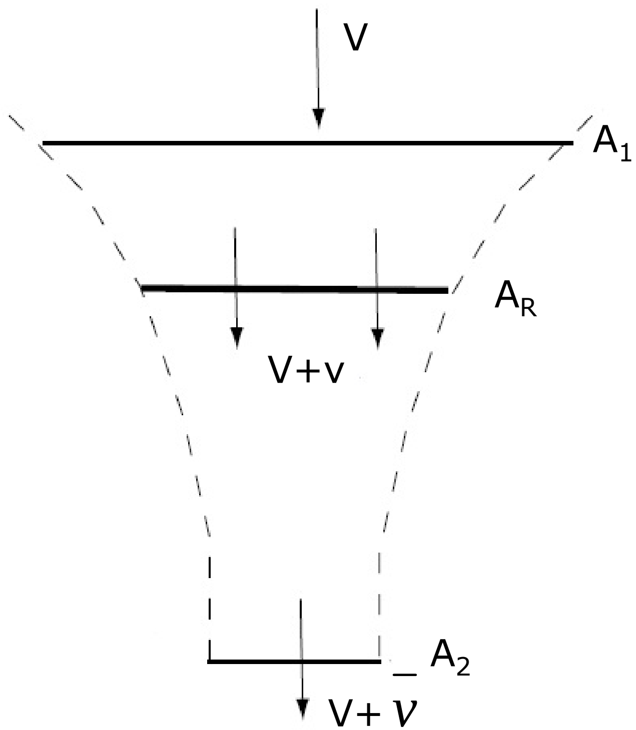
$$dL = \frac{I_B r \Omega^2}{R}\left(\theta - \frac{\dot{\beta}_s}{\Omega} + \frac{\lambda}{x}\right)x^2 dx$$

$$= \frac{I_B r \Omega^2}{R}\left(\frac{C_{d0}}{C_{L_\alpha}}\right)x^2 dx$$



From total thrust 'T'

$$T = b\int_0^l dL$$ b – No. of blades

$$\frac{2C_T}{C_{L_\alpha}\sigma} = \frac{\theta}{3} + \frac{\lambda}{2}$$ where, $\sigma = \frac{bc}{\pi R}$: Solidity ratio

$$C_T = \frac{T}{\rho\pi R^2(\Omega R)^2}$$: Thrust co-efficient

Momentum theory, $\lambda = -\sqrt{\frac{C_T}{2}}$

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$$\int_0^R \vec{r} \times d\overrightarrow{F_A} = -\frac{\lambda\Omega}{8}\left(\theta + \frac{4\lambda}{3} - \frac{\dot{\beta}_s}{\Omega}\right)\hat{j}_B$$

$$+\frac{I_B\gamma^2}{8}\left(-\frac{C_{d0}}{C_{L_\alpha}} + \frac{\dot{\beta}_s}{\Omega}\left(\frac{\dot{\beta}_s}{\Omega} - \theta\right) + \frac{4}{3}\left(\theta - \frac{2\dot{\beta}_s}{\Omega}\right)\lambda + 2\lambda^2\right)\hat{k}_B$$

Flapping eqn. of motion

$$\ddot{\beta}_s + \frac{\sigma\Omega}{8}\dot{\beta}_s + \Omega^2 \cos\beta_s \sin\beta_s = \frac{\gamma\Omega^2}{8}\left[\theta + \frac{4}{3}\lambda\right]$$

and linearizing it

$$\ddot{\beta}_s + \underbrace{\frac{\gamma\Omega}{8}}\dot{\beta}_s + \underbrace{\Omega^2}\beta_s = \frac{\gamma\Omega^2}{8}\left[\theta + \frac{4}{3}\lambda\right]$$

< Hover >

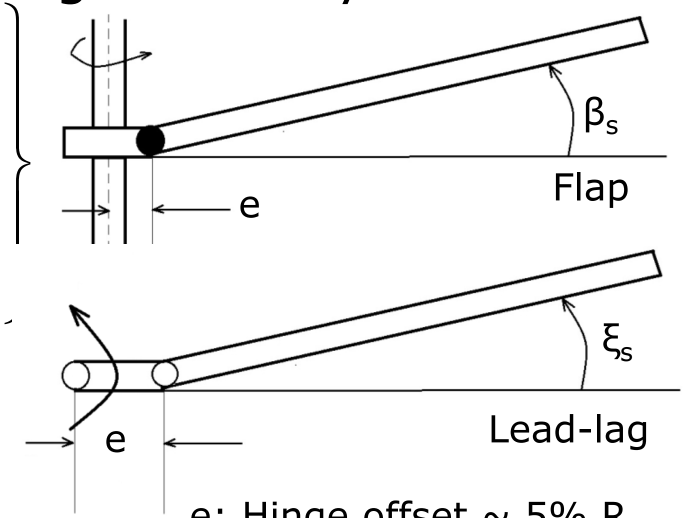
Damping ratio $\xi = \dfrac{\sigma}{16}$

