If the shear stress at the design section calculated in accordance with Article 5.8.2.9 exceeds $0.18f'_c$ and the beam-type element is not built integrally with the support, its end region shall be designed using the strut-and-tie model specified in Article 5.6.3.

Where a beam is loaded on top and its end is not built integrally into the support, all the shear funnels down into the end bearing. Where the beam has a thin web so that the shear stress in the beam exceeds $0.18f'_c$, there is the possibility of a local diagonal compression or horizontal shear failure along the interface between the web and the lower flange of the beam. Usually the inclusion of additional transverse reinforcement cannot prevent this type of failure and either the section size must be increased or the end of the beam designed using a strut-and-tie model.

5.8.3.3 Nominal Shear Resistance

The nominal shear resistance, V_n, shall be determined as the lesser of:

$$V_n = V_c + V_s + V_p \qquad (5.8.3.3\text{-}1)$$

$$V_n = 0.25 f'_c b_v d_v + V_p \qquad (5.8.3.3\text{-}2)$$

in which:

$V_c = 0.083\beta\sqrt{f'_c}\, b_v d_v$, if the procedures of Articles 5.8.3.4.1 or 5.8.3.4.2 are used (5.8.3.3-3)

V_c = the lesser of V_{ci} and V_{cw}, if the procedures of Article 5.8.3.4.3 are used

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} \qquad (5.8.3.3\text{-}4)$$

where:

b_v = effective web width taken as the minimum web width within the depth d_v as determined in Article 5.8.2.9 (mm)

d_v = effective shear depth as determined in Article 5.8.2.9 (mm)

s = spacing of stirrups (mm)

β = factor indicating ability of diagonally cracked concrete to transmit tension as specified in Article 5.8.3.4

θ = angle of inclination of diagonal compressive stresses as determined in Article 5.8.3.4 (°); if the procedures of Article 5.8.3.4.3 are used, cot θ is defined therein

α = angle of inclination of transverse reinforcement to longitudinal axis (°)

C5.8.3.3

The shear resistance of a concrete member may be separated into a component, V_c, that relies on tensile stresses in the concrete, a component, V_s, that relies on tensile stresses in the transverse reinforcement, and a component, V_p, that is the vertical component of the prestressing force.

The expressions for V_c and V_s apply to both prestressed and nonprestressed sections, with the terms β and θ depending on the applied loading and the properties of the section.

The upper limit of V_n, given by Eq. 2, is intended to ensure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement.

where $\alpha = 90°$, Eq. 4 reduces to:

$$V_s = \frac{A_v f_y d_v \cot\theta}{s} \qquad (C5.8.3.3\text{-}1)$$

The angle θ is, therefore, also taken as the angle between a strut and the longitudinal axis of a member.

A_v = area of shear reinforcement within a distance s (mm^2)

V_p = component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (N); V_p is taken as zero if the procedures of Article 5.8.3.4.3 are used

5.8.3.4 Procedures for Determining Shear Resistance

Design for shear may utilize any of the three methods identified herein provided that all requirements for usage of the chosen method are satisfied.

Three complementary methods are given for evaluating shear resistance. Method 1, specified in Article 5.8.3.4.1, as described herein, is only applicable for nonprestressed sections. Method 2, as described in Article 5.8.3.4.2, is applicable for all prestressed and nonprestressed members, with and without shear reinforcement, with and without axial load. Method 3, specified in Article 5.8.3.4.3, is applicable for both prestressed and nonprestressed sections in which there is no net axial tensile load and at least minimum shear reinforcement is provided. Axial load effects can otherwise be accounted for through adjustments to the level of effective precompression stress f_{pc}. In regions of overlapping applicability between the latter two methods, Method 3 will generally lead to somewhat more shear reinforcement being required, particularly in areas of negative moment and near points of contraflexure. Method 3 provides a direct capacity rating while Method 2 may require iterative evaluation. If Method 3 leads to an unsatisfactory rating, it is permissible to use Method 2.

