Reference : Geotechnical Earthquake Engineering (Steven L. Kramer, University of Washington, Prentice Hall, 1996)

resist those loads. Because the actual loading on retaining walls during earthquakes is extremely complicated, seismic pressures on retaining walls are usually estimated using simplified methods.

11.6.1 Yielding Walls

Retaining walls that can move sufficiently to develop minimum active and/or maximum passive earth pressures are referred to as *yielding walls*. The dynamic pressures acting on yielding walls are usually estimated by pseudostatic procedures that share many features of those described for seismic slope stability analysis in Section 10.6.1.1. More recently, a pseudodynamic procedure that accounts, in an approximate manner, for the dynamic response of the backfill has been developed.

11.6.1.1 Mononobe–Okabe Method

Okabe (1926) and Mononobe and Matsuo (1929) developed the basis of a pseudostatic analysis of seismic earth pressures on retaining structures that has become popularly known as the Mononobe–Okabe (M-O) method. The M-O method is a direct extension of the static Coulomb theory to pseudostatic conditions. In a M-O analysis, pseudostatic accelerations are applied to a Coulomb active (or passive) wedge. The pseudostatic soil thrust is then obtained from force equilibrium of the wedge.

Active Earth Pressure Conditions. The forces acting on an active wedge in a dry, cohesionless backfill are shown in Figure 11.11a. In addition to the forces that exist under static conditions (Figure 11.7), the wedge is also acted upon by horizontal and vertical pseudostatic forces whose magnitudes are related to the mass of the wedge by the pseudostatic accelerations $a_h = k_h g$ and $a_v = k_v g$. The total active thrust can be expressed in a form similar to that developed for static conditions, that is,

$$P_{AE} = \frac{1}{2} K_{AE} \gamma H^2 (1 - k_v) \tag{11.15}$$

where the dynamic active earth pressure coefficient, K_{AE}, is given by

$$K_{AE} = \frac{\cos^2(\phi - \theta - \psi)}{\cos\psi \cos^2\theta \cos(\delta + \theta + \psi)\left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi - \beta - \psi)}{\cos(\delta + \theta + \psi)\cos(\beta - \theta)}}\right]^2} \tag{11.16}$$

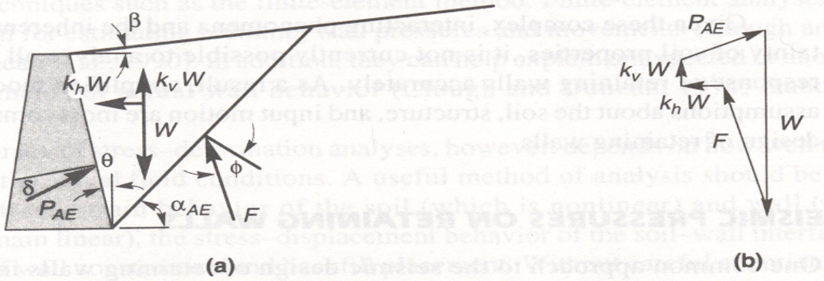

Figure 11.11 (a) Forces acting on active wedge in Mononobe–Okabe analysis, (b) force polygon illustrating equilibrium of forces acting on active wedge.

where $\phi - \beta \geq \psi$, $\gamma = \gamma_d$, and $\psi = \tan^{-1}[k_h/(1-k_v)]$. The critical failure surface, which is flatter than the critical failure surface for static conditions, is inclined (Zarrabi-Kashani, 1979) at an angle

$$\alpha_{AE} = \phi - \psi + \tan^{-1}\left[\frac{-\tan(\phi-\psi-\beta)+C_{1E}}{C_{2E}}\right] \tag{11.17}$$

where

$$C_{1E} = \sqrt{\tan(\phi-\psi-\beta)\,[\tan(\phi-\psi-\beta)+\cot(\phi-\psi-\theta)]\,[1+\tan(\delta+\psi+\theta)\cot(\phi-\psi-\theta)]}$$

$$C_{2E} = 1 + \{\tan(\delta+\psi+\theta)\,[\tan(\phi-\psi-\beta)+\cot(\phi-\psi-\theta)]\}$$

