Design of Piezoelectric Active Structures

Lecture 5:

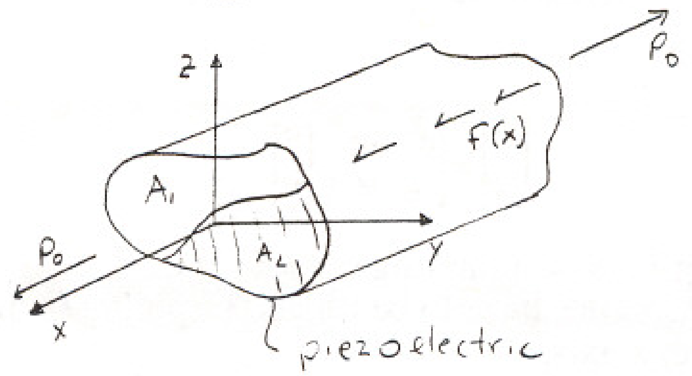
Examples of Actuated Structures

Outline

- Extension of Rods
- Bending Beams
- Transverse Shear Model
- Torsion of Beams
- Anisotropic Plates
- Shells

Extension of Rods - I

- Consider inhomogeneous rod



- Let piezo be poled in x direction. "Longitudinal case"
- Strain Displacement: axial extension

$$u = u_0 (x)$$

$$S_1 = \frac{du_0}{dx}$$

$$E_1 = \frac{v_0}{\ell} \cdot x$$

> all others zero ? may allow poisson contraction $S_2 = S_3 = \nu S_1$

- Stress Resultants (interior)

$$P = \int_A T_1 \; dA = P_1 + P_2$$

Note: equilibrium says $P_o = P$

Extension of Rods II

- Constitutive Relations

  full blown

$$\begin{bmatrix} T \\ D \end{bmatrix} = \begin{bmatrix} c^E & -e_t \\ e & \varepsilon^S \end{bmatrix} \begin{bmatrix} S \\ E \end{bmatrix}$$

  assuming $S_1 \neq 0$, assuming only $E_1 \neq 0$
  Note: properties have to be rotated so poling (3 direction) lines up with x axis.

- Energy Principle

$$\int_\ell \int_A \{-(T\,\delta S) + D\,\delta E)\}\, dA\, dx + P_o \delta u(x=\ell)$$
$$+ \int_\ell f_x\, \delta u\, dx = 0$$

  since only S_1 and E_1 exist, this an be simplified. Also: material properties are different in each section so

$$\int_A (\cdot)\, da = \int_{A_1} da + \int_{A_2} da$$

  plugging in assuming A_1 = structure, A_2 = piezo

$$\int_\ell \Big[\int_A (-c_{11}\, S_1\, \delta S_1\,) da + \int_{A_2} \{ -(c^E_{33}\, S_1 - \rho_{33}\, E_1)\, \delta S_1 + (\rho_{33}\, S_1 + \varepsilon^S_{33}\, E_1)\, \delta E_1 \}\, da +$$
$$+ f_x\, du\Big]\, dx + P_o\, \delta u\, (x=\ell) = 0$$

Extension of Rods - III

- Integrating over cross section

  (introduce section properties)

$$\bar{c} = A_1 c_{11} + A_2 c_{33}^E$$
$$\bar{e} = A_2 e_{33}$$
$$\bar{\varepsilon} = A_2 \varepsilon_{33}^3$$

get

$$\int_\ell [-(\bar{c}\, S_1 - \bar{e}\, E_1)\, \delta S_1 + (\bar{e}\, S_1 + \bar{\varepsilon}\, E_1)\, \delta E_1 + f_x\, \delta u] dx$$
$$+ P_o\, \delta u\, (x{=}\ell) = 0$$

- 2 options:

  1) continuous - $u(x)$ $\Rightarrow$ differential equation for rod

  2) discrete - assume

$$u(x) = \frac{u_o\, x}{\ell} \quad \Rightarrow \delta S_1 = \frac{\delta u_o}{\ell}$$
$$\varphi = \frac{v_o\, x}{\ell} \quad \Rightarrow \delta E_1 = \frac{\delta v_o}{\ell}$$

Discrete Representation

$$\int_\ell \left\{ -\left(\bar{c}\frac{u_o}{\ell} - \bar{e}\frac{v_o}{\ell}\right)\frac{\delta u_0}{\ell} + \left(\bar{e}\frac{u_o}{\ell} + \bar{\varepsilon}\frac{v_o}{\ell}\right)\frac{\delta v_o}{\ell} + f_x \frac{\delta u_0}{\ell} x \right\} dx + P_0\,\delta u_0 = 0$$

arbitrary variations

you get

$$\delta u_o : -\frac{\bar{c}}{\ell} u_o + \bar{e}\frac{v_o}{\ell} + \int_x f_x \cdot \frac{x}{\ell}\, dx + P_0 = 0$$ Actuator Eqn.

$$\delta v_o : \frac{\bar{e}}{\ell} u_o + \frac{\bar{\varepsilon}}{\ell} v_o = 0$$ Sensor Eqn

or in matrix form

total stiffness

$$\begin{bmatrix} \bar{c} & -\bar{e} \\ \bar{e} & \bar{\varepsilon} \end{bmatrix} \begin{bmatrix} \frac{u}{\ell} \\ \frac{v}{\ell} \end{bmatrix} = \begin{bmatrix} P_o + \int_x f_x \cdot \frac{x}{\ell}\, dx \\ 0 \end{bmatrix}$$

(no applied charge

- Free Expansion

$$\frac{u}{\ell} = S_o = \frac{\bar{e}}{\bar{c}}\frac{v}{\ell} = \underbrace{\left(\frac{A_2}{A_1 c_{11} + A_2 c_{33}}\right)}_{\text{stiffness cut down}} \underbrace{e_{33} E_o}_{\text{induced stress}}$$