C.F. : Nat. freq. $\omega^2 = \Omega^2$ ⟹ Resonant System

Helicopter Aeroelasticity

- Coupled Flap-Lag Equation ➡ "Lead-lag instability"

$$\begin{Bmatrix} p_B \\ q_B \\ r_B \end{Bmatrix} = \begin{bmatrix} \cos\xi_s & \sin\xi_s & 0 \\ -\sin\xi_s & \cos\xi_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \Omega \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \dot{\xi}_s \end{Bmatrix}$$



e: Hinge offset ~ 5% R

As before

$$\dot{\vec{H}}_P + \vec{\Omega}_B \times \vec{H}_P = \int_0^R \vec{r} \times d\vec{F}_A + \underbrace{\vec{E} \times M_B \vec{a}_P}_{}$$

This accounts for the acceleration at hinge point

where, M_B : mass of the blade

$\vec{a}_P$: acceleration of hinge point $= \dfrac{d^{I2}\vec{E}}{dt^2}$

$$= \underbrace{\cancel{\dot{\vec{E}}}}_{0} \times \dot{\vec{\Omega}} \times \vec{E} + \vec{\Omega} \times (\underbrace{\cancel{\dot{\vec{E}}}}_{0} \times \vec{\Omega} \times \vec{E}) = \vec{\Omega} \times (\vec{\Omega} \times \vec{E}) + \dot{\vec{\Omega}} \times \vec{E}$$

- Neglect the hinge offset for aerodynamic calculation

Then,

$$dL = \frac{1}{2}\rho[(\Omega + \dot{\xi}_s) r]^2 c \cdot dr \cdot c_{l_\alpha}\left[\theta - \frac{r\dot{\beta}_s + v}{(\Omega \dot{\xi}_s) r}\right]$$

$$\left[\begin{aligned}
&(1)\ \vec{E}\ ,\ \vec{a}_p = \ ? \\
&\vec{v}_p = \frac{d^I \vec{E}}{dt} = \cancelto{0}{\dot{\vec{E}}} + \vec{\Omega} \times \vec{E} = \vec{\Omega} \times \vec{E} \\
&\vec{a}_p = \frac{d^I \vec{v}_p}{dt} = \dot{\vec{v}}_p + \vec{\Omega} \times \vec{v}_p \\
&\qquad = \dot{\vec{\Omega}} \times \vec{E} + \vec{\Omega} \times \cancelto{0}{\dot{\vec{E}}} + \vec{\Omega} \times (\vec{\Omega} \times \vec{E})
\end{aligned}\right]$$

Helicopter Aeroelasticity

The eqn. of motion for 2 D.O.F system

$$\ddot{\beta}_s+\Omega^2\left(1+\frac{3}{2}\bar{e}\right)\beta_s+2\beta_s\underbrace{\dot{\xi}_s\Omega}_{\text{Coriolis' coupling}}$$

$$=\frac{\gamma\Omega^2}{8}\left[\theta+\frac{4}{3}\lambda-\frac{\dot{\beta}_s}{\Omega}+\left(2\theta+\frac{4}{3}\lambda\right)\frac{\dot{\xi}_s}{\Omega}\right] \quad \text{where, } \bar{e}=\frac{e}{R}$$

$$\ddot{\xi}_s+\frac{3}{2}\bar{e}\Omega^2\xi_s-2\beta_s\dot{\beta}_s\Omega$$

$$=\frac{\gamma\Omega^2}{8}\left[-(\theta+\frac{8}{3}\lambda)\frac{\dot{\beta}_s}{\Omega}-\left(2\frac{C_{d0}}{C_{l_\alpha}}-\frac{4}{3}\lambda\theta\right)\frac{\dot{\xi}_s}{\Omega}-\frac{C_{d0}}{C_{l_\alpha}}+\frac{4}{3}\lambda\theta+2\lambda^2\right]$$

