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Mechanics of Bond and Slip of Deformed Bars in Concrete

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The action of bonding forces, and the associated slip and cracking are examined for bars with various surface properties. The mechanics of slip of deformed bars in concrete is discussed, with the support of experimental data. The stresses and deformations in the concrete, caused by the bonding forces, are presented. The ACI Building Code shear stress requirements are studied in terms of the corresponding limiting bond stresses.

Keywords: beams (structural); bond (concrete to reinforcement); crack width and spacing; cracking (fracturing); deformation; deformed reinforcement; reinforced concrete; reinforcing steel; research; shear; shear stress; slippage; stresses.

■ THE PURPOSE OF THIS PAPER is to explain some basic aspects of the action of bonding forces and the associated slip and cracking. A section on the interaction of bond and shear in beams is also included. Subsequent papers will report the experimental and analytical studies that formed the basis of this paper.

Bond can be thought of as the shearing stress or force between a bar and the surrounding concrete. The force in the bar is transmitted to the concrete by bond, or vice versa. Bond is made up of three components:

(a) Chemical adhesion
(b) Friction
(c) Mechanical interaction between concrete and steel.

Bond of plain bars depends primarily on the first two elements, although there is some mechanical interlocking due to the roughness of the bar surface. Deformed bars, however, depend primarily on mechanical interlocking for superior bond properties. This does not mean friction and chemical adhesion are negligible in case of deformed bars, but that they are secondary.

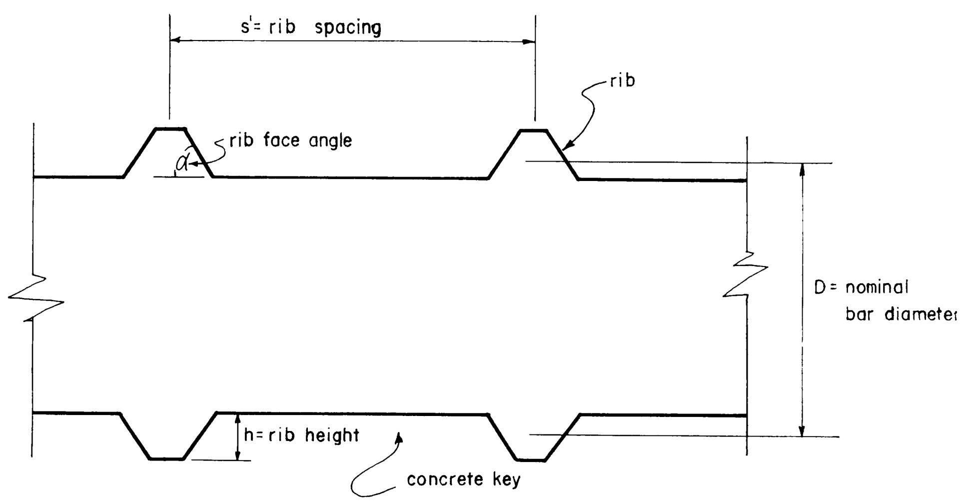


Fig. 1—Geometry of bar deformations

Deformed bars have often been considered simply as plain bars with superior bond properties. However, the fundamental differences in the action of deformed bars as compared with plain bars require that the deformed bars be treated separately. This paper discusses the nature of bond only for deformed bars having ribs oriented transversely to the bar axis.

THE FUNDAMENTALS OF BOND AND SLIP

To obtain a clear understanding of the bond of a deformed bar in concrete, the bond forces, slip, and cracking of the concrete in the vicinity of the bar will be examined. Unless the strain of the concrete and steel is the same and constant over a length, a deformed bar attempts to move or slip in relation to the surrounding concrete. Initially, chemical adhesion combined with mechanical interaction prevents slip. After adhesion is destroyed, and slip occurs, the ribs of a bar restrain this movement by bearing against the concrete between the ribs; this concrete will be called the concrete key (Fig. 1). Friction which would occur after the slip of plain bars does not occur here because of the presence of the ribs.

Slip of a deformed bar can occur in two ways: (1) the ribs can push the concrete away from the bar (wedging action), and (2) the ribs can crush the concrete. Pullout tests by Rehm[1] and Lutz[2] on bars with only a single concrete key (Fig. 2) indicated that the movement of the ribs is about the same for all ribs with face angles greater than about 40 deg (the face angle of ribs is measured with respect to the bar axis, as shown in Fig. 1). These tests included specimens with ribs having face angles of 90 deg (and even 105 deg) in which case the rib cannot push the concrete outward (no wedging action). The friction between the rib face and the concrete is evidently sufficient to prevent relative movement at the interface when face angles are larger than about 40 to 45 deg. It follows that slip is due almost entirely to the crushing of the porous concrete paste (mortar) in front of the ribs if the rib face angles are larger than about 40 deg. It is understandable that crushing of the concrete does occur, since the average bearing pressure exerted by these ribs is very large.

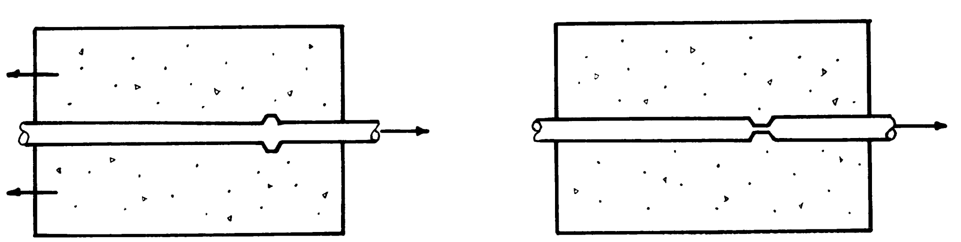
Bars with ribs having face angles less than about 30 deg exhibit markedly different load-slip relationship (Fig. 3). Here the friction between the rib face and the concrete is not sufficient to prevent relative movement. Thus, slip is due primarily to the relative movement between the concrete and steel along the face of the rib, and secondarily to some crushing of the mortar.

Slip caused by relative movement (in addition to that caused by crushing) also occurs when the frictional properties of the rib face are reduced by grease. That is, bars with ribs having face angles greater than about 45 deg but with friction impaired exhibit load-slip curves which are similar to that for a rib with flat rib face angle.[2] Recent tests by Lutz indicate that the extent to which slip properties are affected by grease depends on the face angle; ribs with flatter face angles are understandably more affected by poor frictional properties.

