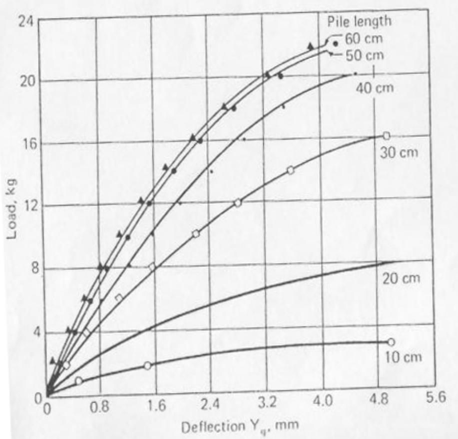
The increase in k with depth in noncohesive soils and normally loaded clays and silts is due to two reasons: These soils frequently exhibit an increase in strength with depth as a result of overburden pressures and natural deposition and consolidation processes; and pile deflections decrease with depth for any given loading, and the corresponding equivalent elastic moduli of soil reaction tend to increase with decreasing deflection.

Prakash and Agarwal (1967) reported test data on model piles subjected to horizontal load at 5 cm above the sand surface. The pile lengths were 10, 20, 30, 40, 50, and 60 cm. The deflections of piles at ground level, are shown in Fig. 7.10. It is seen that as the pile length increases, the ground deflection of a pile of constant cross section decreases at the same load. This decrease in deflection occurs first at a very rapid rate. Subsequently, this rate decreases, and, beyond a characteristic length of the pile, the deflections are not materially affected. This length L_α is defined as the *infinite length of the pile*. This length would obviously be a function of pile stiffness EI and soil stiffness k. The greater the pile stiffness, the greater L_α, and the greater the soil stiffness, the smaller L_α.

The concept of an "infinitely long pile" can also be derived from consideration of Fig. 7.7*b*. Because soil reactions are introduced only upon deflection of the pile, it is obvious that, for a given load and pile section, there is a characteristic pile length beyond which the deflections of the pile are negligible. This length corresponds to L_α.

It is easy to imagine that for the deflection of the pile y_x under a given load and at any depth, x is composed of the rigid-body movement and that which is due to curvature of the pile. The latter decreases as the pile stiffness EI increases, while other things remain the same. If the pile becomes rigid enough so that its deflections, due to curvature, may be neglected as compared with its rigid-body movements, it is called a *rigid pile* or a *pole*. Analytical expressions



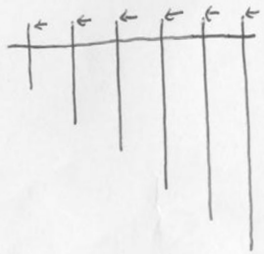


Figure 7.10 Lateral load vs. deflection with different pile embedment. *(After Prakash and Agarwal, 1967.)*

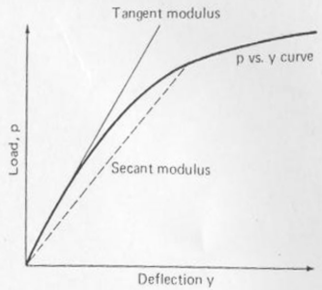


Figure 7.11 Typical soil reaction deflection curve.

Soil Dynamics
by S. Prakash
McGraw-Hill Book Company
(1981)

will be developed subsequently in the chapter for classifying piles as infinitely long or rigid.

Stress, strain, and load-deformation relations in soils are invariably nonlinear (Fig. 7.11). For loads of less than one-third to one-half their ultimate values, the load-deflection curve can be adequately expressed by a tangent modulus. For larger loads, a secant modulus is more appropriate. The secant modulus is dependent upon load level. Therefore, application of the theory of linear elasticity to the solution of pile problems is not strictly justified; however, to account for the nonlinearity between the load and pile deflection (or soil resistance and pile deflection), two approaches may be adopted:

1. One may employ repeated application of the elastic theory. Soil resistance moduli are adjusted upon completion of each trial run until satisfactory compatibility is obtained between the predicted soil behavior and the load-deflection relationships required by an elastic pile (Matlock and Reese, 1962).
2. The relationship between the secant modulus and the tangent modulus can be defined in terms of strain level, as was the case of the relationship between shear moduli at different strains in Chap. 4. However, the strain levels in soils along the length of the pile have not been defined, and a considerable amount of research is needed to solve this problem. Guidelines on n_h or k_x values to be adopted for a practice problem are discussed in a subsequent section. In any case, the final computed values of deflections and bending moments are not very sensitive to changes in soil modulus values.