5.8.3.4.1 Simplified Procedure for Nonprestressed Sections

For concrete footings in which the distance from point of zero shear to the face of the column, pier or wall is less than $3d_v$ with or without transverse reinforcement, and for other nonprestressed concrete sections not subjected to axial tension and containing at least the minimum amount of transverse reinforcement specified in Article 5.8.2.5, or having an overall depth of less than 400 mm, the following values may be used:

$\beta = 2.0$
$\theta = 45°$

C5.8.3.4.1

With β taken as 2.0 and θ as 45°, the expressions for shear strength become essentially identical to those traditionally used for evaluating shear resistance. Recent large-scale experiments (*Shioya et al. 1989*), however, have demonstrated that these traditional expressions can be seriously unconservative for large members not containing transverse reinforcement.

5.8.3.4.2 General Procedure

For sections containing at least the minimum amount of transverse reinforcement specified in Article 5.8.2.5, the values of β and θ shall be as specified in Table 1. In using this table, ε_x shall be taken as the calculated longitudinal strain at the middepth of the member when the section is subjected to M_u, N_u, and V_u as shown in Figure 1.

For sections containing less transverse reinforcement than specified in Article 5.8.2.5, the values of β and θ shall be as specified in Table 2. In using this table, ε_x shall be taken as the largest calculated longitudinal strain which occurs within the web of the member when the section is subjected to N_u, M_u, and V_u as shown in Figure 2.

Unless more accurate calculations are made, ε_x shall be determined as:

- If the section contains at least the minimum transverse reinforcement as specified in Article 5.8.2.5:

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{2(E_sA_s + E_pA_{ps})} \quad (5.8.3.4.2\text{-}1)$$

The initial value of ε_x should not be taken greater than 0.001.

- If the section contains less than the minimum transverse reinforcement as specified in Article 5.8.2.5:

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{E_sA_s + E_pA_{ps}} \quad (5.8.3.4.2\text{-}2)$$

The initial value of ε_x should not be taken greater than 0.002.

- If the value of ε_x from Eqs. 1 or 2 is negative, the strain shall be taken as:

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right)}{2\left(E_cA_c + E_sA_s + E_pA_{ps}\right)} \quad (5.8.3.4.2\text{-}3)$$

C5.8.3.4.2

The shear resistance of a member may be determined by performing a detailed sectional analysis that satisfies the requirements of Article 5.8.3.1. Such an analysis, see Figure C1, would show that the shear stresses are not uniform over the depth of the web and that the direction of the principal compressive stresses changes over the depth of the beam. The more direct procedure given herein assumes that the concrete shear stresses are uniformly distributed over an area b_v wide and d_v deep, that the direction of principal compressive stresses (defined by angle θ) remains constant over d_v, and that the shear strength of the section can be determined by considering the biaxial stress conditions at just one location in the web. See Figure C2.

Members containing at least the minimum amount of transverse reinforcement have a considerable capacity to redistribute shear stresses from the most highly strained portion of the cross-section to the less highly strained portions. Because of this capacity to redistribute, it is appropriate to use the middepth of the member as the location at which the biaxial stress conditions are determined. Members that contain no transverse reinforcement, or contain less than the minimum amount of transverse reinforcement, have less capacity for shear stress redistribution. Hence, for such members, it is appropriate to perform the biaxial stress calculations at the location in the web subject to the highest longitudinal tensile strain, see Figure 2.

The longitudinal strain, ε_x, can be determined by the procedure illustrated in Figure C3. The actual section is represented by an idealized section consisting of a flexural tension flange, a flexural compression flange, and a web. The area of the compression flange is taken as the area on the flexure compression side of the member, i.e., the total area minus the area of the tension flange as defined by A_c. After diagonal cracks have formed in the web, the shear force applied to the web concrete, $V_u - V_p$, will primarily be carried by diagonal compressive stresses in the web concrete. These diagonal compressive stresses will result in a longitudinal compressive force in the web concrete of $(V_u - V_p)\cot\theta$. Equilibrium requires that this longitudinal compressive force in the web needs to be balanced by tensile forces in the two flanges, with half the force, that is $0.5\ (V_u - V_p)\cot\theta$, being taken by each flange. To avoid a trial and error iteration process, it is a convenient simplification to take this flange force due to shear as $V_u - V_p$. This amounts to taking $0.5 \cot\theta = 1.0$ in the numerator of Eqs. 1, 2 and 3. This simplification is not expected to cause a significant loss of accuracy. After the required axial forces in the two flanges are calculated, the resulting axial strains, ε_t and ε_c, can be calculated based on the axial force-axial strain relationship shown in Figure C4.

where:

