5.2 THEORETICAL METHODS OF ANALYSIS

5.2.1 Load-Transfer Method

This method, proposed by Coyle and Reese (1966), utilizes soil data measured from field tests on instrumented piles and laboratory tests on model piles. The relevant soil data required in this method are curves relating the ratio of the adhesion (or load transfer) and the soil shear strength to the pile movement. Such curves were first developed by Seed and Reese (1957), and a typical relationship is shown in Fig. 5.1. In actual problems, a number of such relationships may be required to describe the load transfer along the whole length of the pile.

The load-transfer method may be summarized as follows:

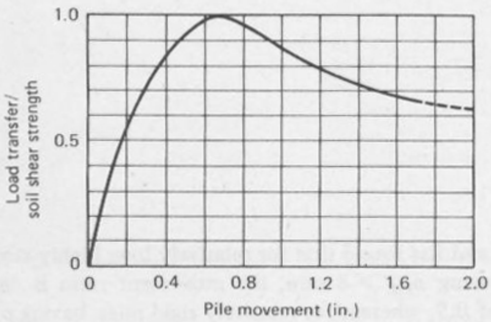

FIGURE 5.1*a* Typical shear stress vs. pile movement curve (after Coyle and Reese, 1966).

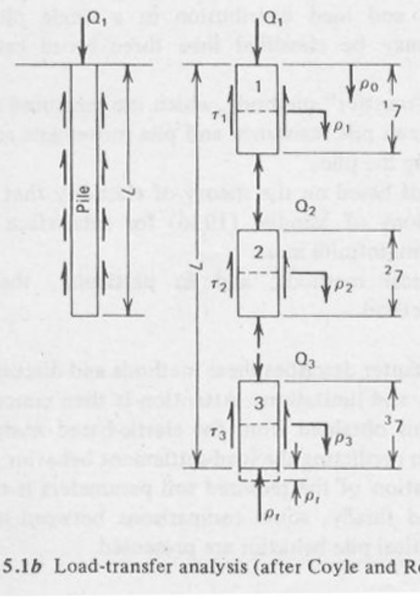

FIGURE 5.1*b* Load-transfer analysis (after Coyle and Reese, 1966)

1. The pile is divided into a number of segments (for simplicity, three segments are shown in Fig. 5.1*b*).
2. A small tip movement, ρ_t, is assumed (zero may be selected, but generally the tip undergoes some movement, except for end-bearing piles on rock).
3. The point resistance, P_t, caused by this movement is calculated. This may be done approximately by assuming the pile tip to be a rigid circular area and employing the Boussinesq theory:

$$P_t = \frac{2dE\rho_t}{(1-\nu^2)} \tag{5.2}$$

where

E, ν are the average deformation parameters of the material beneath the tip, estimated from triaxial tests or other data

4. A movement, ρ_3, in the bottom segment at midheight is assumed (for the first trial, assume $\rho_3 = \rho_t$).
5. Using the estimated ρ_3, the appropriate curve of load-transfer/soil-shear-strength versus pile movement (e.g., as in Fig. 5.1) is used to find the appropriate ratio.
6. From a curve of shear-strength versus depth, the strength of the soil at the depth of the segment is obtained.
7. The load transfer or adhesion is then calculated as τ_a = (ratio × shear strength). The load Q_3 on the top of segment 3 can then be calculated as

$$Q_3 = P_t + \tau_a L_3 P_3 \tag{5.3}$$

where

L_3 = length of segment 3
P_3 = average perimeter of segment 3

8. The elastic deformation at the midpoint of the pile segment (assuming a linear variation of load in the segment) is calculated as

$$\Delta'\rho_3 = \left(\frac{Q_m + P_t}{2}\right)\left(\frac{L_3}{2A_3E_p}\right) \tag{5.4}$$

where

$$Q_m = \frac{Q_3 + P_t}{2}$$

A_3 = area of segment 3
E_p = pile modulus

9. The new midheight movement is then given by

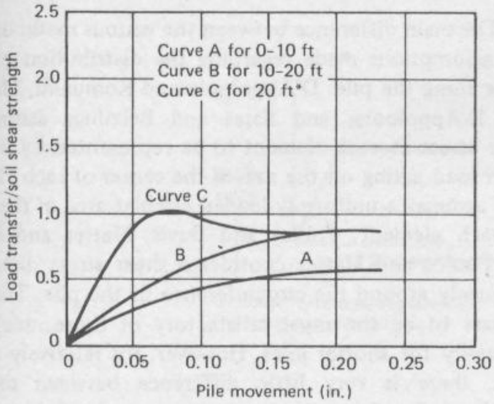

FIGURE 5.2 Design load-transfer curves for pipe piles in clay (Coyle and Reese, 1966).

