factors) given in Table 4.8. The higher values in Table 4.7 are applied to the normal loads and service conditions, while the lower values are applied to the maximum loads and worst environmental conditions.

The basic philosophy using total factors of safety is that the foundation should be capable of resisting a load F_s times greater than the design load. The load and resistance factor design (LRFD) method applies separate or partial factors to the loads and soil resistance. The load factors are provided mainly for variability and pattern of loading, which differ for dead loads, live loads, environmental loads, and water pressures. The resistance factors consider the variability and uncertainty of assessment of soil resistance, which differ for the cohesive and friction components. Thus, the factored shear strength of soil at the ultimate limit state may be expressed as

$$\tau = f_c c + \sigma_n f_\phi \tan\phi \tag{4.33}$$

for the Coulomb criterion. The factors f_c and f_ϕ are the resistance factors for the cohesive and friction components, respectively. It is evident from Equation 4.33 that the total factor of safety obtained will depend on the relative contributions of the cohesive and friction components.

Whitman (1984) has recently reviewed the application of the related topic of risk analysis to geotechnical engineering.

4.13 EXAMPLE PROBLEMS

EXAMPLE 4.1

A rectangular footing (Fig. 4.21) 28 ft wide and 84 ft long is to be placed at a depth of 10 ft in a deep stratum of soft, saturated clay (bulk unit weight 105 lb/ft^3). The water table is at 8 ft below ground surface. Find the ultimate bearing capacity under the following two conditions:

a. assuming that the rate of application of dead and live loads is fast in comparison with the rate of dissipation of excess pore-water pressures caused by loads, so that undrained conditions prevail at failure;

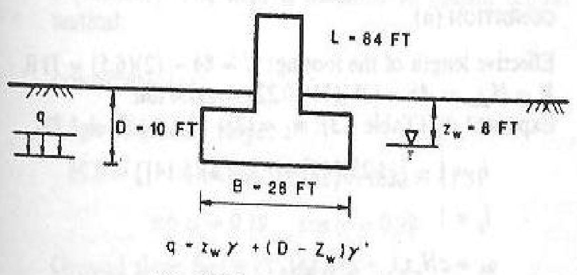

Fig. 4.21 Footing geometry.

b. assuming, as the other extreme, that the rate of loading is slow enough that no excess pore-water pressures are introduced in the foundation soil.

The strength parameters of the soil, obtained from unconsolidated, undrained tests are $c_u = 0.22$ ton/ft^2, $\phi_u = 0$. Consolidated, drained tests give $c_d = 0.04$ ton/ft^2, $\phi_d = 23°$.

CONDITION (a)

Submerged unit weight of soil: $\gamma' = 105 - 62 = 43$ lb/ft^3.
Overburden stress: $q = [(8)(105) + (2)(43)]/(2000)$
$= 0.463$ ton/ft^2.
Bearing capacity factors (Table 4.1): $N_c = 5.14$; $N_q = 1$; $N_\gamma = 0$.
Shape factors (Table 4.4, Brinch Hansen):

$$s_c = 1 + \frac{B}{L}\frac{N_q}{N_c} = 1 + (1/3)(0.19) = 1.065$$

$$s_q = 1.00$$

Ultimate bearing pressure (Eq. 4.24):

$$q_0 = cN_c s_c + qN_q s_q$$

$$q_0 = (0.22)(5.14)(1.065) + (0.463)(1)(1.00)$$
$$= 1.21 + 0.46 = 1.67 \text{ ton/ft}^2$$

CONDITION (b)

Bearing capacity factors: $N_c = 18.05$; $N_q = 8.66$; $N_\gamma = 9.70$.
Shape factors:

$$s_c = 1 + \frac{B}{L}\frac{N_q}{N_c} = 1 + (1/3)(0.48) = 1.16$$

$$s_q = 1 + \frac{B}{L}\tan\phi = 1 + (1/3)(0.42) = 1.14$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 1 - (0.4)(1/3) = 0.87$$