For the usual case of good frictional properties and a rib face angle greater than 40 deg, slip occurs by progressive crushing of the porous concrete paste structure in front of the rib. This does not appear to produce significant wedging action until considerable crushing has occurred, at which time a wedge of crushed concrete (compacted powder) becomes lodged in front of the rib and moves along with it. This, in effect, produces a rib with a face angle of 30 to 40 deg. Such wedges of compacted concrete powder have been observed in front of ribs that had carried high bond forces; Rehm[1] states that crushing extends in front of the rib for a length of 5 to 7 times the height of the rib, although the compacted powder that moves with the rib extends at most 2 times the height of the rib.

It is interesting to note (Fig. 3) that the slip resistance (slope of bond-slip curve) upon reloading is considerably higher than the slip resistance found initially. This was observed by Rehm[1] and in recent unpublished tests by Lutz. The reason for this is that during the second loading the rib bears against a compacted nonporous crushed concrete as compared with the porous intact concrete during the initial loading.

It should be pointed out here that it is not possible to produce extensive crushing and wedge



(a) (b)

Fig. 2—(a) Single-rib tests by Lutz; (b) single concrete key test by Rehm

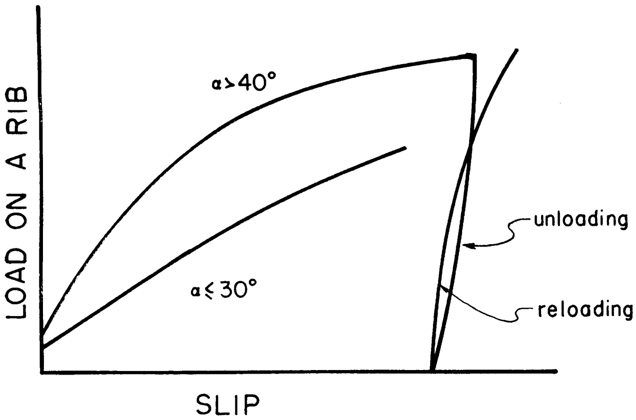


Fig. 3—Sketch of load-slip curves

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action in front of every rib without having transverse and longitudinal cracking, caused by the forces produced by the ribs bearing on the concrete.

BOND IN A BEAM

The behavior of a beam will be examined numerically at various stages of the loading and cracking in order to obtain a clear idea of the bond forces that exist in a beam as well as to develop an insight into how cracking affects the bond stress and the stress distribution in the concrete. The beam used is shown in Fig. 4.

Bond before cracking and slip

Initially the bar adheres chemically to the concrete. This adhesion acts until slip or movement of the steel relative to the adjacent concrete occurs. It is difficult to measure chemical adhesion strength since any test designed to measure the load necessary to cause slip also measures the apparent adhesion caused by the roughness (actually mechanical interlocking) of the bar surface. Chemical adhesion can best be measured by loading the adhered surfaces in tension. Representative values of the adhesion properties of the mortar or cement paste to the bar may be estimated from adhesion tests of nonreactive aggregates to mortar. From shear tests[3] adhesion strength of 280 to 600 psi was obtained, while tensile bond tests[4] yielded 190 to 240 psi for saturated specimens. dropping to about 50 psi for saturated-surface-dry specimens. From these results it appears much more likely that adhesion will be lost in tension rather than in shear. The existence of substantial tension stresses normal to the bar surface will be examined in the following sections.

Before slip and cracking occur, tension can exist on the interface between the bar and the concrete as a result of the larger Poisson's ratio of the steel as compared with that of concrete. An elastic finite-element analysis[2] revealed that the interfacial tensile stress is about 11 percent of the tensile stress in the concrete parallel to a 1 in. diameter bar when both the concrete and the steel are strained the same amount in the direction of the bar (for Poisson's ratios of 0.30 and 0.15 for the steel and concrete, respectively). However, the shrinkage of the concrete on drying will cause compressive stresses between the steel and concrete. A free shrinkage strain of 0.0003 applied to the specimen mentioned above, produces a compressive interfacial stress of about 80 psi. This is more than enough to keep the stresses compressive between the concrete and steel prior to cracking of the concrete.

The bond stresses in the beam of Fig. 4 before the first cracking occurs will now be examined. Using $f_c' = 4000$ psi and a modular ratio $n = 8$, an elastic analysis of the uncracked section gives the position of the neutral axis as 6.47 in. from the compression face of the beam. Due to the loading, no bond stresses exist between the concrete and the reinforcing bar in the middle third of the beam. In the outer thirds of the beam, where constant shear force and a corresponding

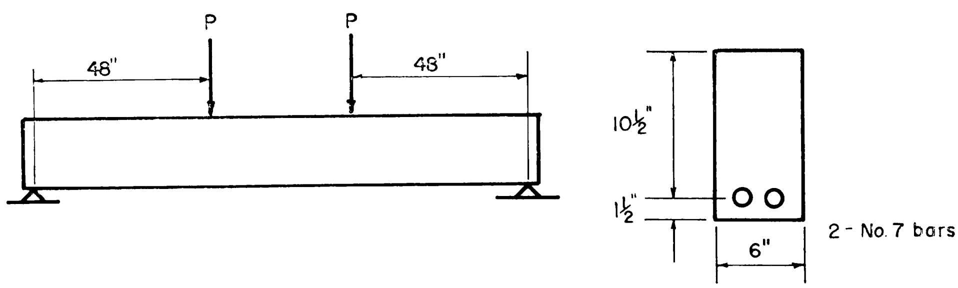

Fig. 4—Beam dimensions for illustrative example

change in moment exist, the steel and the concrete elongate the same amount over any given length, but due to the difference in the elastic moduli of the two materials, a larger change in stress occurs in the steel than in the concrete over a unit length, leading to a bond stress of:

$$u = 0.0065V$$

where V is the shear force at the point considered.

First cracking occurs at a concrete stress of about $7.5\sqrt{f_c'}$, which is caused by third point loadings of 1820 lb. Therefore, the constant bond stress in the outer thirds of the beam just before the first flexural cracking occurs is:

$$u = 11.8 \text{ psi}$$

At this low bond stress it is unlikely that any slip will occur between the reinforcing bar and the concrete.