7.7 SOLUTION FOR PILES IN NONCOHESIVE SOILS

A typical foundation pile of length L_s and flexural stiffness EI is shown in Fig. 7.12. The depth x is measured downward from the ground line. The boundary conditions at the top consist of an imposed moment M_g and a shear Q_g, and each is shown acting in a positive sense. The soil modulus variation for which

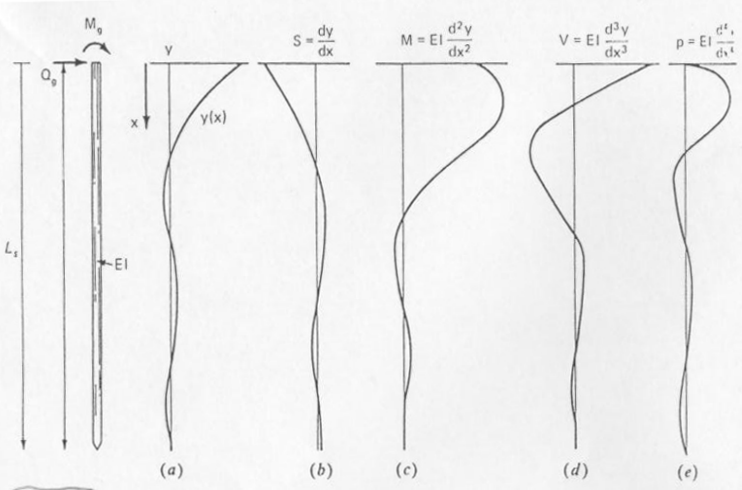


Figure 7.12 A pile of length L_s fully embedded in soil and acted upon by loads Q_g and M_g. (a) Deflection y. (b) Slope dy/dx. (c) Moment $EI\,d^2y/dx^2$. (d) Shear $EI\,d^3y/dx^3$. (e) Soil reaction $EI\,d^4y/dx^4$.

the solution is available is defined by

$$k_x = n_h x \tag{7.9a}$$

Figure 7.12a shows the $y(x)$ curve of the pile. Once this curve is known, its derivatives yield slope, moment, shear, and soil reaction, which are shown in Fig. 7.12b, c, d, and e, respectively. The factors on which the deflection of the pile $y(x)$ depends are:

Q_g, M_g, EI of pile, k_x of soil, L_s, x and L_a, the characteristic length of the infinitely long pile.

∴ $T = f(EI, k)$ k: soil stiffness, EI: pile structural stiffness

Instead of L_a, a quantity T, termed the *relative stiffness factor*, may be used. T has units of length and is related with L_a through the following relationship:

$$T = \frac{L_a}{\lambda} \tag{7.10}$$

in which λ is a positive integer. Now $y(x)$ may be expressed as

$$y(x) = Y(x, T, L_s, k, EI, Q_g, M_g) \tag{7.11}$$

If the deflections are small and if an elastic behavior is assumed, the principle of superposition may be applied. Therefore, the effects of an imposed lateral load Q_g and imposed moment M_g may be considered separately, as shown in Fig 7.13a

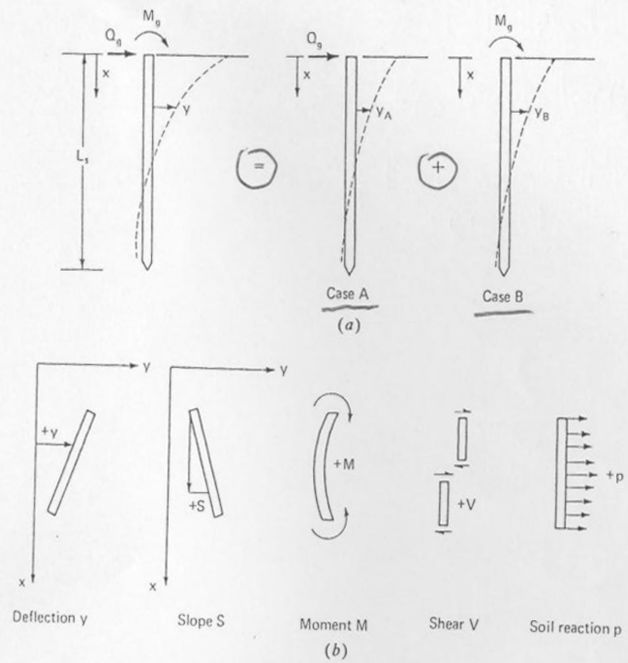


Figure 7.13 (a) Application of the principle of superposition to the laterally-loaded-pile problem. (b) Sign convention. *(After Matlock and Reese, 1962.)*

If y_A represents the deflection caused by the lateral load Q_g and y_B the deflection caused by M_g, the total deflection is

$$y = y_A + y_B \tag{7.12}$$

Furthermore, it is the ratios y_A/Q_g and y_B/M_g that are important in the elastic solution. Thus the solutions for Cases A and B may be expressed as

$$\frac{y_A}{Q_g} = f_A(x, T, L_s, k, EI) \tag{7.13a}$$

and

$$\frac{y_B}{M_g} = f_B(x, T, L_s, k, EI) \tag{7.13b}$$

in which f_A and f_B are two different functions of the same terms. In each case, there are six terms and two dimensions (force and length) involved. There are arrangements chosen are (Matlock and Reese, 1962):

Case A:

$$\frac{y_A EI}{Q_g T^3},\ \frac{x}{T},\ \frac{L_s}{T},\ \frac{kT^4}{EI} \tag{7.14a}$$