$$\rho_3' = \rho_t + \Delta\rho_3' \tag{5.5}$$

10. ρ_3' is compared with the estimated value of ρ_3 from step (4).
11. If the computed movement ρ_3' does not agree with ρ_3 within a specified tolerance, steps (2) to (10) are repeated and a new midpoint movement calculated.
12. When convergence is achieved, the next segment up is considered, and so on, until a value of load (Q_0) and displacement (ρ_0) for the top of the pile are obtained.

The procedure is then repeated using different assumed tip movements until a series of values of Q_0 and ρ_0 are obtained. These values can then be used to plot a computed load-settlement curve.

On the basis of field data on instrumented piles and laboratory tests on model piles, Coyle and Reese derived a series of three average curves of load transfer, shear-strength versus pile-movement curves for various depths, which are shown in Fig. 5.2. The interpretation of the tests on instrumented piles to obtain these curves is described in detail by Coyle and Reese (1966). The curves in Fig. 5.2 are limited to the case of steel-pipe friction piles in a clay soil with an embedded depth not exceeding 100 ft, and the soil shear strengths in these curves have been obtained from unconfined compression tests.

From a series of tests on instrumented pile in sand, Coyle and Sulaiman (1967) have presented data on the load-transfer-versus-movement characteristics for steel piles in saturated sand, a summary of which is shown in Fig. 5.3. This data suggests that for depths of 0 to 20 ft, curve *A*, with an upper limit of skin friction of twice the shear strength, can be used (considerably higher values were obtained at shallow depths). For depths greater than 20 ft, the measured relationships approach curve *B*, with an upper limit of skin friction of 0.5 times the shear strength.

Reese et al. (1969) carried out load tests to study the load transfer along bored piles in clay. On the basis of a curve-fitting analysis of these test results, the following relationship between load transfer (adhesion) and pile movement was developed:

$$\tau_{az} = \tau_{a\max}\left[2.0\sqrt{\frac{\rho}{s_0}} - \left(\frac{\rho}{s_0}\right)\right] \tag{5.6}$$

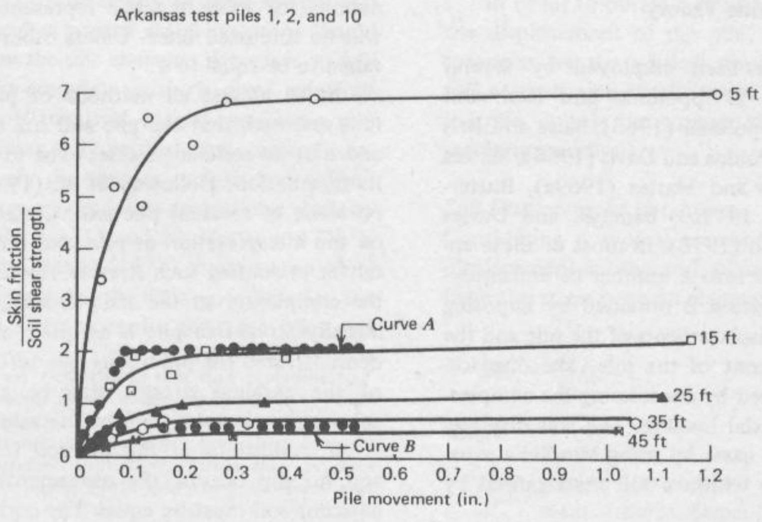

FIGURE 5.3 Design load-transfer curves for piles in sand (after Coyle and Sulaiman, 1967).