Bond after formation of flexural cracks

If a crack forms in the constant moment region of the beam under third point loads of 1820 lb the neutral axis moves from 6.47 to 4.41 in. at the crack (measured from the compression face). The steel stress increases from 2770 psi before cracking to 8050 psi at the crack, while the maximum elastic compressive concrete stress changes from 555 to 720 psi. Hence, the steel stress varies from 8050 psi at the crack to about 2770 psi away from the crack in the constant moment region of the beam where the shear force is zero. (The steel stress due to shrinkage is not considered here.) The corresponding bond stress distribution is sketched in Fig. 5. This type of bond stress distribution was measured by Mains.[5]

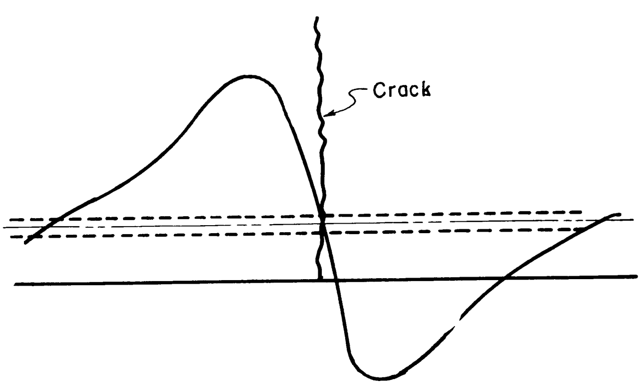

Fig. 5—Bond stress distribution in constant moment region

The stress condition in the vicinity of the crack is very complex. The tensile stress that existed in the concrete at the section of crack in the direction of the bar has disappeared. This unloading of tensile concrete allows the crack to open at the surface of the beam. What is the crack width at the surface of the bar? Prior to cracking, perfect bond existed between the steel and the concrete; if perfect bond still exists after cracking, the crack width at the bar surface would be zero.

An elastic finite element analysis was made of the stresses that exist in the concrete next to a bar at a crack by evaluating the stresses near the end of cylinder of finite length containing a concentrically embedded bar[2] (Fig. 6). Assuming that perfect bond does exist between the reinforcing bar and the concrete, an interfacial tensile stress (normal to the bar surface) of 31 percent of the steel stress was found to exist between the steel and the concrete at the crack. For the beam in Fig. 4, an interfacial tensile stress of about 2500 psi would exist at the cracking load. Since this stress far exceeds the tensile adhesion strength (even when considering the compression due to shrinkage), separation of the bar and the concrete will occur in the vicinity of the crack.

In the case of plain bars this separation would mean complete loss of bond in the region next to the crack. The concrete stress at the bar surface and parallel to the bar would disappear, and the crack width at the bar would be essentially the same as the width at the surface of the concrete. This was found experimentally by Mathey and

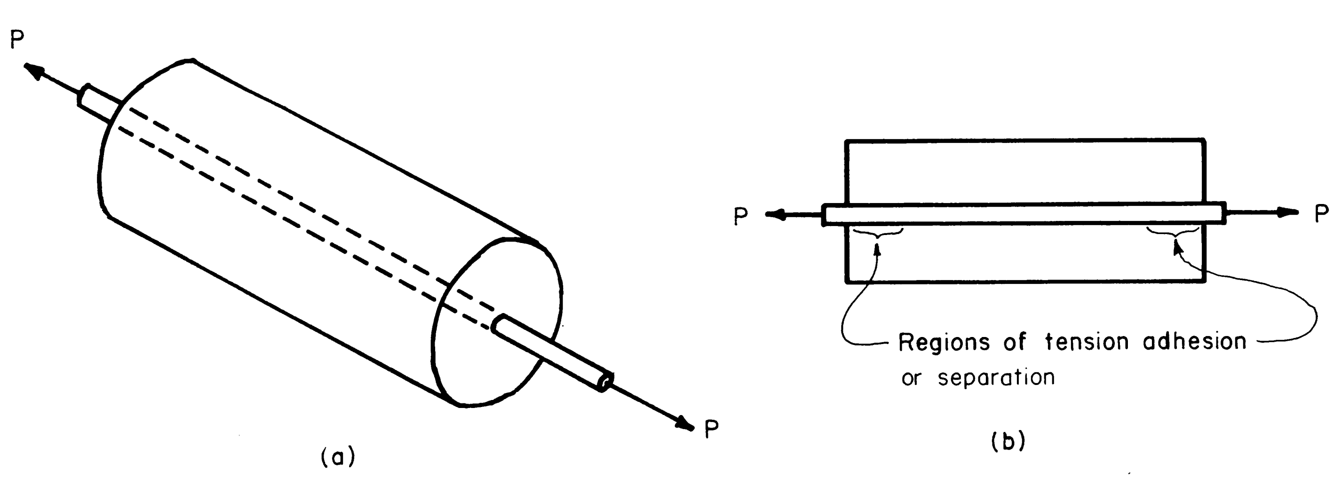

Fig. 6—Sketch of pullout specimen for experimental and analytical studies

Watstein[6] on tensile bond specimens similar to the one shown on Fig. 6.

However, when the reinforcing bar has transverse ribs, separation does not produce complete unloading of the concrete adjacent to the bar inasmuch as the bar ribs prevent much of the opening of the crack at the bar (Fig. 7a). Some unloading does occur, allowing the crack to open at the surface of the bar. This opening (or slip) is caused partly by the unloading of the concrete between the crack and the nearest bar rib when the crack forms, producing a relative contraction of the concrete, and partly by the bearing deformation (crushing) under the ribs. Another factor contributing to the opening is the inclination of bar ribs, since the separation of the bar and the concrete is accompanied by a movement along the inclined bar rib face. The bond near a transverse crack is transferred solely by bearing of the concrete against the face of the ribs, as contact between the cylindrical surfaces no longer exists.

With a moderate increase in the load above the cracking load, additional external cracks form in the constant moment region and in the shear zone. (The formation of internal cracks will be discussed later.) At each of these surface cracks the steel stress will reach a local peak; between cracks the steel stress is lower as part of the tension force is carried by the concrete. The transfer of forces produces bond stresses. The stresses and deformations of the concrete between two of these surface cracks will be examined next, including the effects of slip at the cracks.