Although the M-O analysis implies that the total active thrust should act at a point $H/3$ above the base of a wall of height, H, experimental results suggest that it actually acts at a higher point under dynamic loading conditions. The total active thrust, P_{AE} [equation (11.15)], can be divided into a static component, P_A [equation (11.9)], and a dynamic component, ΔP_{AE}:

$$P_{AE} = P_A + \Delta P_{AE} \tag{11.18}$$

The static component is known to act at $H/3$ above the base of the wall. Seed and Whitman (1970) recommended that the dynamic component be taken to act at approximately $0.6H$. On this basis, the total active thrust will act at a height

$$h = \frac{P_A H/3 + \Delta P_{AE}(0.6H)}{P_{AE}} \tag{11.19}$$

above the base of the wall. The value of h depends on the relative magnitudes of P_A and P_{AE}—it often ends up near the midheight of the wall. M-O analyses show that k_v, when taken as one-half to two-thirds the value of k_h, affects P_{AE} by less than 10%. Seed and Whitman (1970) concluded that vertical accelerations can be ignored when the M-O method is used to estimate P_{AE} for typical wall designs.

Example 11.1

Compute the overturning moment about the base of the wall shown below for $k_h = 0.15$ and $k_v = 0.075$.

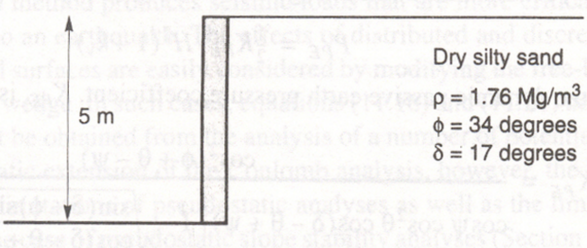

Figure E11.1

Solution First, estimate the static active thrust on the wall. Because the wall is not smooth ($\delta > 0$), Coulomb theory should be used. From equations (11.9) and (11.10),

$$K_A = \frac{\cos^2(34^\circ - 0^\circ)}{\cos^2(0^\circ)\cos(17^\circ + 0^\circ)\left[1 + \sqrt{\dfrac{\sin(17^\circ + 34^\circ)\sin(34^\circ - 0^\circ)}{\cos(17^\circ + 0^\circ)\cos(0^\circ - 0^\circ)}}\right]^2} = 0.256$$

and

$$P_A = \frac{1}{2}K_A\gamma H^2 = \frac{1}{2}(0.256)(1.76\ \text{Mg/m}^3)(9.81\ \text{m/sec}^2)(5\ \text{m})^2 = 55.3\ \text{kN/m}$$

Now, the total active thrust can be computed from equations (11.15) and (11.16). The angle, ψ, is given by

$$\psi = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) = \tan^{-1}\left(\frac{0.15}{1-0.075}\right) = 9.2°$$

and

$$K_{AE} = \frac{\cos^2(34° - 0° - 9.2°)}{\cos(9.2°)\cos^2(0°)\cos(17° + 0° + 9.2°)\left[1 + \sqrt{\dfrac{\sin(17° + 34°)\sin(34° + 0° - 9.2°)}{\cos(17° + 0° + 9.2°)\cos(0° - 0°)}}\right]^2}$$

$$= 0.362$$

and

$$P_A = \frac{1}{2}K_{AE}\gamma H^2(1-k_v) = \frac{1}{2}(0.362)(1.76\ \text{Mg/m}^3)(9.81\ \text{m/sec}^2)(5\ \text{m})^2(1-0.075)$$

$$= 72.3\ \text{kN/m}$$

The dynamic component of the total thrust is

$$\Delta P_{AE} = P_{AE} - P_A = 72.3\ kN/\text{m} - 55.3\ \text{kN/m} = 17\ \text{kN/m}$$

From equation (11.19), the total thrust acts at a point

$$h = \frac{P_A\frac{H}{3} + \Delta P_{AE}(0.6H)}{P_{AE}} = \frac{55.3\ \text{kN/m}\ \frac{5\ \text{m}}{3} + 17\ \text{kN/m}(0.6)(5\ \text{m})}{72.3\ \text{kN/m}} = 1.98\ \text{m}$$

above the base of the wall. Because only the horizontal component of the total active thrust contributes to the overturning moment about the base, the overturning moment is given by

$$M_o = (P_{AE})_h\, h = (72.3\ \text{kN/m})\cos(17°)(1.98\ \text{m}) = 137\ \frac{\text{kN-m}}{\text{m}}$$