where

τ_{az} = adhesion at depth z (tons/ft^2)
$\tau_{a\max}$ = maximum adhesion that can occur at any depth (tons/ft^2)
ρ = downward movement of pile at depth z (in.)
s_0 = $2d\epsilon$ (in.)
d = pile diameter
ϵ = average failure strain, in percent, obtained from stress-strain curves for unconfined compression tests run on soil samples near the pile tip

Although the load-transfer method has gained quite wide acceptance, the following theoretical and practical limitations should be recognized:

(a) In using the load-transfer curves, it is inherently assumed that the movement of the pile at any point is related only to the shear stress at that point and is independent of the stresses elsewhere on the pile. This inherent assumption is equivalent to that made when the theory of subgrade reaction is used to analyze laterally loaded piles. Thus, no proper account is taken of the continuity of the soil mass.
(b) The load-transfer method, because of its inherent disregard for continuity of the soil, is not suitable for analyzing load-settlement characteristics of pile groups.
(c) In order to obtain load-transfer curves at a site, considerably more instrumentation is required on a pile than for a normal pile-load test. Extrapolation of test data from one site to another may not always be entirely successful.

5.2.2. Analysis Based on Elastic Theory

Elastic-based analyses have been employed by several investigators: for example, D'Appolonia and Romualdi (1963), Thurman and D'Appolonia (1965), Salas and Belzunce (1965), Nair (1967), Poulos and Davis (1968), Mattes and Poulos (1969), Poulos and Mattes (1969a), Butterfield and Banerjee (1971*a*, 1971*b*), Banerjee and Davies (1977), Randolph and Wroth (1978). In most of these approaches, the pile is divided into a number of uniformly-loaded elements, and a solution is obtained by imposing compatibility between the displacements of the pile and the adjacent soil for each element of the pile. The displacements of the pile are obtained by considering the compressibility of the pile under axial loading. The soil displacements are obtained in most cases by using Mindlin's equations for the displacements within a soil mass caused by loading within the mass.

The main difference between the various methods lies in the assumptions made regarding the distribution of shear stress along the pile. D'Appolonia and Romualdi, Thurman and D'Appolonia, and Salas and Belzunce assume the shear stress at each element to be represented by a single-point load acting on the axis at the center of each element. Nair assumes a uniformly-loaded circular area at the center of each element. Poulos and Davis, Mattes and Poulos, and Poulos and Mattes consider a shear stress distributed uniformly around the circumference of the pile. The latter appears to be the most satisfactory of those mentioned, especially for shorter piles. However, for relatively slender piles, there is very little difference between solutions based on the three above representations of shear stress. In the derivations described below, the method of Poulos and Davis (1968), among others, is followed. The basic problems of a floating or friction pile in a semi-infinite mass and an end-bearing pile are considered in detail and modifications to these analyses are described.

5.2.2.1 BASIC ANALYSIS FOR SINGLE FLOATING PILE

The pile is considered to be a cylinder, of length L, shaft diameter d, and base diameter d_b, and loaded with an axial force P at the ground surface. For the purposes of the analysis, the pile is acted upon by a system of uniform vertical shear stresses p around the periphery, and the base is acted upon by a uniform vertical stress p_b, as shown in Fig. 5.4. The sides of the pile are assumed to be rough. The soil is initially considered to be an ideal homogeneous isotropic elastic half-space, having elastic parameters E_s and ν_s that are not influenced by the presence of the pile. Modifications for more realistic representation of soil behavior will be discussed later. Unless otherwise stated, d_b will be taken to be equal to d.

As in almost all methods of pile-settlement analysis, it is assumed, that the pile and soil are initially stress-free, and that no residual stresses exist in the pile resulting from its installation. Holloway et al. (1975) emphasize the importance of residual pile-soil stresses on pile behavior and on the interpretation of pile-load tests, and suggest a method for evaluating such stresses. However, in order to reduce the complexity of the analysis here, the assumption of an initially stress-free pile is adopted; as subsequently will be demonstrated for predicting pile settlements, the influence of the residual stresses may be adequately taken into account by choosing appropriate values of the soil modulus.

If conditions at the pile-soil interface remain elastic and no slip occurs, the movements of the pile and the adjacent soil must be equal. The correct values of the stress