An elastic finite element analysis was made of a cylinder of concrete with a concentrically embedded reinforcing bar, as shown in Fig. 6a. One

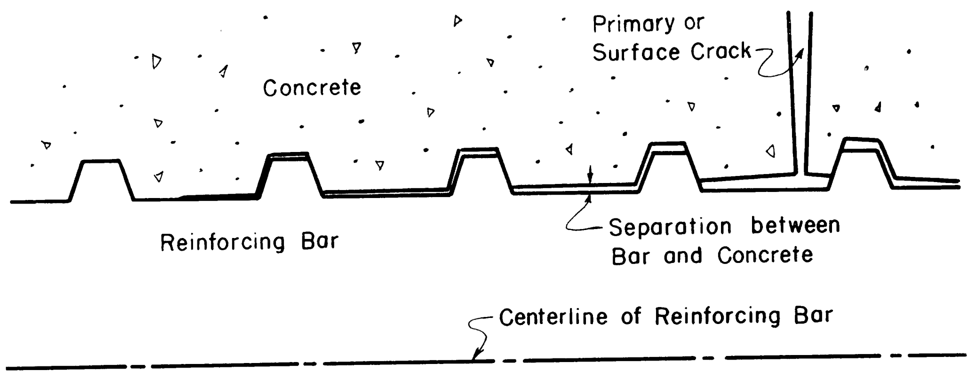

Fig. 7a—Section through reinforcing bar and concrete showing separation that occurs near a primary crack

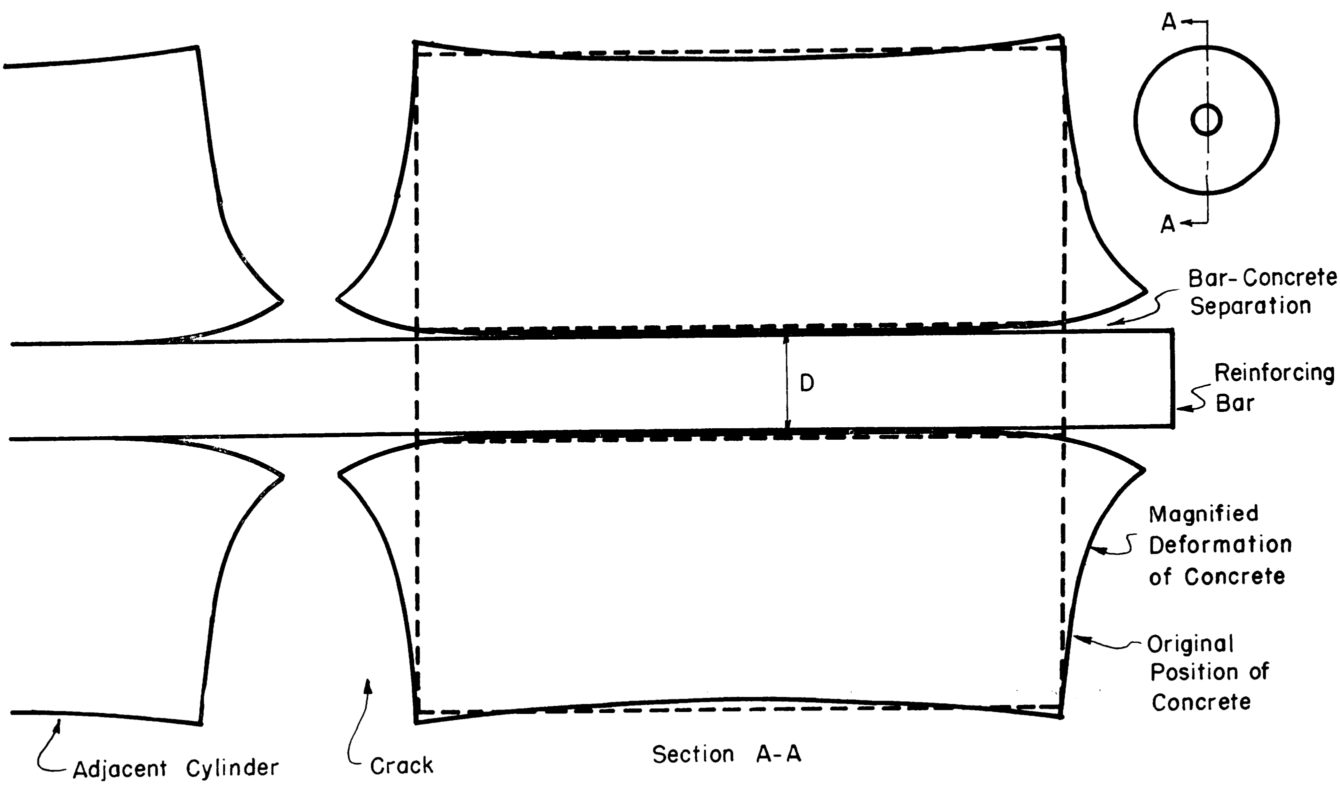

Fig. 7b—Magnified deformation of *6Dx6D* diameter cylinder with separation over 1.5*D* from each crack: from a finite-element analysis

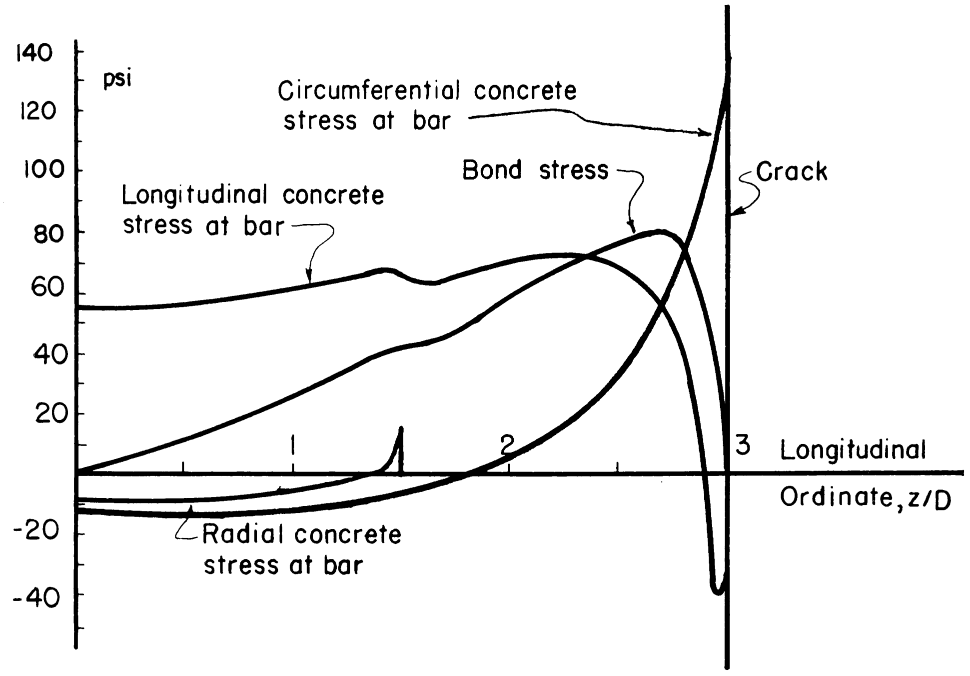

Fig. 8—Longitudinal variation of stresses in *6Dx6D* diameter cylinder with separation, at $f_s = 1000$ psi