Linearize about equilibrium (small perturbation)

$$\beta_s = \beta_0 + \beta$$

$$\xi_s = \xi_0 + \xi$$

(β_0 = coning angle)

$$\beta_0 = \frac{\gamma}{8\left(1+\frac{3}{2}\bar{e}\right)}\left[\theta + \frac{4}{3}\lambda\right]$$

$$\xi_0 = \frac{\gamma}{12\bar{e}}\left[-\frac{C_{d0}}{C_{l_\alpha}} + \frac{4}{3}\lambda\theta + 2\lambda^2\right] = \frac{1}{3}\frac{\gamma}{\bar{e}}\left(\frac{2C_Q}{C_{l_\alpha}\sigma}\right)$$

C_Q : rotor torque co-efficient

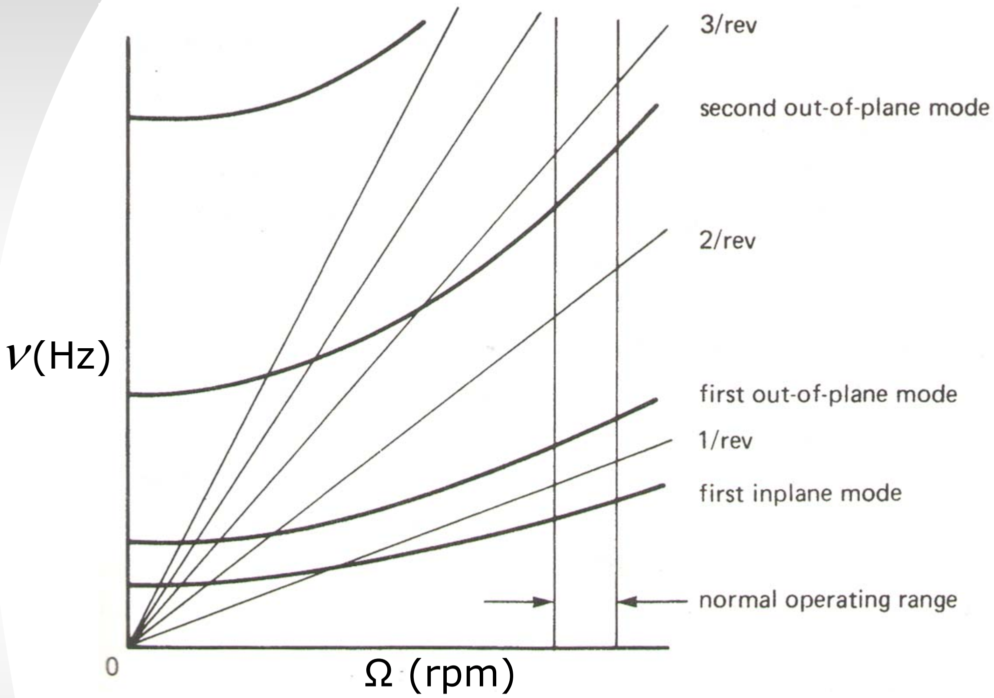
The perturbed equations are

$$\ddot{\beta}_s + \frac{\gamma\,\dot{\beta}}{8} + \left(1 + \frac{3}{2}\bar{e}\right)\beta + \left[2\beta_0 - \frac{\gamma}{8}\left(2\theta + \frac{4}{3}\lambda\right)\dot{\xi}\right] = 0$$