Case B:

$$\frac{y_B EI}{M_g T^2},\ \frac{x}{T},\ \frac{L_s}{T},\ \frac{kT^4}{EI} \tag{7.14b}$$

Each of the nondimensional terms formulated above can be assigned the following names and symbols:

$$\frac{x}{T} = Z \quad \text{(depth coefficient)} \tag{7.15}$$

$$\frac{L_s}{T} = Z_{\max} \quad \text{(maximum depth coefficient)} \tag{7.16}$$

$$\frac{kT^4}{EI} = \phi(z) \quad \text{(soil modulus function)} \tag{7.17}$$

$$\frac{y_A EI}{Q_g T^3} = A_y \quad \text{(deflection coefficient for Case A)} \tag{7.18}$$

$$\frac{y_B EI}{M_g T^2} = B_y \quad \text{(deflection coefficient for Case B)} \tag{7.19}$$

Thus, from Eqs. (7.18) and (7.19),

$$y_x = y_A + y_B = A_y \frac{Q_g T^3}{EI} + B_y \frac{M_g T^2}{EI} \tag{7.20}$$

Proceeding in a similar manner, the solutions for other quantities may be expressed as follows:

Slope:

$$S_x = S_A + S_B = A_s \frac{Q_g T^2}{EI} + B_s \frac{M_g T}{EI} \tag{7.21}$$

Moment:

$$M_x = M_A + M_B = A_m Q_g T + B_m M_g \tag{7.22}$$

Shear:

$$V_x = V_A + V_B = A_V(Q_g) + B_V \frac{M_g}{T} \tag{7.23}$$

Soil reaction:

$$p_x = p_A + p_B = A_p \frac{Q_g}{T} + B_p \frac{M_g}{T^2} \tag{7.24}$$

From the theory of a beam on an elastic foundation, the equation of the deflected beam is

$$EI \frac{d^4 y}{dx^4} = p \tag{7.25}$$

From Winkler's hypothesis, $p = -ky$. Soil reaction is always in a direction opposite to deflection; hence the negative sign is used. The basic equation for a beam on an elastic foundation, or for a laterally loaded pile, may then be written as

$$\frac{d^4 y}{dx^4} + \frac{k}{EI} y = 0 \tag{7.26}$$

Because the applied lateral load Q_g and an applied moment M_g have been considered separately, according to the principle of superposition, Eq. (7.26) becomes

Case A:

$$\frac{d^4 y_A}{dx^4} + \frac{k}{EI} y_A = 0 \tag{7.27a}$$

Case B:

$$\frac{d^4 y_B}{dx^4} + \frac{k}{EI} y_B = 0 \tag{7.27b}$$

Substituting for y_A, $\frac{k}{EI}$, and $\frac{x}{T}$ in nondimensional forms from Eqs. (7.18), (7.17), and (7.15), respectively, in Eq. (7.27*a* and *b*), we obtain

Case A:

$$\frac{d^4 A_y}{dz^4} + \phi(z) A_y = 0 \tag{7.28}$$

Case B:

$$\frac{d^4 B_y}{dz^4} + \phi(z) B_y = 0 \tag{7.29}$$

To obtain a particular set of nondimensional A and B coefficients, it is necessary to specify $\phi(z)$ and to define T.

For sands and other soils whose soil modulus may be assumed to increase linearly with depth, $\phi(z)$ may be equated to Z. Hence, in Eq. (7.17), by substituting for k from Eq. (7.9), we get

$$\frac{n_h x T^4}{EI} = \frac{x}{T} \tag{7.30}$$

Eq.(7.15)

or

$$T = \sqrt[5]{\frac{EI}{n_h}} \tag{7.31}$$

Reese and Matlock (1956), who defined $\phi(z)$ and T, as above, obtained the solution of Eqs. (7.28) and (7.29) by using the finite-difference method for the coefficients A and B. Deflection (y), slope (s), moment (M), shear (V), and soil reaction (p) coefficients for Q_g and M_g are shown in Tables 7.3 and 7.4 respectively (Matlock and Reese, 1961).

Based on the boundary conditions Q_g and M_g and the resulting A and B coefficients, relations can be derived so that problems involving other boundary conditions (such as a partially or completely restrained top) may be solved (Reese and Matlock, 1956; Matlock and Reese, 1961).

Matlock and Reese (1962) showed that by considering the soil modulus variation of the form given in Eq. (7.8) for $n = \frac{1}{2}$, 1, and 2, the difference caused by the applied shear and moment at the ground surface in the deflection and moment in a long pile is not appreciable. It was further shown that one can make good predictions of the moment curves by using $k_x = n_h x$, even though variations may be quite nonlinear with respect to depth. However, the effect of the soil modulus variation close to the ground surface on the computed moments is very large.