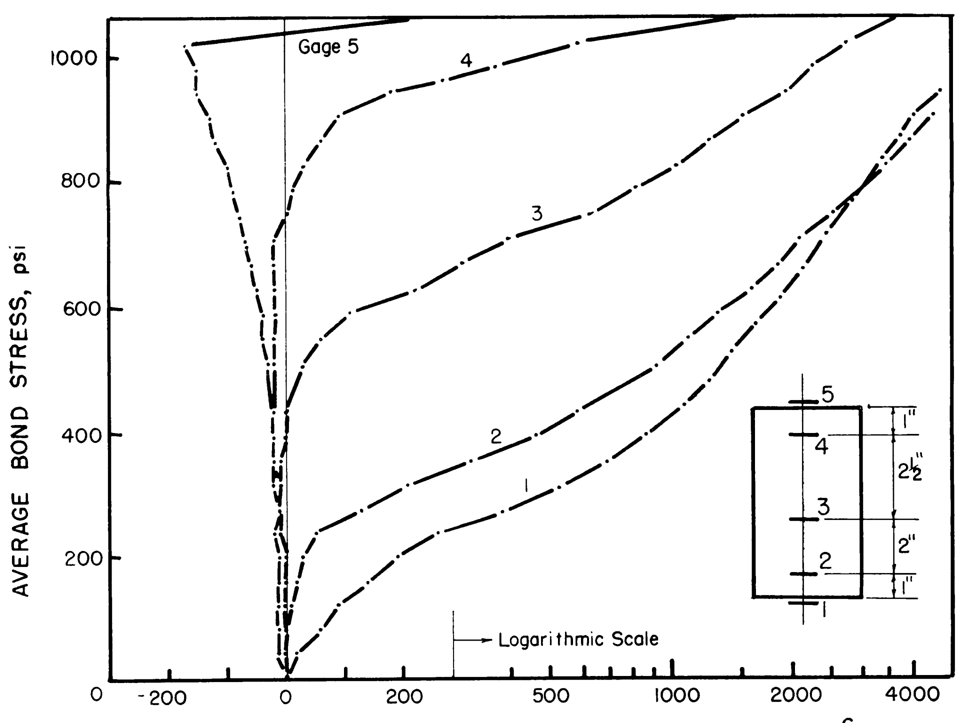

Fig. 9—Transverse strains on concrete surface of eccentric pullout specimen

of the cases analyzed was a 6 in. long cylinder (representing a 6 in. crack spacing) with a diameter of 6 in. and a 1 in. diameter reinforcing bar (producing a 2.5 in. cover). In this case it was assumed that separation has occurred between the steel and the concrete for a distance of 1.5 in. from each end of the cylinder (Fig. 6b). In the regions where separation was assumed, only bond stresses, transmitted by the ribs, were allowed to act. Normal stresses were not permitted in order to represent the separation of the cylindrical surfaces of the bar and the surrounding concrete. Slip resulted from the inclination of the rib (the rib face angle was assumed to be 59 deg) and from the separation of the steel and concrete; i.e., the concrete bearing on the rib face slipped relative to the rib as separation occurred, shown in Fig. 7a. Loading of the reinforcing bar caused the cylinder to deform into the shape shown in Fig. 7b. The bond stresses resulting from this analysis are shown in Fig. 8, along with the longitudinal, radial and circumferential stresses that occurred in the concrete adjacent to the bar.

The results of the finite element analysis of this model, which represents quite closely the situation existing between two flexural cracks near a reinforcing bar in a reinforced concrete member, indicate: (1) a large variation in crack width from the bar to the concrete surface (Fig. 7b); (2) slip due to the rib inclinations amounting to about one-quarter of the elongation of the steel between the two primary flexural cracks; (3) separation of the bar from the concrete, equal to about 0.3 times the elongation of the steel between cracks; and (4) negligible deformation of the concrete at the outside surface. (Very small longitudinal tensile and compressive stresses were found at the surface in the analysis, which agrees with the experimental results of Clark,[7] Mathey and Watstein,[5] and Broms.[8])

The circumferential concrete tensile stresses at the crack cause the circumferential expansion of the concrete, as shown in Fig. 7b. The concrete bends away from the bar. Similar behavior was measured in eccentric pullout tests[2] as illustrated by the surface strains plotted in Fig. 9. Large circumferential tensile strains and subsequent longitudinal cracking is indicated by Gages 1 and 2 (Gages 1 and 5 were located on the end faces, 1 in. from the bar and ¾ in. from the edges) while Gage 4 at low loads and Gage 5 show compression. The circumferential stresses shown in Fig. 8 and the formation of splitting cracks as indicated by the strains in Fig. 9 suggest that splitting should occur as flexural cracks form. Yet there is little or no visual indication of the existence of splitting cracks between flexural cracks until much higher loads. Of course, splitting cracks at flexural cracks may occur in the vicinity of the bar without being visible at the surface of the concrete member. The circumferential (splitting) strains are relieved to some extent by the slip resulting from crushing of the concrete in front of the ribs. The tests by Mathey and Watstein[6] mentioned above indicated that the slip at a steel stress of 8000 psi, corresponding to the cracking load in the beam example, is about 60 percent of the steel elongation. This means that about 35 percent (60 percent less the 25 percent mentioned above) of the steel elongation results from bearing of the ribs. Even with a total slip of 60 percent of the steel elongation, the longitudinal stresses will be sufficiently large to cause internal transverse cracks to form between the ends of the cylinder, i.e., between flexural cracks. The existence of such internal transverse cracks has been demonstrated by Broms.[8] Pullout tests indicated[2] that internal transverse cracks tend to occur before longitudinal