Passive Earth Pressure Conditions. The total passive thrust on a wall retaining a dry, cohesionless backfill (Figure 11.12) is given by

$$P_{PE} = \tfrac{1}{2}K_{PE}\gamma H^2(1-k_v) \tag{11.20}$$

where the dynamic passive earth pressure coefficient, K_{PE}, is given by

$$K_{PE} = \frac{\cos^2(\phi + \theta - \psi)}{\cos\psi\cos^2\theta\cos(\delta - \theta + \psi)\left[1 - \sqrt{\dfrac{\sin(\delta + \phi)\sin(\phi + \beta - \psi)}{\cos(\delta - \theta + \psi)\cos(\beta - \theta)}}\right]^2} \tag{11.21}$$

The critical failure surface for M-O passive conditions is inclined from horizontal by an angle

$$\alpha_{PE} = \psi - \phi + \tan^{-1}\left[\frac{\tan(\phi + \psi + \beta) + C_{3E}}{C_{4E}}\right] \tag{11.22}$$

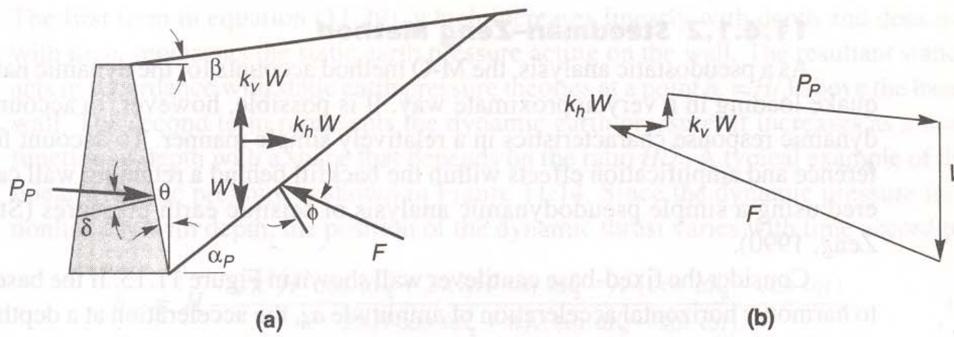

Figure 11.12 (a) Forces acting on passive wedge in Mononobe–Okabe analysis; (b) force polygon illustrating equilibrium of forces acting on passive wedge.

where

$$C_{3E} = \sqrt{\tan(\phi+\beta-\psi)\,[\tan(\phi+\beta-\psi)+\cot(\phi+\theta-\psi)]\,[1+\tan(\delta+\psi-\theta)\cot(\phi+\theta-\psi)]}$$

$$C_{4E} = 1+\{\tan(\delta+\psi-\theta)\,[\tan(\phi+\beta-\psi)+\cot(\phi+\theta-\psi)]\}$$

The total passive thrust can also be divided (Towhata and Islam, 1987) into static and dynamic components:

$$P_{PE} = P_P + \Delta P_{PE} \tag{11.23}$$

where P_{PE} and P_P are computed from equations (11.20) and (11.12), respectively. Note that the dynamic component acts in the opposite direction of the static component, thus reducing the available passive resistance.

Discussion. Although conceptually quite simple, the M-O analysis provides a useful means of estimating earthquake-induced loads on retaining walls. A positive horizontal acceleration coefficient causes the total active thrust to exceed the static active thrust and the total passive thrust to be less than the static passive thrust. Since the stability of a particular wall is generally reduced by an increase in active thrust and/or a decrease in passive thrust, the M-O method produces seismic loads that are more critical than the static loads that act prior to an earthquake. The effects of distributed and discrete surface loads and irregular backfill surfaces are easily considered by modifying the free-body diagram of the active or passive wedge. In such cases, equations (11.16) and (11.21) no longer apply—the total thrusts must be obtained from the analysis of a number of potential failure planes.

As a pseudostatic extension of the Coulomb analysis, however, the M-O analysis is subject to all of the limitations of pseudostatic analyses as well as the limitations of Coulomb theory. As in the case of pseudostatic slope stability analyses (Section 10.6.1.1), determination of the appropriate pseudostatic coefficient is difficult and the analysis is not appropriate for soils that experience significant loss of strength during earthquakes (e.g., liquefiable soils). Just as Coulomb theory does under static conditions, the M-O analysis will overpredict the actual total passive thrust, particularly for $\delta > \phi/2$. For these reasons the M-O method should be used and interpreted carefully.