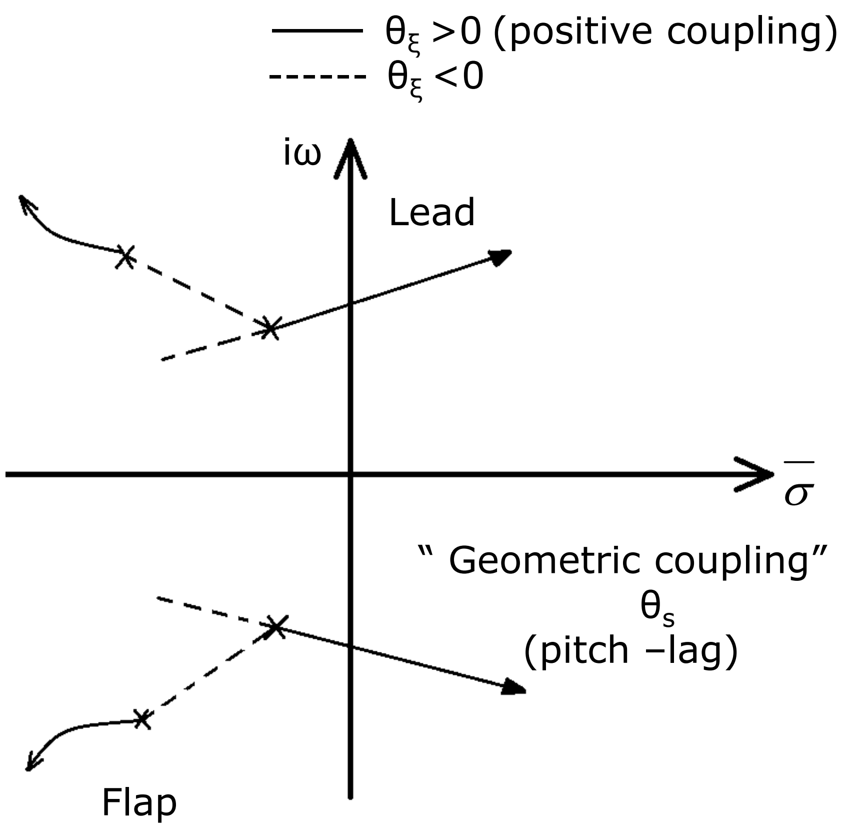
$$\ddot{\xi} + \frac{\gamma}{8}\left(2\frac{C_{d0}}{C_{l_\alpha}} - \frac{4}{3}\lambda\theta\right)\dot{\xi} + \frac{3}{2}\bar{e}\xi + \left[-2\dot{\beta}_0 + \left(\theta + \frac{8}{3}\lambda\right)\right]\dot{\beta} = 0$$

$$\frac{\omega_\beta^{\;2}}{\Omega^2} = 1 + \frac{3}{2}\bar{e}, \qquad \frac{\omega_\xi^{\;2}}{\Omega^2} = \frac{3}{2}\bar{e}, \qquad \bar{e} = \frac{e}{R}$$

Helicopter Aeroelasticity



Fan Diagram



" Geometric coupling"
θ_s
(pitch –lag)

