Proposition. The image of a convex set under the perspective function is convex.
Proof. Let C be a convex set, and f be a perspective function.
We want to show that $f(C)$ is convex, that is, for any $\lambda \in [0,1]$ and for any $(x,s),(y,t) \in C$,

$$\lambda\frac{x}{s} + (1-\lambda)\frac{y}{t} \in f(C),$$

where $x, y \in \mathbb{R}^n$ and $s, t \in \mathbb{R}_{++}$.

$$\lambda\frac{x}{s} + (1-\lambda)\frac{y}{t} = \frac{\lambda tx + (1-\lambda)sy}{st} = \frac{\frac{\lambda tx+(1-\lambda)sy}{\lambda t+(1-\lambda)s}}{\frac{st}{\lambda t+(1-\lambda)s}}$$

$$= \frac{\frac{\lambda t}{\lambda t+(1-\lambda)s}x + \frac{(1-\lambda)s}{\lambda t+(1-\lambda)s}y}{\frac{\lambda t}{\lambda t+(1-\lambda)s}s + \frac{(1-\lambda)s}{\lambda t+(1-\lambda)s}t}$$

Since C is convex, both of the enumerator and the denominator also belong to C, and thus, $\lambda\frac{x}{s} + (1-\lambda)\frac{y}{t} \in f(C)$, which means that $f(C)$ is convex.