In Figs. 7.14 and 7.15, A_y and B_y coefficients are plotted for values of Z_{max} 2, 3, 4, 5, and 10. It can be seen that the coefficients A_y and B_y vary almost in a linear fashion with the depth coefficient z for $Z_{max} = 2$. Because the deflections

Table 7.3 Coefficients A for long piles under lateral loads*

Z	A_y	A_S	A_M	A_V	A_p
0.0	2.435	−1.623	0.000	1.000	0.000
0.1	2.273	−1.618	0.100	0.989	−0.227
0.2	2.112	−1.603	0.198	0.956	−0.422
0.3	1.952	−1.578	0.291	0.906	−0.586
0.4	1.796	−1.545	0.379	0.840	−0.718
0.5	1.644	−1.503	0.459	0.764	−0.822
0.6	1.496	−1.454	0.532	0.677	−0.897
0.7	1.353	−1.397	0.595	0.585	−0.947
0.8	1.216	−1.335	0.649	0.489	−0.973
0.9	1.086	−1.268	0.693	0.392	−0.977
1.0	0.962	−1.197	0.727	0.295	−0.962
1.2	0.738	−1.047	0.767	0.109	−0.885
1.4	0.544	−0.893	0.772	−0.056	−0.761
1.6	0.381	−0.741	0.746	−0.193	−0.609
1.8	0.247	−0.596	0.696	−0.298	−0.445
2.0	0.142	−0.464	0.628	−0.371	−0.283
3.0	−0.075	−0.040	0.225	−0.349	0.226
4.0	−0.050	0.052	0.000	−0.106	0.201
5.0	−0.009	0.025	−0.033	0.013	0.046

Table 7.4 Coefficients B for long piles under lateral loads*

Z	B_y	B_S	B_M	B_V	B_p
0.0	1.623	−1.750	1.000	0.000	0.000
0.1	1.453	−1.650	1.000	−0.007	−0.145
0.2	1.293	−1.550	0.999	−0.028	−0.259
0.3	1.143	−1.450	0.994	−0.058	−0.343
0.4	1.003	−1.351	0.987	−0.095	−0.401
0.5	0.873	−1.253	0.976	−0.137	−0.436
0.6	0.752	−1.156	0.960	−0.181	−0.451
0.7	0.642	−1.061	0.939	−0.226	−0.449
0.8	0.540	−0.968	0.914	−0.270	−0.432
0.9	0.448	−0.878	0.885	−0.312	−0.403
1.0	0.364	−0.792	0.852	−0.350	−0.364
1.2	0.223	−0.629	0.775	−0.414	−0.268
1.4	0.112	−0.482	0.688	−0.456	−0.157
1.6	0.029	−0.354	0.594	−0.477	−0.047
1.8	−0.030	−0.245	0.498	−0.476	0.054
2.0	−0.070	−0.155	0.404	−0.456	0.140
3.0	−0.089	0.057	0.059	−0.213	0.268
4.0	−0.028	0.049	−0.042	0.017	0.112
5.0	0.000	0.011	−0.026	0.029	−0.002

*After Matlock and Reese (1961 and 1962).

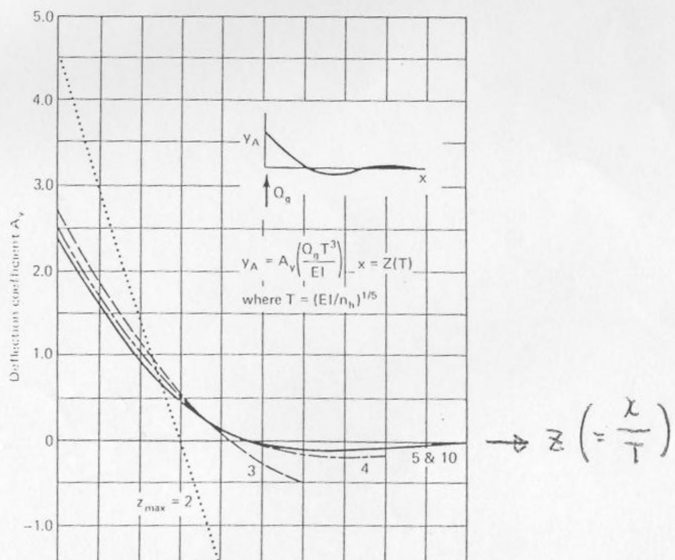


Figure 7.14 Deflection coefficien
A_y due to lateral load at grounc

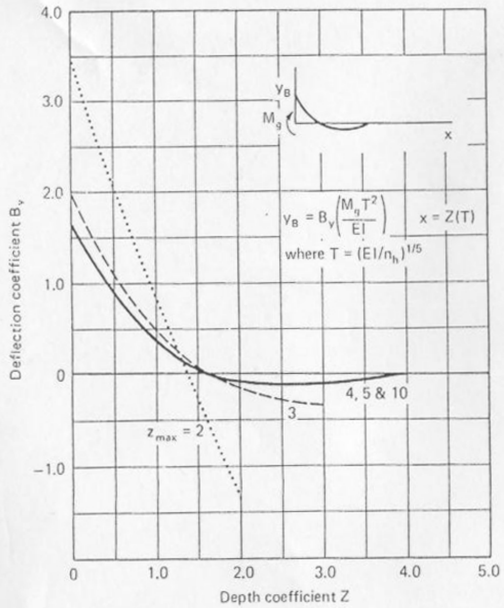


Figure 7.15 Deflection coefficient B_y due to moment at ground surface. *(After Reese and Matlock, 1956.)*

y_A and y_B are directly proportional to A_y and B_y for a given set of conditions, it is evident that both deflections y_A and y_B are linear with depth x. This shows that the pile undergoes only rigid-body deflections and that deflections caused by curvature are negligible. Hence, the piles with $Z_{max} \leq 2$ behave as rigid piles or poles.