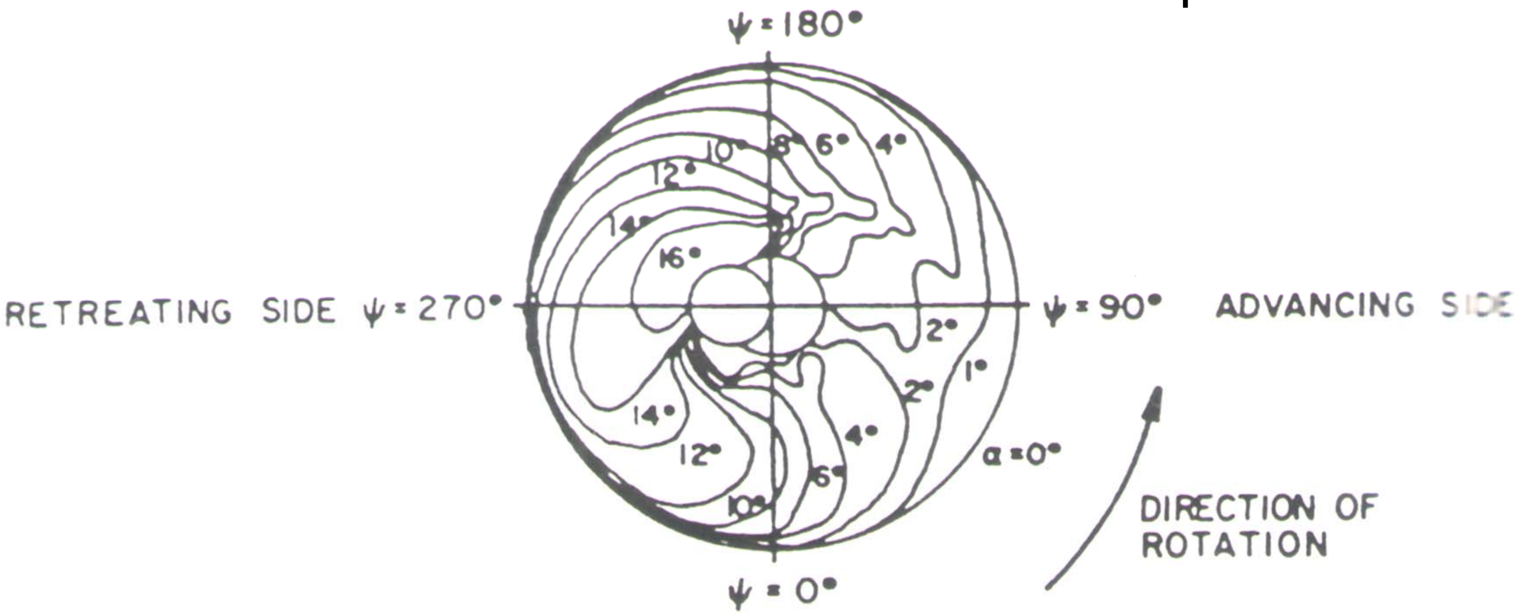
Helicopter Aeroelasticity

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)
  - Single DOF instability in rotorcraft ….. " Stall flutter"
  - primarily associated with high speed flight, maneuvering
  - does not constitute a destructive instability, but rather produces a limit cycle behavior
  - Rotor disc in forward flight ….. AOA on the advancing side is considerably smaller than those on the retreating side (Typical AOA distribution at $\mu = 0.33$ …. Fig 7.13
  - Retreating side ….. large AOA and changes rapidly with azimuth angle airload prediction needs to include unsteady effects

Helicopter Aeroelasticity

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)
  - Since the stalled region is only encountered over a portion of the rotor disk, it will not be continuing unstable motion
  - complexity of the flow field around a stalled airfoil

→ experimental data



A.O.A distribution of helicopter rotor at 140 knots ($\mu = 0.33$)

Helicopter Aeroelasticity

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)
  - Single DOF blade motion assumed

$$\ddot{\alpha} + \omega_\theta^2 \alpha = \left(\frac{\rho (\Omega R)^2 c^2}{2 I_\theta} \right) C_M(\dot{\alpha})$$

→ Energy eqn. multiply by $d\alpha$ and integrating over one cycle

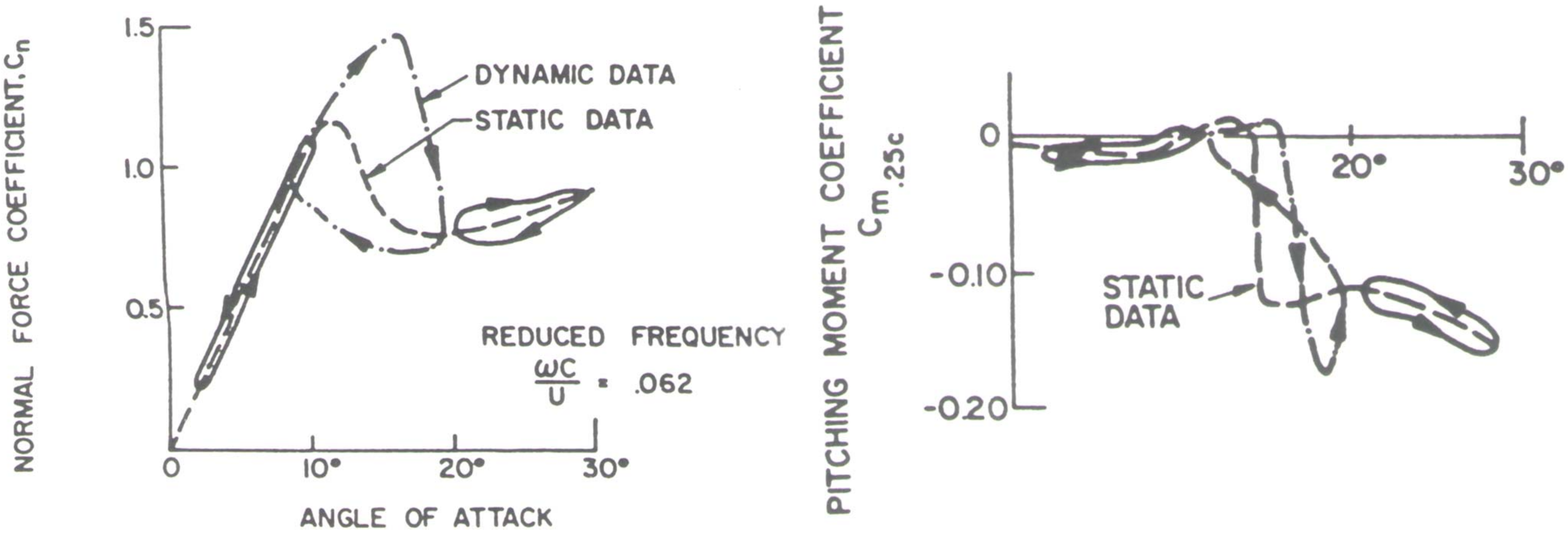
$$\Delta \left\{ \frac{\dot{\alpha}^2}{2} + \omega_\theta^2 \frac{\alpha^2}{2} \right\} = \left(\frac{\rho (\Omega R)^2 c^2}{2 I_\theta} \right) \int C_M(\dot{\alpha})\, d\alpha$$