Ultimate bearing pressure:

$$q_0 = cN_c s_c + qN_q s_q + \frac{\gamma B}{2} N_\gamma s_\gamma$$

$$q_0 = (0.04)(18.05)(1.16) + (0.463)(8.66)(1.14)$$
$$+ (1/2)(43)(28)(9.7)(0.87)/(2000)$$
$$= 0.72 + 4.57 + 2.54 = 7.83 \text{ ton/ft}^2$$

EXAMPLE 4.2

Solve the problem described in Example 4.1 if the footing is placed at the same depth (10 ft) in a deep stratum of medium dense sand. Assume for sand a saturated unit weight of 118 lb/ft^3 and an average moist unit weight above the water table of 100 lb/ft^3. Drained triaxial tests on sand samples show that the angle ϕ of shearing resistance of sand varies with mean normal stress σ_0 according to the equation

$$\phi = \phi_1 - (5.5°)\log_{10}(\sigma_0/\sigma_1)$$

where $\phi_1 = 38°$ is the angle of shearing resistance at a mean normal stress $\sigma_1 = 1$ ton/ft^2.

Submerged unit weight of sand: $\gamma' = 118 - 62 = 56$ lb/ft^3.
Overburden stress: $q = [(8)(100) + (2)(56)]/(2000) = 0.456$ ton/ft^2.

To find the mean normal stress a preliminary estimate of bearing capacity is needed. It is assumed for this preliminary analysis that $\phi = 34°$.

Bearing capacity factors: $N_q = 29.44$; $N_\gamma = 52.18$.
Shape factors:

$$s_q = 1 + \frac{B}{L}\tan\phi = 1 + (1/3)(0.67) = 1.22$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4(1/3) = 0.87$$

Ultimate bearing pressure (Eq. 4.24):

$$q_0 = qN_q s_q + \frac{\gamma B}{2}N_\gamma s_\gamma$$

$$q_0 = (0.456)(29.44)(1.22) + (1/2)(56)(28)(52.18)(0.87)/(2000)$$

$$= 16.4 + 17.8 = 34.2 \text{ ton/ft}^2$$

Mean normal stress along the slip surface (De Beer, 1965):

$$\sigma_0 = \tfrac{1}{4}(q_0 + 3q)(1 - \sin\phi)$$

$$\sigma_0 = \tfrac{1}{4}[34.2 + (3)(0.456)](1 - 0.559) = 3.92 \text{ ton/ft}^2$$

Representative angle of shearing resistance:

$$\phi = 38° - (5.5°)(0.593) = 34.7 \cong 35°$$

The analysis is now repeated with $\phi = 35°$:

$$N_q = 33.3 \quad N_\gamma = 62.3$$

$$s_q = 1 + \frac{B}{L}\tan\phi = 1 + (1/3)(0.7) = 1.23$$

$$s_\gamma = 0.87$$

$$q_0 = qN_q s_q + \frac{\gamma B}{2}N_\gamma s_\gamma$$

$$q_0 = (0.456)(33.3)(1.23) + (1/2)(56)(28)(62.3)(0.87)/(2000)$$

$$= 18.7 + 21.2 = 39.9 \text{ ton/ft}^2$$

In view of small change in mean normal stress from the previously found value, this answer is retained.

EXAMPLE 4.3

For the footing discussed in Example 4.1, find the ultimate bearing capacity in conditions (a) and (b) if the footing reaction acts 3 ft off-center in the direction of the short side B ($e_B = 3$ ft) and if the inclination of the reaction is in the same direction. Assume that the horizontal component of the reaction is equal to half of the ultimate value given by

$$H = V\tan\phi + Ac$$

CONDITION (a)

Effective width of the footing: $B' = 28 - (2)(3) = 22$ ft.
Horizontal reaction: $H = 0.5H_{max} = (0.5)(22)(84)(0.22) = 203.3$ ton.
Exponent m_B (Table 4.3): $m_B = [2 + (1/3)]/[1 + (1/3)] = 1.75.$