A_c = area of concrete on the flexural tension side of the member as shown in Figure 1 (mm^2)

A_{ps} = area of prestressing steel on the flexural tension side of the member, as shown in Figure 1 (mm^2)

A_s = area of nonprestressed steel on the flexural tension side of the member at the section under consideration, as shown in Figure 1. In calculating A_s for use in this equation, bars which are terminated at a distance less than their development length from the section under consideration shall be ignored (mm^2)

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (MPa). For the usual levels of prestressing, a value of $0.7 f_{pu}$ will be appropriate for both pretensioned and post-tensioned members

N_u = factored axial force, taken as positive if tensile and negative if compressive (N)

M_u = factored moment, not to be taken less than $V_u d_v$ (N-mm)

V_u = factored shear force (N)

Within the transfer length, f_{po} shall be increased linearly from zero at the location where the bond between the strands and concrete commences to its full value at the end of the transfer length.

The flexural tension side of the member shall be taken as the half-depth containing the flexural tension zone, as illustrated in Figure 1.

The crack spacing parameter s_{xe}, used in Table 2, shall be determined as:

$$s_{xe} = s_x \frac{35}{a_g + 16} \leq 2000 \text{ mm} \qquad (5.8.3.4.2\text{-}4)$$

where:

a_g = maximum aggregate size (mm)

s_x = the lesser of either d_v or the maximum distance between layers of longitudinal crack control reinforcement, where the area of the reinforcement in each layer is not less than $0.003 b_v s_x$, as shown in Figure 3 (mm)

For members containing at least the minimum amount of transverse reinforcement, ε_x can be taken as:

$$\varepsilon_x = \frac{\varepsilon_t + \varepsilon_c}{2} \qquad (C5.8.3.4.2\text{-}1)$$

where ε_t and ε_c are positive for tensile strains and negative for compressive strains. If, for a member subject to flexure, the strain ε_c is assumed to be negligibly small, then ε_x becomes one half of ε_t. This is the basis for the expression for ε_x given in Eq. 1. For members containing less than the minimum amount of transverse reinforcement, Eq. 2 makes the conservative simplification that ε_x is equal to ε_t.

In some situations, it will be more appropriate to determine ε_x using the more accurate procedure of Eq. C1 rather than the simpler Eqs. 1 through 3. For example, the shear capacity of sections near the ends of precast, pretensioned simple beams made continuous for live load will be estimated in a very conservative manner by Eqs. 1 through 3 because, at these locations, the prestressing strands are located on the flexural compression side and, therefore, will not be included in A_{ps}. This will result in the benefits of prestressing not being accounted for by Eqs. 1 through 3.

Absolute value signs were added to Eqs. 1 through 3 in 2004. This notation replaced direction in the nomenclature to take M_u and V_u as positive values. For shear, absolute value signs in Eqs. 1 through 3 are needed to properly consider the effects due to V_u and V_p in sections containing a parabolic tendon path which may not change signs at the same location as shear demand, particularly at midspan.

For pretensioned members, f_{po} can be taken as the stress in the strands when the concrete is cast around them, i.e., approximately equal to the jacking stress. For post-tensioned members, f_{po} can be conservatively taken as the average stress in the tendons when the post-tensioning is completed.

Note that in both Table 1 and Table 2, the values of β and θ given in a particular cell of the table can be applied over a range of values. Thus from Table 1, θ=34.4° and β=2.26 can be used provided that ε_x is not greater than 0.75×10^{-3} and v_u/f'_c is not greater than 0.125. Linear interpolation between the values given in the tables may be used, but is not recommended for hand calculations. Assuming a value of ε_x larger than the value calculated using Eqs. 1, 2 or 3, as appropriate, is permissible and will result in a higher value of θ and a lower value of β. Higher values of θ will typically require more transverse shear reinforcement, but will decrease the tension force required to be resisted by the longitudinal reinforcement. Figure C5 illustrates the shear design process by means of a flow chart. This figure is based on the simplified assumption that $0.5 \cot \theta = 1.0$.