- Fig. 7.14 ….. time history of the $\left\{ \begin{array}{l} \text{pitching moment coeff.} \\ \text{normal force coeff.} \end{array} \right.$

as a function of AOA for an airfoil oscillating at reduced frequency typical of 1/rev at three mean AOA's.

Helicopter Aeroelasticity

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)



  - large hysteresis loop ….. in the dynamic case when the mean AOA is small

Helicopter Aeroelasticity

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)
  - Pitching moment behaviour ….. in the vicinity of lift stall

    ⟶ average pitching moment increased markedly

    ("moment stall")

    ⟶ time history looks like figure 'eight 8'

  change in energy over one cycle ~ $\int C_M(\dot{\alpha})\,d\alpha$

  - Value of this integral = area enclosed by the loop

    loop is traversed in a counter-clockwise direction …..

    integral is negative, dissipation, positive damping.

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)

  Moment stall occurs statically … no net dissipation of energy

  or energy is being fed into the structure (integral is positive)

  → "stall flutter"

  -loss of damping at stall

  -marked change in average

  pitching moment

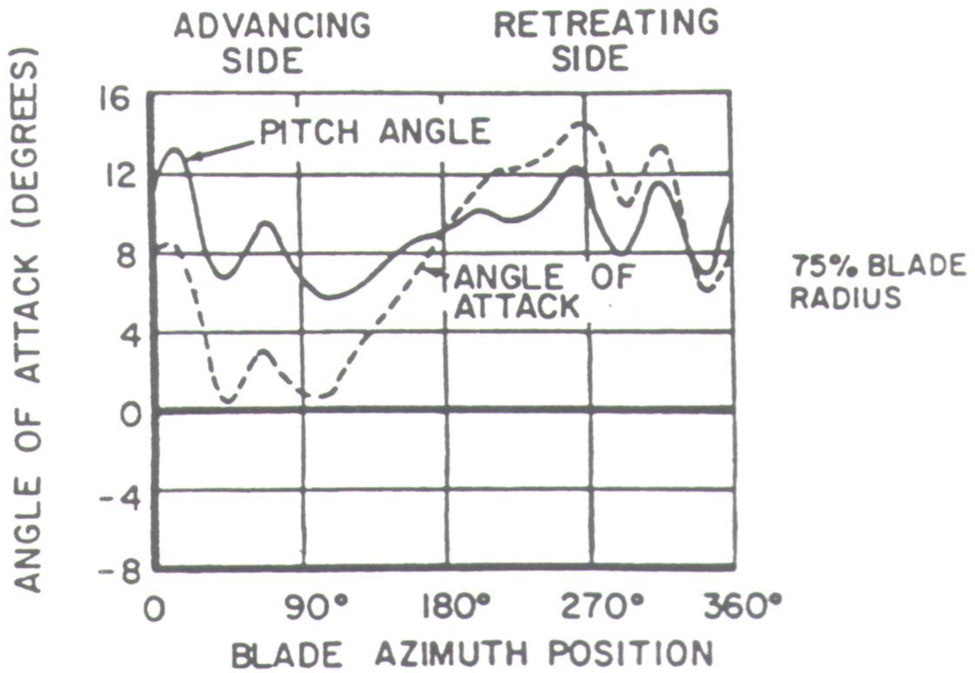
  large torsional motion , 2.3 cycles of the torsion

Helicopter Aeroelasticity

- **Stall flutter in rotorcraft aeroelasticity** (Dowell Sec. 7.2)

  Fig. 7.15 ….. Typical time history of blade torsional motion when stall flutter is encountered.

  → marked increase in the vibratory loads in the blade pitch control system



Helicopter Aeroelasticity

- **Aeromechanical instability in rotorcraft** (Dowell Sec. 7.3)

Fig. 7.28 ….. Typical Coleman plot of the rotor-fuselage coupling. Will induce ground/air resonance.