Inclination factors (Table 4.3):

$$i_q = 1$$

$$i_c = 1 - [(1.75)(203.3)/(406.6)(5.14)] = 0.83$$

Ultimate bearing pressure (see calculations from Example 4.1):

$$q_0 = cN_c s_c i_c + qN_q s_q i_q$$

$$q_0 = (1.21)(0.83) + (0.46)(1)$$

$$= 1.00 + 0.46 = 1.46 \text{ ton/ft}^2$$

CONDITION (b)

Assume $\tan\delta = \tan\phi_d = 0.42$, $c_a = 0$.

$$H = (0.5)(0.42)V = 0.21V$$

$$i_q = (1 - 0.21)^{1.75} = 0.66 \quad \text{(Table 4.3)}$$

$$i_c = 0.66 - [(1 - 0.66)/(18.05)(0.42)] = 0.62$$

$$i_\gamma = (1 - 0.21)^{2.75} = 0.52$$

$$q_0 = cN_c s_c i_c + qN_q s_q i_q + \frac{\gamma B}{2}N_\gamma s_\gamma i_\gamma$$

$$q_0 = (0.72)(0.62) + (4.57)(0.66) + (2.54)(0.52)$$

$$= 0.45 + 3.02 + 1.32 = 4.79 \text{ ton/ft}^2$$

EXAMPLE 4.4

For the same footing find the ultimate bearing capacity if the reaction acts 6.5 ft off-center in the direction of the long side, and if the inclination is in the same direction. Assume that the horizontal component is equal to the ultimate value given by

$$H = V\tan\phi + Ac$$

CONDITION (a)

Effective length of the footing: $L' = 84 - (2)(6.5) = 71$ ft.
$H = H_{max} = Ac = (28)(71)(0.22) = 437.4$ ton.
Exponent m_L (Table 4.3): $m_L = (2 + 3)/(1 + 3) = 1.25.$

$$i_c = 1 - [(1.25)(437.4)/(437.4)(5.14)] = 0.76$$

$$i_q = 1$$

$$q_0 = cN_c s_c i_c + qN_q s_q i_q$$

$$q_0 = (1.21)(0.76) + (0.46)(1) = 1.38 \text{ ton/ft}^2$$

CONDITION (b)

$$H = 0.42V$$

$$i_q = (1 - 0.42)^{1.25} = 0.51$$

$$i_c = 0.51 - [(1 - 0.51)/(18.05)(0.42)] = 0.45$$

$$i_\gamma = (1 - 0.42)^{2.25} = 0.29$$

$$q_0 = cN_c s_c i_c + qN_q s_q i_q + \frac{\gamma B}{2} N_\gamma s_\gamma i_\gamma$$

$$q_0 = (0.72)(0.45) + (4.57)(0.51) + (2.54)(0.29)$$

$$= 0.32 + 2.33 + 0.74 = 3.39 \text{ ton/ft}^2$$

EXAMPLE 4.5

For the footing discussed in preceding examples, find the ultimate bearing capacity if the footing base is tilted 1 (vertical) to 4 (horizontal). Assume, as in Example 4.3, that the reaction is 3 ft off-center in the direction of the short side B, inclined in the same direction, with a horizontal reaction equal to one-half the ultimate value from

$$H = V \tan \phi + Ac$$

CONDITION (a)

Angle of base tilt: $\alpha = \tan^{-1}(1/4) = 0.245$.
Base tilt factors (Table 4.5):

$$b_c = 1 - (2)(0.245)/(3.14 + 2) = 0.90$$

$$b_q = 1$$

Ultimate bearing pressure (see calculations from Example 4.3):

$$q_0 = cN_c s_c b_c + qN_q s_q b_q$$

$$q_0 = (1.00)(0.90) + 0.46 = 0.90 + 0.46 = 1.36 \text{ ton/ft}^2$$

CONDITION (b)

Base tilt factors (Table 4.5)

$$b_q = b_\gamma = [1 - (0.245)(0.42)]^2 = 0.80$$

$$b_c = 0.80 - [(1 - 0.80)/(18.05)(0.42)] = 0.77$$

$$q_0 = cN_c s_c b_c + qN_q s_q b_q + \frac{\gamma B}{2} N_\gamma s_\gamma b_\gamma$$

$$q_0 = (0.45)(0.77) + (3.02)(0.80) + (1.32)(0.80)$$

$$= 0.35 + 2.42 + 1.06 = 3.83 \text{ ton/ft}^2$$

EXAMPLE 4.6

For the footing discussed in Example 4.1, find the ultimate bearing capacity if the ground slopes 5 (horizontal) to 1 (vertical). The load is assumed to remain central and vertical.