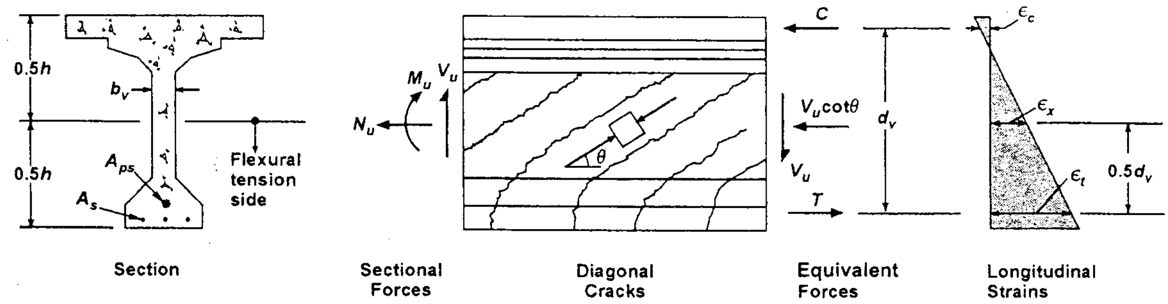

Figure 5.8.3.4.2-1 Illustration of Shear Parameters for Section Containing at Least the Minimum Amount of Transverse Reinforcement, V_p=0.

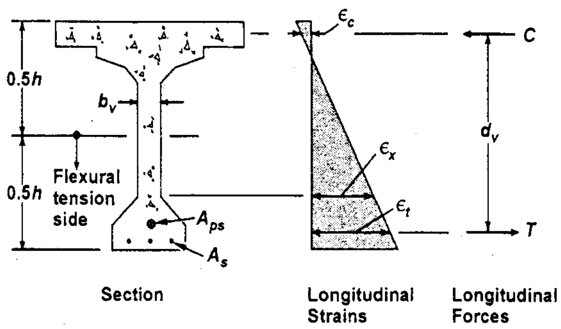

Figure 5.8.3.4.2-2 Longitudinal Strain, ε_x for Sections Containing Less than the Minimum Amount of Transverse Reinforcement.

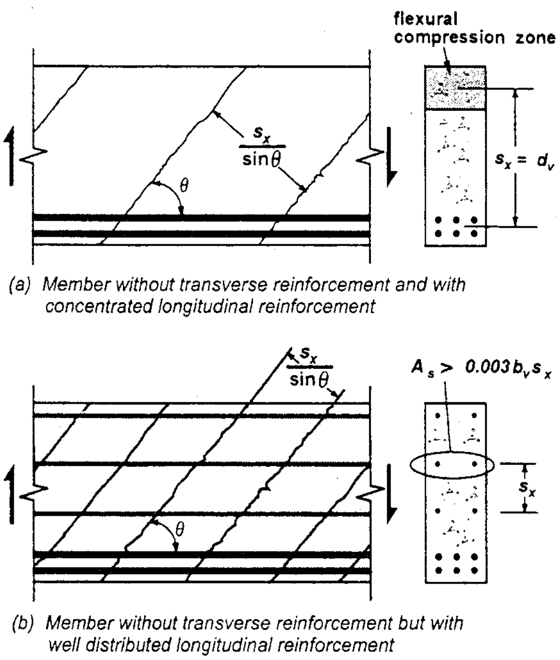

Figure 5.8.3.4.2-3 Definition of Crack Spacing Parameter s_x.

For sections containing a specified amount of transverse reinforcement, a shear-moment interaction diagram, see Figure C6, can be calculated directly from the procedures in this Article. For a known concrete strength and a certain value of ε_x, each cell of Table 1 corresponds to a certain value of v_u/f'_c, i.e., a certain value of V_n. This value of V_n requires an amount of transverse reinforcement expressed in terms of the parameter $A_v f_y/(b_v s)$. The shear capacity corresponding to the provided shear reinforcement can be found by linearly interpolating between the values of V_n corresponding to two consecutive cells where one cell requires more transverse reinforcement than actually provided and the other cell requires less reinforcement than actually provided. After V_n and θ have been found in this manner, the corresponding moment capacity M_n can be found by calculating, from Eqs. 1 through 3, the moment required to cause this chosen value of ε_x, and calculating, from Eq. 5.8.3.5-1, the moment required to yield the reinforcement. The predicted moment capacity will be the lower of these two values. In using Eqs. 5.8.2.9-1, 5.8.3.5-1 and Eqs. 1 through 3 of the procedure to calculate a $V_n - M_n$ interaction diagram, it is appropriate to replace V_u by V_n, M_u by M_n and N_u by N_n and to take the value of ϕ as 1.0. With an appropriate spreadsheet, the use of shear-moment interaction diagrams is a convenient way of performing shear design and evaluation.