Further, from the same figures, it can be seen that the deflection coefficients A_y and B_y for Z_{max} of 5 and 10 are identical. This means that pile length beyond $Z_{max} = 5$ is not effective in altering the deflections of the pile. Therefore,

$$\boxed{L_\alpha = 5T} \tag{7.32}$$

It will be seen that, in practice, most of the piles satisfy the condition $L_s > 5T$. Therefore, solutions for long piles are applicable. Coefficients A and B in Tables 7.3 and 7.4 can then be used. Also, the pile heads are usually fixed: that is, at $Z = 0$, $S = 0$. Therefore, Eq. (7.21) gives

↑ slope

$$S_n = S_A + S_B = A_S \frac{Q_g T^2}{EI} + B_S \frac{M_g T}{EI} = 0 \tag{7.33a}$$

which gives

$$\frac{M_g}{Q_g T} = -\left(\frac{A_S}{B_S}\right)_{(Z=0)} \tag{7.33b}$$

By substituting the values of A_S and B_S at $z = 0$ from Tables 7.3 and 7.4, we get

$$\boxed{\frac{M_g}{Q_g T}} = -0.93 \tag{7.33c}$$

Quantity $M_g/Q_g T$ has been defined as the *nondimensional fixity factor* by Prakash (1962). The expression for deflection [Eq. (7.20)] is modified for this case as

$$y = (A_y - 0.93B_y)\frac{Q_g T^3}{EI}$$
$$= C_y \frac{Q_g T^3}{EI} \tag{7.34a}$$

in which

$$\boxed{C_y = A_y - 0.93B_y} \tag{7.34b}$$

In a similar manner, the C coefficients for moment and soil reaction may be defined as

$$C_m = A_m - 0.93B_m \tag{7.35a}$$

which gives

$$M = C_m Q_g T \tag{7.35b}$$

and

$$C_p = A_p - 0.93B_p \tag{7.36a}$$

which gives

$$p = C_p \frac{Q}{T} \tag{7.36b}$$

The piles may undergo some rotation at the joints where their heads meet the caps. This results in partial fixity. In this case, the nondimensional fixity factor (NDFF) may be defined suitably, and the coefficients 'C' would also be modified accordingly.

Design Procedure

Based upon the discussion presented above, the following design procedure is recommended:

1. Determine the loads on the top of the pile.
2. Determine the soil profile and estimate a proper value of k or n_h for the type of soil.

3. Select a trial section with known EI and its width.
4. Compute

$$T = \sqrt[5]{\frac{EI}{n_h}} \quad \text{and} \quad Z_{max} = \frac{L_s}{T}$$

The term Z_{max} in practice is always greater than 5. Consideration of the vertical-load-carrying capacity determines the length.

5. Estimate the fixity λ of the pile head.
6. Compute the deflections from

$$y = C_y \frac{Q_g T^3}{EI} \tag{7.34a}$$

in which

Shear moment

$$C_y = A_y - (\lambda \times 0.93 B_y)$$

The maximum deflection occurs at the top of the pile and should not be greater than the permissible value. In the absence of a specified permissible value, $\frac{1}{2}$ in (1.25 cm) may be adopted as a reasonable permissible value.

7. Determine the bending moment along the length of the pile by using

$$M_x = C_m Q T \tag{7.35b}$$

in which

$$C_m = A_m - 0.93\lambda B_m$$

The following tabulation format is convenient for recording these computations.

Table for computation of M_x

Z	x	A_m	$0.93\lambda B_m$	C_m	M_x	Remarks
1	2	3	4	5	6	7

Plot the bending moment along the depth of the pile and determine M_{max}. Check the stresses in the section and compare them with the allowable stresses in the material of the pile.

8. Determine the soil reaction along the length of the pile by using the following equation:

$$P_x = C_p \frac{Q}{T} \tag{7.36b}$$

in which

$$C_p = A_p - 0.93\lambda B_p$$

Table of computations for p_x

Z	x	A_p	$0.93\lambda B_p$	C_p	p_x	Remarks
1	2	3	4	5	6	7

Compare with

Plot the soil reaction diagram along the depth of the pile. The permissible soil reaction at any depth x is given by

$$p_a = \frac{1 + \sin\phi}{1 - \sin\phi} \gamma \times b \tag{7.37}$$

in which γ = unit weight of soil and b = width of pile.