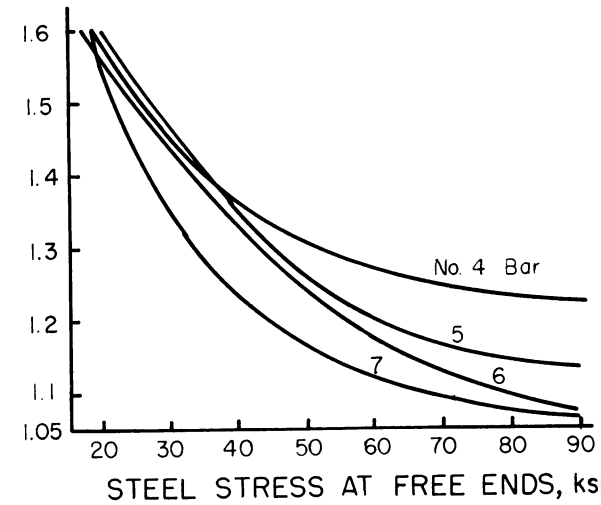

Fig. 10—Ratio of extensions of free and embedded bars from tests by Mathey and Watstein

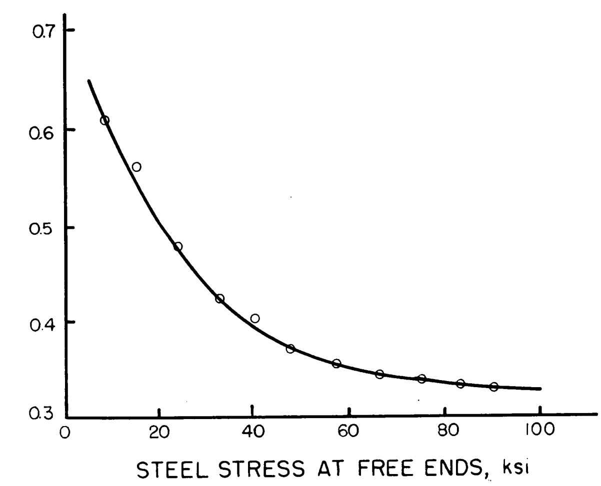

Fig. 11—Ratio of crack widths at the steel and at the concrete surface, from tests by Mathey and Watstein

cracks if the steel stress is high relative to the bond stress (in the flexural zone), while splitting would occur first when the bond stress is large relative to the steel stress (in the anchorage zone). Formation of internal transverse cracks adjacent to the reinforcement undoubtedly relieves the circumferential stresses.

The pullout tests by Mathey and Watstein[6] show that the ratio of the elongation of the free bar to the elongation of the bar embedded in concrete for various bar sizes decreases substantially with increasing steel stresses in a manner that indicates that there is little change in the magnitude of the average bond stress with increasing load (Fig. 10). A plot of the ratio of the crack width adjacent to the bar to the crack width at the concrete surfaces shows[6] a large variation in the crack width from the bar to the concrete surface with increasing steel stress (Fig. 11). Both of these phenomena are explained by the addition, widening, and radial extension of internal cracks between two surface cracks,[8] and have little to do with the slip caused by the crushing of the concrete key.

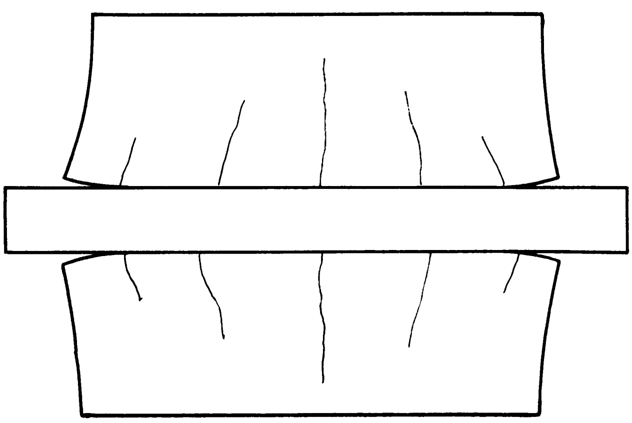

Fig. 12—Deformed concrete between transverse cracks, showing internal cracks

The formation of internal transverse cracks increases the total elongation along the specimen. However, calculations and measurements show[2] that the longitudinal strain on the concrete surface is nearly zero (or may even be compressive); hence, the cracks must widen at the outside surface to satisfy compatibility. The longitudinal straining of the concrete near the bar due to increasing load is accomplished primarily by the internal cracking (Fig. 12); thus the longitudinal strains in the concrete remain relatively small.

BOND AND SHEAR

The fundamentals of the bond of a deformed bar and the interrelationship between bond and cracking have been presented. The interrelationship between bond and shear will now be examined, not by the study of the fundamentals of this interaction, but by a comparison of current shear and bond equations and bond test results.

Flexural cracking completely changes the bond stress distribution in a beam as has been shown above. A corresponding large change in shear stress also results. The situation becomes even more complex with the formation of diagonal cracks. Due to the complex state of stress that develops, it is necessary to use and compare the nominal bond and shear stresses which, at best, represent average values over a region.

Two kinds of bond stresses exist in a beam—flexural bond, due to shear:

$$u = \frac{V}{N \pi D j d}$$

where N is the number of bars, and the anchorage bond stress:

$$u = \frac{A_s f_s}{N \pi D L} = \frac{f_s D}{4L}$$

These equations give the same bond stress if the shear V is constant. The anchorage bond stress in the beam shown in Fig. 4 would be evaluated at the third points where $L = 48$ in.

The nominal shear stress $v = V/(bjd)$ can be combined with the flexural bond expression above to give a relationship between the bond and shear stresses:

$$u = \frac{b}{N \pi D} v \tag{1}$$

According to ACI 318-63,[9] the critical diagonal tension crack forms when the nominal shear stress is equal to:

$$v_c = 1.9\sqrt{f_c'} + 2500 \left(\frac{p_w V_d}{M}\right) = \frac{1.9\sqrt{f_c'}}{1 - \dfrac{2850}{f_s}}$$

(plotted as Line b, Fig. 13); or alternately: $2\sqrt{f_c'}$ (Line a).

Using Eq. 1, the bond stress that exists when $v = v_c$ will be:

$$u_v = \frac{1.9b\sqrt{f_c'}}{N \pi D} \Big/ \left(1 - \frac{2850}{f_s}\right)$$

or

$$\frac{1.9b\sqrt{f_c'}}{N \pi D} + 713\,\frac{D}{L}$$

The latter expression results from the former if $f_s = 4Lu_v/D$ from above is substituted in the former. L in this case should be taken to the point along the beam at which u_v is being evaluated. These two quantities are equal when the shear is constant over the length L.