The values of β and θ listed in Table 1 and Table 2 are based on calculating the stresses that can be transmitted across diagonally cracked concrete. As the cracks become wider, the stress that can be transmitted decreases. For members containing at least the minimum amount of transverse reinforcement, it is assumed that the diagonal cracks will be spaced about 300 mm apart. For members without transverse reinforcement, the spacing of diagonal cracks inclined at θ° to the longitudinal reinforcement is assumed to be $s_x/\sin\theta$, as shown in Figure 3. Hence, deeper members having larger values of s_x are calculated to have more widely spaced cracks and hence, cannot transmit such high shear stresses. The ability of the crack surfaces to transmit shear stresses is influenced by the aggregate size of the concrete. Members made from concretes that have a smaller maximum aggregate size will have a larger value of s_{xe} and hence, if there is no transverse reinforcement, will have a smaller shear strength.

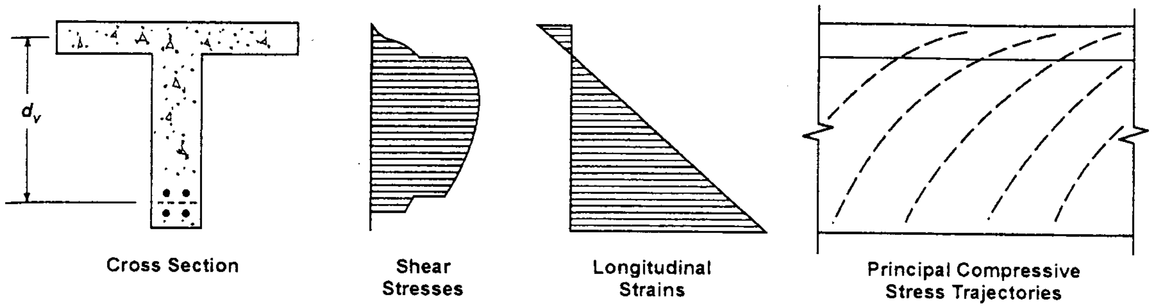

Figure C5.8.3.4.2-1 Detailed Sectional Analysis to Determine Shear Resistance in Accordance with Article 5.8.3.1.

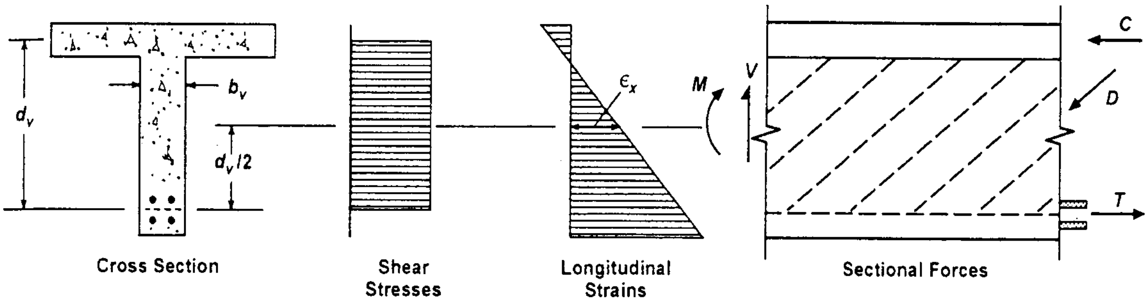

Figure C5.8.3.4.2-2 More Direct Procedure to Determine Shear Resistance in Accordance with Article 5.8.3.4.2.