9. If the deflection, stress in the pile, and soil reactions indicate that the section adopted is safe and is not overly conservative, the selection of the section is permissible. Otherwise, select a new section and repeat steps 3 through 8.

you do it yourself

Example 7.2 A pile in sand, 10 m long with head free to deflect and rotate is acted upon by a lateral load of 3000 kg. The EI of the section is 3.5×10^{10} kg·cm². The pile undergoes a deflection of 12 mm. If the piles in the group were restrained against rotation to the extent of 50 percent of the fully restrained piles, determine the maximum bending moment and the soil reaction on the piles. Also plot the deflected shape of the pile. Assume that the width of the pile is 30 cm and that the unit weight of soil is 1.8 g/cm³. Neglect group action.

SOLUTION From the test data,

$$y_g = \frac{A_y Q_g T^3}{EI}$$

Therefore,

$$T = \sqrt[3]{\frac{EIy_g}{A_y Q_g}} = \sqrt[3]{\frac{(1.2)(3.5 \times 10^{10})}{2.435 \times 3000}} = 179.15 \text{ cm}$$

Because the pile head is restrained only 50 percent of the full restraint, the negative moment at the top of the pile is† $-0.465 Q_g T$

therefore

$$M_g = -0.465 \times 3000 \times 179.15 \text{ kg·cm}$$

Now,

$$y = A_y \frac{Q_g T^3}{EI} + B_y \frac{M_g T^2}{EI} = (A_y - 0.465 B_y) \frac{Q_g T^3}{EI}$$

Table 7.5 Computation of deflection y, bending moment M, and soil reaction p in Example 7.2

x, m	$Z = \frac{x}{T}$	A_y	$0.465B_y$	C_y	y, cm	A_m	$0.465B_m$	C_m	M, t·cm,	A_p	$0.465B_p$	C_p	p, kg/cm,
1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.0	2.435	0.755	1.680	0.828	0	− 0.465	− 0.465	− 250	0.000	0	0	0
0.36	0.2	2.112	0.601	1.511	0.745	0.198	0.465	− 0.267	− 1,43	− 0.422	− 0.120	− 0.302	− 5.06
0.72	0.4	1.796	0.466	1.330	0.655	0.379	0.459	− 0.080	− 43	− 0.718	− 0.186	− 0.532	− 8.91
1.07	0.6	1.496	0.350	1.146	0.565	0.532	0.446	0.086	46	− 0.897	− 0.210	− 0.687	− 11.50
1.43	0.8	1.216	0.251	0.965	0.476	0.649	0.425	0.244	1,20	− 0.973	− 0.201	− 0.772	− 12.93
1.79	1.0	0.962	0.169	0.793	0.391	0.727	0.396	0.331	1,77	− 0.962	− 0.169	− 0.793	− 13.28
2.15	1.2	0.738	0.104	0.634	0.312	0.767	0.360	0.407	2,19	− 0.885	− 0.125	− 0.760	− 12.73
2.87	1.6	0.381	0.013	0.368	0.181	0.746	0.276	0.470	2,55	− 0.609	− 0.022	− 0.587	− 9.83
3.58	2.0	0.142	− 0.032	0.175	0.086	0.628	0.188	0.440	2,36	− 0.283	0.065	− 0.318	− 5.83
5.37	3.0	− 0.075	− 0.041	− 0.034	− 0.017	0.225	0.027	0.198	1,06	0.226	0.125	0.101	1.69
7.16	4.0	− 0.050	− 0.013	− 0.037	− 0.018	0.000	− 0.019	0.020	10	0.201	0.052	0.149	2.50
8.96	5.0	− 0.009	0.000	− 0.009	− 0.004	− 0.033	− 0.012	− 0.021	− 11	0.046	− 0.001	0.047	0.79

Similarly,

$$M = (A_m - 0.465B_m)Q_g T$$

and

$$p = (A_p - 0.465B_p)\frac{Q_g}{T}$$

The values of y, m, and p are computed in tabular form as shown in Table 7.5 and have been plotted in Fig. 7.16b.

The maximum M = 255 t·cm and p_{max} = 13.5 kg/cm.

