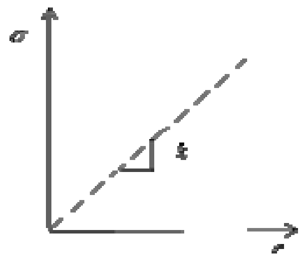
Visco – elastic models

(Reological -)

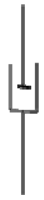
● Basic units

- Spring

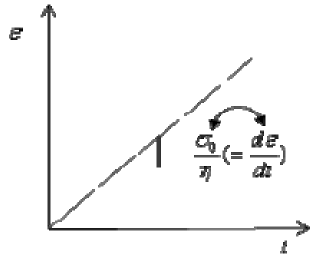
$$k = \frac{\sigma}{\varepsilon}$$



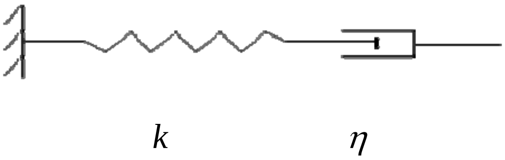
- Dashpot

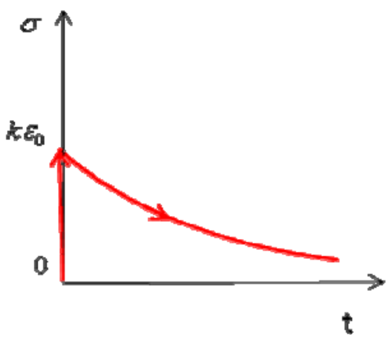


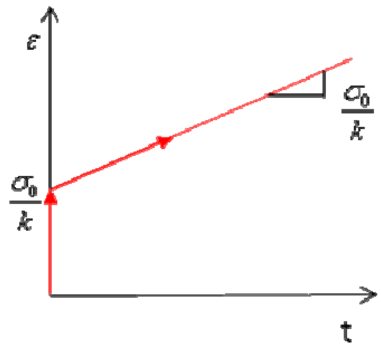
$$\eta = \frac{\sigma}{\dot{\varepsilon}}$$ (viscous damping)



● Maxwell model







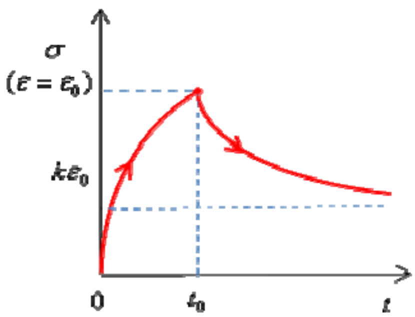
$$\sigma = \frac{k\eta\varepsilon_0}{\eta + kt}$$

$$\varepsilon = \frac{\sigma}{k} + \frac{\sigma_0}{\eta}t$$

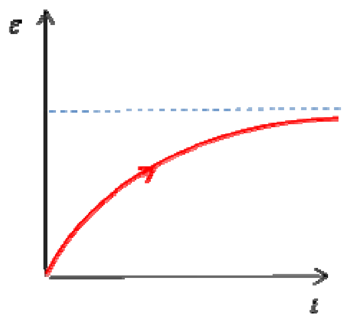
- Relaxation : apply initial strain & hold it → stress released
- Creep : apply initial stress & maintain it → deformation occurs

- Kelvin-Voigt model

Relaxation



Creep



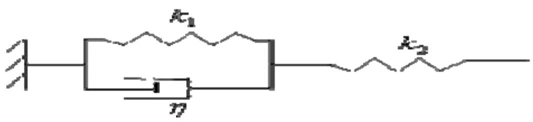
$$\sigma = \frac{k\varepsilon_0}{1-e^{-kt/\eta}}$$

$$(\sigma_{t=\infty} = k\varepsilon_0)$$

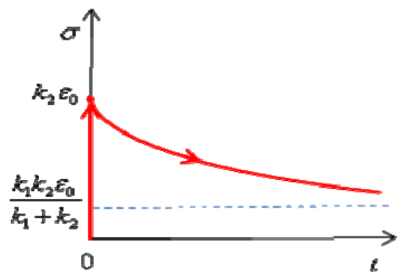
$$\varepsilon = \frac{\sigma_0}{k}(1-e^{-kt/\eta})$$

$$\varepsilon_{t=\infty} = \sigma_0 / k$$

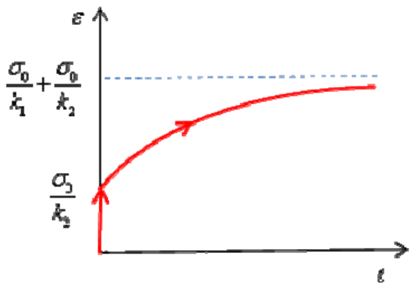
- Generalized Kelvin



Relaxation



Creep

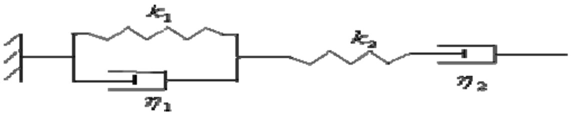


$$\sigma = \frac{k_1 k_2 \varepsilon_0}{k_1 + k_2(1-e^{k_1 t/\eta})}$$

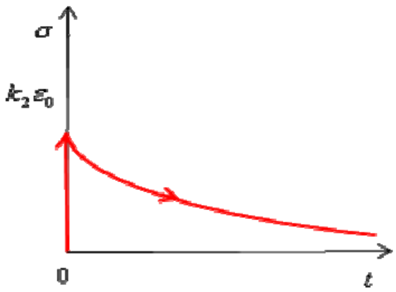
$$\varepsilon = \frac{\sigma_0}{k_2} + \frac{\sigma_0}{k_1}(1-e^{-k_1 t/\eta})$$

- Burger model :

(Kelvin – Voigt + Maxwell)

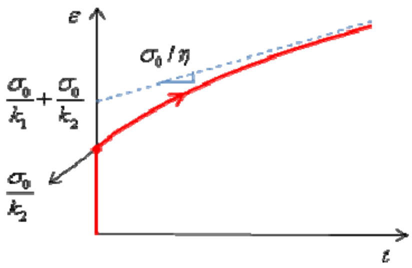


Relaxation



$$\sigma = \frac{\varepsilon_0 k_1 k_2 \eta_2}{k_1\eta_2 + k_1 k_2 t + k_2\eta_2(1-e^{-k_1 t/\eta_1})}$$

Creep



$$\varepsilon = (\frac{\sigma_0}{k_2} + \frac{\sigma_0 t}{\eta_2}) + \frac{\sigma_0}{k_1}(1-e^{-k_1 t/\eta_1})$$