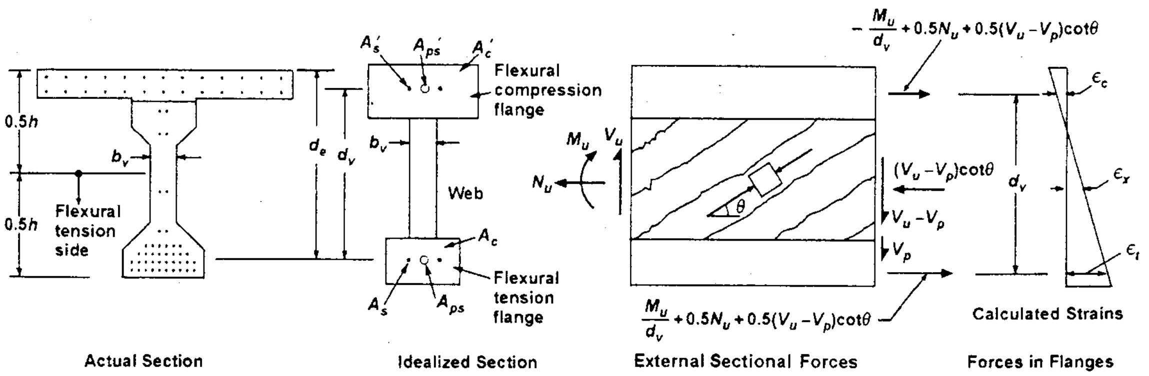

Figure C5.8.3.4.2-3 More Accurate Calculation Procedure for Determining ε_x.

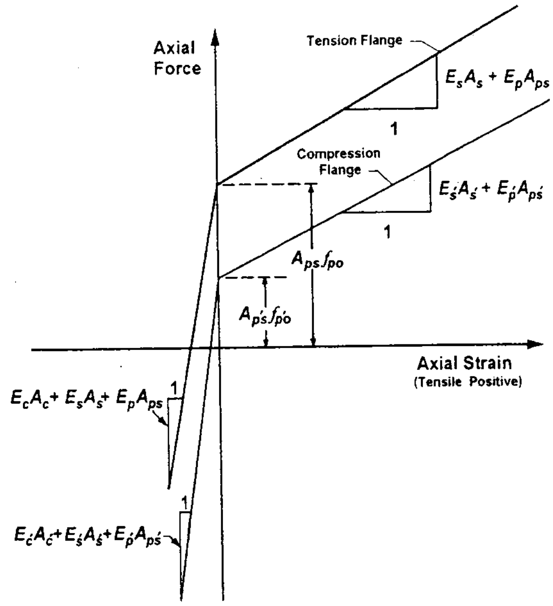

Figure C5.8.3.4.2-4 Assumed Relations Between Axial Force in Flange and Axial Strain of Flange.

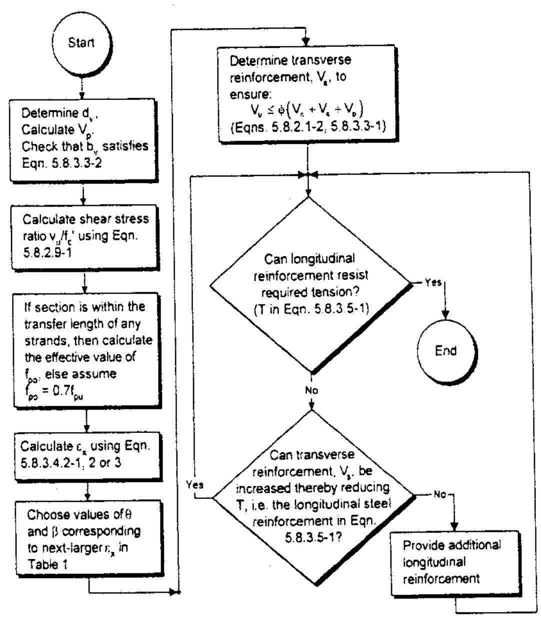

Figure C5.8.3.4.2-5 Flow Chart for Shear Design of Section Containing at Least Minimum Transverse Reinforcement.

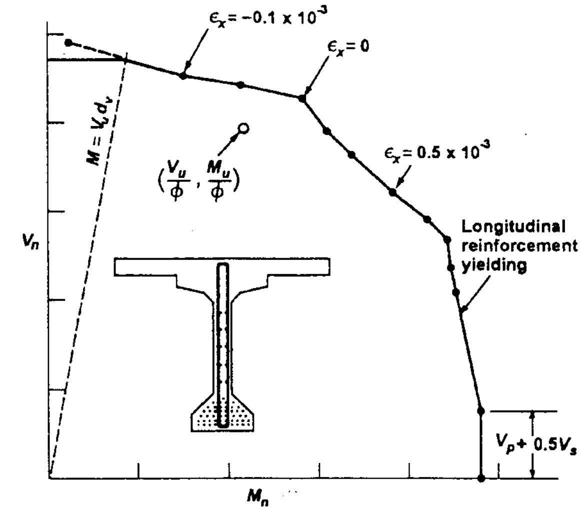

Figure C5.8.3.4.2-6 Typical Shear-Moment Interaction Diagram.