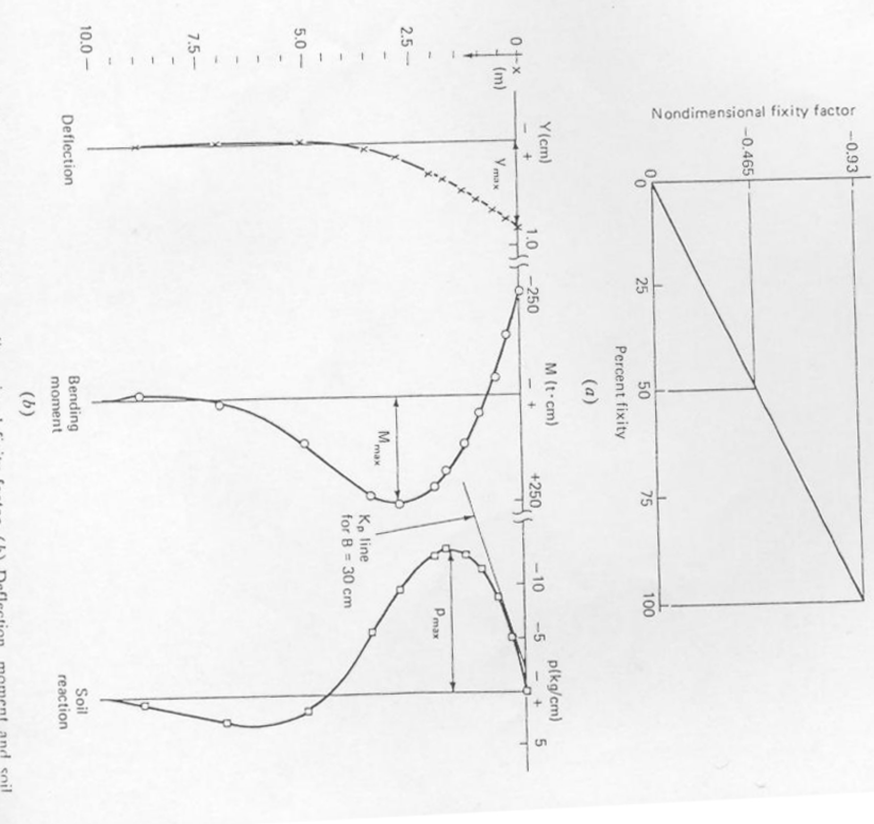


Figure 7.16 (a) Percent fixity vs nondimensional fixity factor (b) Deflection, moment and soil

k_p line for $\phi = 30°$ and $\gamma = 1.8\ \text{g/cm}^3$ [Eq. (7.37)] has been plotted on the soil reaction diagram.

7.8 PILES IN CLAY

Solutions similar to the ones presented in Sec. 7.7 have been obtained by Davisson (1962) and by Davisson and Gill (1963) for clays that occur in two layers. The stiffnesses of the top and bottom layers are defined by k_1 and k_2 respectively. For a particular case of $k_1 = k_2$, by using R in place of T for clay, letting $\phi(z) = 1$, and with A_{yc} in place of A_y in Eq. (7.28), we obtain the following equation:

$$\frac{d^4 A_{yc}}{dz^4} + \phi(z) A_{yc} = 0 \tag{7.38}$$

$$\phi(z) = \frac{kR^4}{EI} = 1 \tag{7.17}$$

Therefore,

$$R = \sqrt[4]{\frac{EI}{k \leftarrow n_h}} \quad \text{(cf. 7.31)} \tag{7.39}$$

The depth coefficient is

$$z = \frac{X}{R} \tag{7.40a}$$

and the maximum depth coefficient is

$$Z_{max} = \frac{L_s}{R} \tag{7.40b}$$

If both shear and moment act on a pile head, and if one uses B_{yc} in Eq. (7.20), we obtain

$$y_x = A_{yc}\frac{Q_g R^3}{EI} + B_{yc}\frac{M_g R^2}{EI} \tag{7.41a}$$

Similarly, Eq. (7.22) becomes

$$M_x = A_{mc} Q_g R + B_{mc} M_g \tag{7.41b}$$

in which

A_{yc} = deflection coefficient for shear load on a free head pile in clay
B_{yc} = deflection coefficient for moment load on a free head pile in clay
A_{mc} = moment coefficient for shear load on free head pile in clay
B_{mc} = moment coefficient for moment load on free head pile in clay

The soil reaction diagram may be obtained by multiplying the y diagram

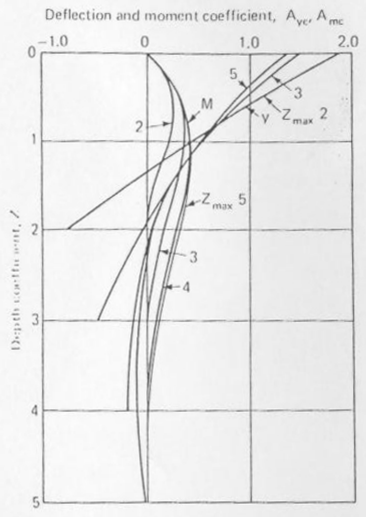


Figure 7.17 Deflection and moment coefficients A_{yc} and A_{mc} due to lateral loads at ground surface. *(After Davisson and Gill, 1963.)*

The solutions for the A and B coefficients may be obtained by using either the technique described by Matlock and Reese (1962) or an analogue computer (Davisson and Gill, 1963). In Figs. 7.17 and 7.18, A_{yc}, A_{mc}, B_{yc}, and B_{mc} have been plotted for piles of different Z_{max}.