CONDITION (a)

Angle of ground slope:

$$\omega = \tan^{-1}(1/5) = 0.20 = 11.3°$$

$$\sin \omega = 0.19 \quad \cos \omega = 0.98$$

Ground slope factor (Table 4.5):

$$g_c = 1 - [(2)(0.20)/(3.14 + 2)] = 0.92$$

Bearing capacity factor (Eq. 4.26):

$$N_\gamma = -2 \sin \omega = -0.38$$

Ultimate bearing pressure (see calculations from Example 4.1):

$$q_0 = cN_c s_c g_c + qN_q s_q \cos \omega + \frac{\gamma B}{2} N_\gamma s_\gamma$$

$$q_0 = (1.21)(0.92) + (0.46)(0.98)$$

$$- [(1/2)(43)(28)(0.38)(0.87)]/(2000)$$

$$= 1.02 + 0.45 - 0.10 = 1.37 \text{ ton/ft}^2$$

CONDITION (b)

Ground slope factors (Table 4.5):

$$g_q = g_\gamma = (1 - 0.20)^2 = 0.64$$

$$g_c = 0.64 - (1 - 0.64)/[(18.05)(0.42)] = 0.59$$

$$q_0 = cN_c s_c g_c + qN_q s_q g_q + \frac{\gamma B}{2} N_\gamma s_\gamma g_\gamma$$

$$q_0 = (0.72)(0.59) + (4.57)(0.98)(0.64) + (2.54)(0.64)$$

$$= 0.42 + 2.87 + 1.62 = 4.91 \text{ ton/ft}^2$$

EXAMPLE 4.7

For a layered footing consisting of a sand layer ($\phi = 45°$) over a weak clay ($c = 20$ kPa), compute the bearing capacity if the footing is loaded vertically. Use the following dimensions for the strip footing on a saturated footing (see Fig. 4.16):

$$B = \tfrac{1}{3}\text{ m} \quad D = \tfrac{1}{3}\text{ m} \quad H = \tfrac{2}{3}\text{ m}$$

and

$$\gamma_1 = 1.9 \text{ g/cm}^3$$

Using Meyerhof and Hanna's procedure,

$$q_1 = 0.5\gamma BN_\gamma$$

$$= 0.5(1.9 - 1)(9.8)(0.33)(382) = 556$$

$$q_2 = cN_c = 20(5.14) = 103 \text{ kPa}$$

$$\frac{q_2}{q_1} = 0.185$$

From Figure 4.17, $\delta/\phi = 0.55$ and $K_s = 10$. Then

$$q_u = q_b + \gamma H^2\left(1 + \frac{2D}{H}\right)K_s \frac{\tan \phi}{B} - \gamma H$$

$$= 103 + (1.9 - 1)(9.8)(0.66)^2[1 + 2(0.33)/0.66]$$

$$\times (10)(\tan 45°)/0.33 - (1.9 - 1)(9.8)(0.66)$$

$$= 103 + 231 - 6 = 328 \text{ kPa}$$

4.14 NUMERICAL EVALUATION OF BEARING CAPACITY

The classical methods of determining bearing capacity have proven very useful. However, under some circumstances it is necessary to resort to alternative procedures for analysis. This may be true when a knowledge of the foundation deformations is required, or when classical limit solutions do not exist for the problem at hand. At the present time, sufficient experience exists with the constitutive modeling of soils that accurate predictions of foundation performance may be obtained for a