More details on the procedures used in deriving the tabulated values of θ and β are given in Collins and Mitchell (*1991*).

Table 5.8.3.4.2-1 Values of θ and β for Sections with Transverse Reinforcement.

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

Table 5.8.3.4.2-2 Values of θ and β for Sections with Less than Minimum Transverse Reinforcement.

s_{xe}	$\varepsilon_x \times 1000$										
(mm)	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤130	25.4 6.36	25.5 6.06	25.9 5.56	26.4 5.15	27.7 4.41	28.9 3.91	30.9 3.26	32.4 2.86	33.7 2.58	35.6 2.21	37.2 1.96
≤250	27.6 5.78	27.6 5.78	28.3 5.38	29.3 4.89	31.6 4.05	33.5 3.52	36.3 2.88	38.4 2.50	40.1 2.23	42.7 1.88	44.7 1.65
≤380	29.5 5.34	29.5 5.34	29.7 5.27	31.1 4.73	34.1 3.82	36.5 3.28	39.9 2.64	42.4 2.26	44.4 2.01	47.4 1.68	49.7 1.46
≤500	31.2 4.99	31.2 4.99	31.2 4.99	32.3 4.61	36.0 3.65	38.8 3.09	42.7 2.46	45.5 2.09	47.6 1.85	50.9 1.52	53.4 1.31
≤750	34.1 4.46	34.1 4.46	34.1 4.46	34.2 4.43	38.9 3.39	42.3 2.82	46.9 2.19	50.1 1.84	52.6 1.60	56.3 1.30	59.0 1.10
≤1000	36.6 4.06	36.6 4.06	36.6 4.06	36.6 4.06	41.2 3.20	45.0 2.62	50.2 2.00	53.7 1.66	56.3 1.43	60.2 1.14	63.0 0.95
≤1500	40.8 3.50	40.8 3.50	40.8 3.50	40.8 3.50	44.5 2.92	49.2 2.32	55.1 1.72	58.9 1.40	61.8 1.18	65.8 0.92	68.6 0.75
≤2000	44.3 3.10	44.3 3.10	44.3 3.10	44.33 3.10	47.1 2.71	52.3 2.11	58.7 1.52	62.8 1.21	65.7 1.01	69.7 0.76	72.4 0.62

5.8.3.4.3 Simplified Procedure for Prestressed and Nonprestressed Sections

For concrete beams not subject to significant axial tension, prestressed and nonprestressed, and containing at least the minimum amount of transverse reinforcement specified in Article 5.8.2.5, V_n in Article 5.8.3.3 may be determined with V_p taken as zero and V_c taken as the lesser of V_{ci} and V_{cw}, where:

V_{ci} = nominal shear resistance provided by concrete when inclined cracking results from combined shear and moment (N)

V_{cw} = nominal shear resistance provided by concrete when inclined cracking results from excessive principal tensions in web (N)

V_{ci} shall be determined as:

$$V_{ci} = 0.0525\sqrt{f'_c}\, b_v d_v + V_d + \frac{V_i M_{cre}}{M_{max}} \geq 0.16\sqrt{f'_c}\, b_v d_v \qquad (5.8.3.4.3\text{-}1)$$

C5.8.3.4.3

Article 5.8.3.4.3 is based on the recommendations of NCHRP Report 549 (*Hawkins et al.*, 2005). The concepts of this Article are compatible with the concepts of ACI Code 318-05 and AASHTO *Standard Specifications for Highway Bridges (2002)* for evaluations of the shear resistance of prestressed concrete members. However, those concepts are modified so that this Article applies to both prestressed and nonprestressed sections.

The nominal shear resistance V_n is the sum of the shear resistances V_c and V_s provided by the concrete and shear reinforcement, respectively. Both V_c and V_s depend on the type of inclined cracking that occurs at the given section. There are two types of inclined cracking: flexure-shear cracking and web-shear cracking for which the associated resistances are V_{ci} and V_{cw}, respectively. Figure C1 shows the development of both types of cracking when increasing uniform load was applied to a 1600-mm bulb-tee girder. NCHRP Report XX2 (*Hawkins et al.*, 2005).