The ultimate nominal bond stress allowed, according to ACI 318-63 is:

$$u = 9.5\sqrt{f_c'} \leq 800 \text{ psi}$$

which for the beam example is 685 psi. The total bond force per unit length of embedment is:

$$\frac{P_u}{L} = u\,\pi\,D = 9.5\,\pi\sqrt{f_c'} \leq 800\,\pi\,D$$

This equation implies (for $f_c' = 4000$ psi) that the bar force per unit length of embedment is independent of the bar diameter for #7 to #11 bars, while it is linearly dependent on the bar diameter for bars smaller than #7.

With little confinement, deformed bars fail in bond by splitting of the concrete, which depends primarily on the load on the concrete and not much on the bar stress and the bar diameter or the bar perimeter. As the confinement improves, usually by the use of stirrups, the ultimate load per unit length depends increasingly on the bar diameter.[2] A bar which is confined to the extent that bond failure must occur by shear failure of the concrete keys instead of splitting, will carry a maximum unit load which should be proportional to the bar perimeter (hence to the bar diameter).

Based on the above reasoning, the maximum bond force per unit length should depend on the bar diameter, the amount of transverse reinforcement (confinement), the amount of confinement provided by the concrete cover, as well as the concrete tensile strength (or $\sqrt{f_c'}$). The ACI 318-63[9] bond equation, which accounts for one or two of the above variables, is based on a fairly well confined bar, and does not indicate the actual bond strength in every situation.

The parameter b/ND relates the bond and shear stresses as can be seen from Eq. (1). The value b/N is a good measure of the lateral spacing of the tensile reinforcement. Fig. 13 shows the relationship between b/ND and the bond stresses that exist at various limiting shear stresses, and compares these bond stresses with results of eccentric bond tests conducted by Ferguson, Turpin, and Thompson.[10]

Eccentric bond tests are pullout tests which allow normal cracking to occur as in a beam, but without the presence of the shear forces that exist in beams. Shear (with diagonal tension cracking) will influence the bond or even cause bond failure as a result of the shear failure.

When no stirrups are used, the bond failure stresses (Line g, Fig. 13) are well above the bond

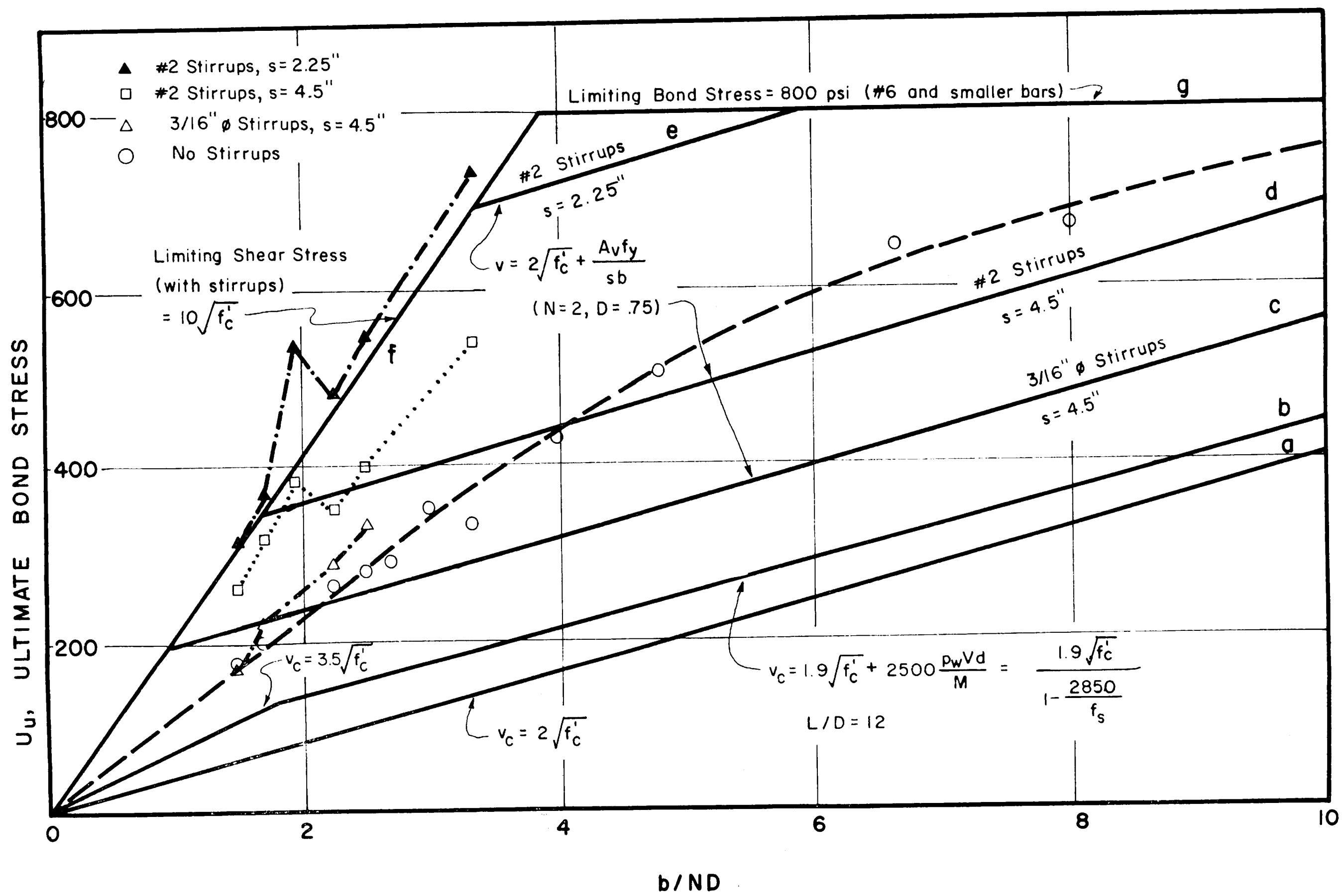

Fig. 13—Bond stress at various limiting shear stresses and test results of Ferguson, Turpin, and Thompson, as functions of b/ND ($f_y = 60{,}000$ psi, $f_c' = 4200$ psi; bar size: #6)

stresses existing at the limiting shear stresses v_c (Lines a and b), therefore shear failure should occur before bond failure. The test points fall between these two lines for the b/ND values shown.