It may be seen from the deflection coefficients in Figs. 7.17 and 7.18 for $Z_{max} = 2$, that the plot is almost a straight line. Hence, the pile may be

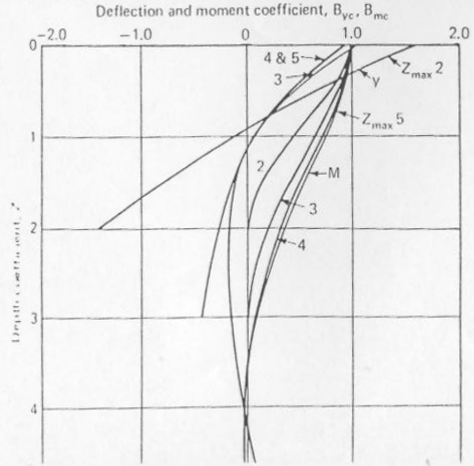


Figure 7.18 Deflection and moment coefficients B_{yc} and B_{mc} due

considered to be a rigid pile if $\boxed{Z_{max} \le 2}$. Similarly, for $\boxed{Z_{max} \ge 4}$, the pile may be considered infinitely long pile. In practice, most piles in clays satisfy this condition.

7.9 DYNAMIC ANALYSIS

In current design practice, the total lateral load applied to a pile foundation is equal to the base shear computed in the dynamic analysis of the superstructure, which is considered to be fixed at the level of the foundation. Hayashi (1973), Prakash and Sharma (1969), and Prakash and Gupta (1970) attempted to determine the natural frequencies of the soil pile system by using an equivalent cantilever method. The soil-pile system is idealized as a massless equivalent cantilever with a single concentrated mass at the top. Its natural frequency is determined by using Rayleigh's method. The exciting frequency is used to check the frequency of the system for resonance. This approach is more or less arbitrary.

Generally, there are three techniques that can be used to solve problems of soil-pile superstructure interaction (Novak, 1977). The first represents soil as a continuum with linear elastic properties. It correctly represents geometric damping as well as soil layer resonance (Novak and Nogami, 1977; Novak, 1977). In the second, the finite element technique is used to represent the pile and the soil. This method offers a maximum flexibility for the variation of soil-pile properties (Novak, 1977; Kuhlemeyer, 1979). The third represents the soil-pile system by a set of discrete (lumped) masses, springs, and dashpots. This approach can be used to incorporate the depth and nonlinearity variations of the soil properties in more detail. These variations depend upon the definition of the local soil stiffness and geometric damping (Penzien et al., 1964; Penzien, 1970; Prakash and Chandrasekaran, 1973, 1977).

A reasonably practical solution for soil-pile interaction under dynamic loads has been proposed by Chandrasekaran (1974; Prakash and Chandrasekaran 1980). This analysis is based on the following assumptions:

1. The pile is divided into a convenient number of segments and mass of each segment is concentrated at its center point (Fig. 7.19).
2. The soil is assumed to act as a linear Winkler's spring. The soil reaction is separated into discrete parts at the center points of the masses in Fig. 7.19. The soil modulus variation is considered both constant with depth and linearly varying with depth (Fig. 7.20).
3. The mass of the superstructure is concentrated at the pile top as M_t.
4. The system is one-dimensional in its behavior.
5. The pile end conditions are either completely free to undergo translation and rotation or completely restrained against rotation but free to undergo translation.

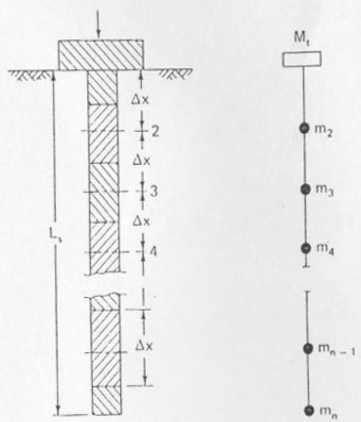


Figure 7.19 Pile-structure idealization. *(After Chandrasekaran, 1974.)*

For evaluating the free-vibration characteristics, the modal analysis is performed by using successive approximations of the natural frequencies of the system with an initially assumed value and related end conditions. The adopted end conditions are also utilized to generate the transfer equations and to evaluate the unknown quantities, either at the pile top or the pile bottom, in terms of the known quantities. These modal quantity values at different station points define the mode shapes. Values at the bottom or top of the piles assist in determining the natural frequencies of vibrations in different modes.

The forces and displacements in two different station points are illustrated in Fig. 7.21 (Prakash and Chandrasekaran, 1977).

The details of the idealization and the method of analysis and detailed parametric studies are presented elsewhere (Chandrasekaran, 1974).

Information has been obtained with these approaches for piles embedded in soils in which the soil modulus can be considered either to remain constant or to vary linearly with depth. In both of these cases, solutions have been obtained for natural frequency, modal displacements, slopes, bending moments, shear forces, and soil reactions along the lengths of the piles in the first three modes of vibrations (Chandrasekaran, 1974; Prakash and Chandrasekaran, 1980). Only typical solutions for handling a practical problem shall be discussed herein.

Natural Frequencies

Based on the parametric study, nondimensional frequency factors have been obtained with respect to the basic soil parameters.

The variables constituting F_{CL_1}, the *nondimensional frequency factor* for piles embedded in soils in which the soil modulus remains constant with depth, is