If stirrups are added to reinforce the web, the additional confinement will improve the bond strength. However, this improvement in bond can occur only if the stirrups can prevent shear failure from occurring first. The question then is whether pullout of the bar will occur before yielding of the stirrups.

The ultimate nominal shear stress is:

$$v_u = 2\sqrt{f_c'} + \frac{A_v f_y}{sb}$$

which occurs when the stirrups yield. Using Eq. (1), the bond stress at this shear stress is:

$$u_v = 2b\sqrt{f_c'}/N\pi D + A_v f_y/sN\pi D$$

(Lines c, d, e). The largest shear stress allowed in a beam is $10\sqrt{f_c'}$; this shear stress produces a bond stress (Line f):

$$u_v = \frac{10b\sqrt{f_c'}}{N\pi D}$$

The maximum value of these bond stress limits is the actual ultimate bond stress value of 800 psi, for a #6 bar in this case (Line g).

When the stirrup reinforcement is relatively light ($s = 4.5$) and b/ND is less than about 2.5, the experimental bond values are even less than the bond corresponding to the shear limitation. This is true for both cases in which $s = 4.5$ in. in Fig. 13. This means that in these instances bond failure may occur before the limiting shear value is obtained in a beam. Thus, data indicates that the code is unconservative in such cases. This would not be the case if f_y were assumed to be 40 ksi rather than the 60 ksi used in Fig. 13.

The actual bond stress limit (Line g) will govern the design when heavy stirrups are used ($s = 2.25$ in Fig. 13) and when b/ND is large (i.e., to the right of the intersection of Line g and a line such as d). This situation was developed in the beams used by Mathey and Watstein[11] and also in the specimens tested by Ferguson and Thompson.[12,13] For low b/ND values the maximum shear stress according to the code (Line f) closely predicts the experimental failure stresses.

Three points should be remembered in connection with Fig. 13: (a) The expressions and data are for one row of tensile reinforcement; the use of two or more tows of bars would alter the situation, (b) the possible effects of dowel forces on bond and shear failures has been ignored, (c) the limiting bond and shear equations in the code may be conservative to various degrees.

The main purpose of the above discussion is to illuminate the relationship between shear and bond, and to show that the maximum bond capacity may be lower than that indicated by ACI 318-63 bond stress equations.

SUMMARY AND CONCLUSIONS

The bond of deformed bars is developed mainly by the bearing pressure of the bar ribs against the concrete. Bars having ribs with steep face angles (larger than about 40 deg with the bar axis) slip mainly by compressing the concrete in front of the bar rib. The concrete is crushed and a concrete wedge forms in front of the bar rib.

Bars with flat ribs, however, slip with the ribs sliding relative to the concrete.

The circumferential tensile stresses in the concrete around the bar are very small prior to flexural cracking. However, near transverse cracks bonding forces cause large circumferential tensile stresses. Also, radial tensile stresses, acting normal to the concrete-steel interface, destroy contact near the crack and allow separation and slip of the bar. These large radial tensile stresses near the crack were confirmed by a finite-element analysis. Due to the large longitudinal concrete tensile stresses, internal transverse cracks form. Radial (splitting) cracks can start at transverse cracks due to the large circumferential stresses. The deformed concrete block is shown in Fig. 7b (without internal cracks) and in Fig. 12 (with internal cracks).

Comparison of the ACI 318-63 bond and shear equations (for unreinforced webs) with bond tests shows that the shear provisions will usually control the design rather than the bond limitations. For reinforced webs, the shear limit will still control the design for small bar spacings. Bond controls only for large bar spacings and heavy stirrups. In some cases (small bar spacing and light stirrups) the beam may fail at bond stresses lower than those given by the Code.

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APPENDIX

NOTATION

A_v = area of stirrups
b = width of beam
d = distance from extreme compression fiber to centroid of tensile reinforcement
D = diameter of bar
e_c, e_f = elongations of embedded and free bars, respectively (Fig. 10)
f_c' = compressive strength of concrete
f_s = steel stress
h = rib height (Fig. 1)
j = ratio of distance between centroid of compression and centroid of tension to depth, d
L = length of anchorage
M = moment on the section
n = ratio of modulus of elasticity of steel to that of concrete
N = number of bars in the tension zone (one row in this case)
P_u = force in a bar at bond failure
p_w = percentage of steel in web
s = spacing of stirrups
s' = bar rib spacing
u = bond stress
u_v = bond stress at limiting shear stress, v_c
v = nominal shear stress
V = total shear force
v_c = nominal shear stress at formation of diagonal tension crack
w_c, w_s = crack widths at the concrete surface and at the steel, respectively (Fig. 11)
α = rib face angle (Fig. 1)

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Sinopsis—Résumé—Zusammenfassung

Mecánica de la Adherencia y Deslizamiento de Barras Corrugadas en el Concreto

Se examina la acción de las fuerzas de adherencia, y el deslizamiento y agrietamiento asociados para barras con varias propiedades superficiales. Se discute la mecánica del deslizamiento de barras corrugadas, y se verifica con datos experimentales. Se presentan los esfuerzos y deformaciones en el concreto causados por la fuerza de adherencia. Se estudian los requisitos de cortante establecidos por el Código de Construcción ACI en términos de los esfuerzos de adherencia límites correspondientes.

Mécanisme de l'Adhérence et du Glissement des Barres à Haute Adhérence dans le Béton

Les auteurs examinent l'action des efforts d'adhérence et le glissement ainsi que la fissuration qui en résultent pour des barres présentant diverses caractéristiques de surface, en s'appuyant sur des résultats expérimentaux. Les contraintes et les déformations du béton causées par les efforts d'adhérence sont présentées. Les prescriptions du Code ACI relatives aux contraintes tangentes sont étudiées par rapport aux contraintes limites d'adhérence correspondantes.

Die Mechanik von Verbund und Schlupf bei Rippenstählen in Beton

Die Wirkung von Verbundkräften und der damit in Zusammenhang stehende Schlupf und die Entwicklung von Rissen wird für Stäbe mit verschiedenen Oberflächeneigenschaften untersucht. Die Mechanik des Schlupfes von Rippenstählen in Beton wird diskutiert, wobei Versuchsergebnisse herangezogen werden. Die Spannungen und Verformungen, die im Beton auf Grund der Verbundkräfte auftreten, werden aufgezeigt. Die Schubspannungsbegrenzungen in den ACI Vorschriften werden im Hinblick auf die entsprechenden Grenzwerte der Verbundspannungen studiert.